Contents 1982

February 20, 22: Circle case: $\mathscr{G} = \text{Maps}(S^1, U_n)$ and $\mathscr{A}$ the unitary connections.

February 23: Meaning of an analytic proof of an index theorem for families.

February 24: Comment on problems encountered with current approach.

February 25: Preparation for lecture 5: ζ-function determinants or analytic torsion.

February 26: More on the problem of finding an analytic formula for the index of a family of operators.

February 27: Formal structure of proof of the index theorem.

February 28: Idea from Bott-Chern paper.

March 1: Bott-Chern formulas in the holomorphic setting.

March 2: Curvature calculations for the determinant line bundle belonging to a family of $\overline{\partial}$ operators over a Riemann surface.

March 3: Generalizing $\text{Tr}(e^{-tD^*D}) = \text{Tr}(e^{-tDD^*})$. Review Schwinger calculation using Witten formulas.

March 6: Preparation for 7th lecture: Computation of the curvature for $\mathscr{L}$ using the analytic torsion metric.

March 5: Bott-Chern theory applied to investigating:

$$[ch(E) - ch(F)] - [ch(Ker(D)) - ch(Coker(D)] = d(???).$$

March 6: Eigenvalue calculations for variations of D^*D.

March 6, 9: Discussion of the problem of finding an analytic proof of an index therem for a family of Dirac operators. Review of spinors, Clifford algebras and their K-theory, Dyer-Kan classification therem for diagrams of simplicial sets.

1983-3

March 10, 11: Calculations for a family of Dirac operators on $\mathbb{R}^p/\mathbb{Z}^p \times \mathbb{R}^q/\mathbb{Z}^q$.

March 11: Notes for 7th lecture.

March 12: Clifford algebras and Dirac operators.

March 13: Dirac operator on $(S^1)^{n-1} \times S^1$, $n = 2m$.

March 14, 15: Kasparov cup product. Further work on familes of Dirac operators.

March 16: Lagrangian viewpoint.

March 17: Path integral approach. Heat kernel over a product of two tori. Path integral formula for the Dirac operator over Euclidean space.

March 18, 20, 23: Calculating diagonal values for the kernel $e^{-t\Box}$.

March 24: Family of Dirac operators and the Connes algebra assigned to a foliation.

March 27: Calculation of the index of the standard harmonic oscillator $d + d^*$ on $\mathbb{R}^n$. Comments on the Patodi approach and the Seeley approach to the asymptotic expansion of $e^{-y\Box}$.

March 28, 29: Physics approach to calculating terms in the heat kernel.

March 30: Further calculations for the heat kernel.

March 31: Review $\zeta(-k)$, $k = 0, 1, 2 \ldots$ and the Adams operations in K-theory.

April 1: Arekelov-Faltings intersection theory on arithmetic surfaces. Related questions and ideas.

April 3: Return to holomorphic vector bundles over Riemann surfaces.

April 4: Calculating constant term in $\mathrm{Tr}(e^{tD^*D}D^{-1}B)$ as $t \to 0$.

1983-4

April8, 9: The fermion C^* algebra.

April 10, 11: A C^* algebra and its K-theory, particularly the Kronecker foliation algebra.

April 12: Cross products and factors in C^* algebras. Atiyah's L^2 index theorem.

April 13: Asymptotic behavious of $\mathrm{Tr}(e^{-tD^*D}D^{-1}\delta D)$ as $t \downarrow 0$ where $D = \overline{\partial}$ on a Riemann surface.

April 14: The determinant line bundle over a space of constant coefficient $\overline{\partial}$-operators on a trivial line bundles over a 2-dimensional torus.

April 16, 17, 18: Review the Ray-Singer calculation of torsion on elliptic curves.

April 19: The GRR formula for a family of constant coefficient $\overline{\partial}$-operators over a torus.

April 20, 22: Connes K-theory of a foliation, especially the Kronecker foliation.

April 23: Identifying $\mathscr{S}(\mathbb{R})$ equipped with the operators translation by 1 and multiplication by $e^{2\pi ix}$ with sections of the trivial line bundle over $\mathbb{T}^2$.

April 24: Calculating **Pic** of the orbit topos of the Kronecker foliation. Review of θ-functions.

April 25, 26: A construction using von Neumann algebras of type II which shows that a flat unitary bundle gives zero in $K(X) \otimes \mathrm{R}$.

April 27, 28: Principal bundles with discrete structure group.

April 29, 30: Equivalence of holomorphic structures and metrics on a compact oriented surface of fixed volume.

May 1: Conversation with George wilson about the Yang-Baxter identity.

May 7: Preparation for talk on Arakelov-Faltings theory and zeta determinants.

May 8: Computing analytic torsion for line bundles. Results from Falting's paper.

May 9: Talk on Arakelov-Faltings theory.

1983-5

May 10, 11: Discussion of the possibility of a K-theory of holomorphic vector bundles.

May 12: K-theory of non-unital rings.

May 18: Preparation for K-theory conference in Luminy.

May 19: Return to discussing families of holomorphic curves, seeking inspiration from the work of Connes work and Feigin- Tsygan.

May 20, 21, 22, 23: Calculations inspired by the Connes results on the Kronecker foliation.

May 25: Notes on cyclic homology. Mixed Hodge structure for a non-singular variety which is not complete. Removing points and discs from a Riemann surface.

May 26: The KMS condition.

may 27: Cyclic homology calculation- forward shift.

May 29: Why $\prod_{n=1}^{\infty}(1-q^n)$ is a modular function.

May 30: Cyclic cohomology and cyclic graphs.

June 1: On cyclic homology for rings without unit.

June 2: Relative cyclic homology of A modulo k.

June 3: Relating the Quillen approach to cyclic homology from Hochschild homology to the Connes approach through the non-commutative de Rham complex. Connes definition as a functor on cyclically ordered finite sets.

June 4: The cyclic category.

June 5: Hsiang and Staffeldt result that $HC_p(T(V), k) = 0$ if $p \neq 1$. Comparison with de Rham homology.

June 6: Cyclic and de Rham homology.

June 9: Preparation for a paper on cyclic homology giving an exposition of some aspects using the double complex.

1983-6

June 30- July 4: Review of progress on the index theorem for families.

July 5: Summary of analytic progress and chang of direction to a geometrical attack. Introduction of form $\mathrm{Tr}(e^{-tL^2+\sqrt{t}dL+\Omega})$.

July 6: Summary of progress on ideas on the cohomology of gauge groups.

July 7: Invariant forms on G give natural transformations from flat connections on $Y \times G/Y$ to $\Omega^*(Y)$.

July 8, 9: Review of Bott-Chern formulas and applications to flat bundles on the trivial principal $\mathscr{G}$-bundle over Y.

July 10: Calculation of characteristic classes using Maurer-Cartan form.

July 14: Using the transgression formula $\int_0^1 dt(e^{td''w+(t^2-t)w^2})$.

July 15: Preparation for letter to Loday on the natural transformation from the Connes complex $\mathscr{C}(A)$ to the filtered de Rham compex based on using $\int_0^1 dt(e^{td''w+(t^2-t)w^2})$.

July 17, 18: Connes periodicity operator. Connes index: $\mathrm{Index}(epe) = \mathrm{tr}((p^{-1}[p,e])^{2m+1})$.

July 19: Loday's conjecture on the filtration of cyclic homology obtained from $\mathfrak{gl}_n$ for different n.

July 20: Defining homology classes for $\tilde{\mathfrak{g}} = \mathfrak{gl}_n(A)$ with values in the filtered de Rham compex using the Chern-Weil curvature process.

July 21, 22: More work on Loday's conjecture. Letter to Loday.

July 23, 24: Fadeev-Popov Ansatz. List of ideas and problems. Formal category of a scheme and related de Rham complexes.

July 25: Review of Feynmann diagrams, effective potentials and vertex functions.

July 26: Review normalization (Lee model).

July 28, 29, 30, August 1: Characteristic classes for $H*(\mathscr{G})$. How to realize $\mathrm{ch}(E_{\mathrm{invar}}$ on $H^*(B\mathscr{G} \times M)$ by equivariant forms on $(\mathscr{G}\backslash\mathscr{A}$.

1983-7

August 7: Determining $H^*(B\mathscr{G})$ and realising primitive generators of $S(\mathfrak{g}^*)$ as differential forms. Contrast between compact group and gauge group cases.

August 8: Comment on continuous cohomology. Is $H_c^*(\mathscr{G}, \mathcal{M}) = H^*((\mathcal{M} \times \Omega^*(\mathscr{A})^{\mathscr{G}})$?

August 9: More work on $B(\mathscr{G})$ and $H^*(\mathscr{G})$.

August 10, 11, 12: The Lie algebra of the gauge group and Gelfand-Fuks cohomology. The map $H^*(\tilde{\mathscr{G}}) \to H^*(\mathscr{G})$. Rational cohomology of $B\mathscr{G}$ and $\mathscr{G}$.

August 13: Continuation of the program to determine the continuous and Lie algebra cohomology of gauge groups. Conjecture on primative generators of the cohomology of $\mathscr{G}$, $\tilde{\mathscr{G}}$, $B_c\mathscr{G}$, and $B\tilde{\mathscr{G}}$.

August 17: On the Ployakov formula for $\det(\not{\partial} + \not{A})$ on $\mathbb{R}^2$.

August 19: On normalization.

August 20: Feynmann's formula for $\frac{1}{ab} = \int_0^1 dt \frac{1}{[ta+(1-t)b]^2}$. Field theory of a real-valued function $\phi(x)$, $x \in \mathbb{R}^n$ given by action $S(\phi) = \int d^n x \{\frac{1}{2}\phi(-\Delta + m^2)\phi + \frac{\lambda}{4!}\phi^4\}$.

1983-8

August 21: Motion of a particle on the line governed by the Hamiltonian (anharmonic oscillator) $H = \frac{p^2}{2} - \frac{w_0^2}{2}x^2 + \frac{\lambda}{4!}x^4$.

August 29, 30: Magnetism.

September 1: Conversation with Jackiw on anomolies and σ-model approximation to low energy QCD.

September 9: BRS and Dixon's work.

September 10: Review problem left over from Loday letter. Review determinant line bundle.

September 15: Discussion of whether there is a direct connection between cyclic cohomology and anomolies.

September 17, 18: Discussion of Connes Λ-interpretation of cyclic cohomology and the discussion of compatibility of two maps from the Lie algebra homology to Deligne cohomology.

1983-9

September 20, 22: Construction of character forms associated to an invariant connection on an equivariant bundle in equivariant cohomology.

September 24-29: Determining $H^*(B\mathscr{G})$.

October 2: Lifting a $\mathfrak{gl}_n(A)$ cycle with values in the De Rham complex to one with values in the complex $\mathscr{B}(A)$. Amitsur compex.

October 4: Observations from a paper of Witten on baryons.

1983-10

October 9: What is $\mathrm{Ext}^*_\lambda(k^\natural, A^\natural)$? Return to problem of letter to Loday on constructing a cocycle for $\mathfrak{gl}_n(A)$ with values in the double compex $\mathscr{C}(A)$.

October 10: Chain complexes $\Omega^*(Y, P \times^G V)$ and $C^*(\mathfrak{g}, V)$, and connection with Lie algebra cohomology. Formulas for the boundary operators b and B in $\mathscr{C}(A)$.

October 11: Karoubi's non-commutative differential algebra of forms $\overline{\Omega}(A) = \Omega(A)/[\ ,\ ]$. Application of Connes theorem to the Loday problem.

October 13, 14: Chern characteristic classes of $\mathfrak{gl}_n(A)$ with values in non-commutative de Rham cohomology of A.

October 17, 18: Curvature of the Grassmannian connection form $\mathrm{ch}_n = \frac{1}{n!}\mathrm{tr}(e(de)^{2n})$. Index formula for F: Index$=(-1)^n\mathrm{tr}(\epsilon e[F,e]^{2n})$.

1983-11

October 20: Maps $K_0(A) \to HC_{2n}(A)$ and $K_1(A) \to HC_{2n-1}(A)$.

October 22, 23: Proving $\mathscr{B}(A)_{\text{red}}$ is quasi-isomorphic to $\overline{A}^{*(*+1)}/(1+t,b)$. Lie algebra cohomology of the gauge group amd how it is related to the index and determinant questions.

October 26, 27: Connes S-operator.

October 29, 30,: Trace for 1-summable Fredholm modlues.

October 31, November 1: Connes S-operator.

November 2: Connes-Karoubi theorem: $H^p(\overline{\Omega}) = \text{Im}\{S : \overline{HC}_{p+2}(A) \to \overline{HC}_p(A)\}$.

1983-12

November 10: Computing transgression $H^*(B\mathscr{G}) \to H^*(\mathscr{G})$. $(BG)^{S^1} \equiv PG \times^G (G_c)$ where G_c denotes the G space with G acting as conjugation.

November 11: Discussion of Singer's approach to calculating transgression.

November 12, 13: Calculating transgression using the Chern-Simons form.

November 14, 15: Transgression formula: $\text{tr}(e^{F_A}) - \text{tr}(e^{F_B}) = d\int_0^1 dt\ \text{tr}(A-B)e^{(1-t)F_B + tF_A - t(1-t)(A-B)^2}$.

November 16: Notes on an anomoly formula: $c_1(\overline{\mathscr{L}}) = \int_{M^{2n}} (\text{ch}(E)\hat{A}(M))_{n+1}$. Connections on the principal bundle.

November 17: Calculations on the principal bundle. Equivariant curvature in both D and A notations.

November 18: Constructing characterisitic classes for $\mathscr{G}$-bundles out of character classes for U-bundles.

November 19, 20: Formulas for gauge transformations on the principal bundle. Action of gauge transformations on connections: $g^{-1} \circ D \circ g$ on $\Omega(M,E) \leftrightarrow d + g^*A$ on $\Omega(P) \times V$. Transgression map: $W(\mathfrak{g})_{\text{basic}} \to \Lambda(\mathfrak{g})^G$.

November 21: Check transgression calculations.

1983-13

November 21: Letter to Singer on transgression formulas. Checking left and right actions of the gauge group.

November 26: Review of local index formulas. Some new ideas.

November 27: Review construction of characterisitic classes for the Lie algebra of gauge trasnformations as in letter to Loday.

November 28: Constructing invariant forms on $\mathscr{G}$.

November 29: On Atiyah's suggestion that Quillen's formula $e^{tL^2 + \sqrt{t}[D,L] + F} = e^{tL^2 + \sqrt{t}d^\mu [D_\mu, L] + \frac{1}{2} dx^\mu dx^\nu F_{\mu\nu}}$ and Getzler's proof should be part of the same framework.

November 30, December 1,2: Comment on Wess-Zumina Lagrangian as described by Witten. Preparation for letter on transgression.

December 3: Comment on Singer's intention on using his vol_B construction and how it works for flat connections.

December 4: Summary of formulas and explanation of apparent paradox.

1983-14

December 6: Talk in Jaffe's seminar. Notes of a conversation with Luis and Ginzpang about anomoly formulas.

December 8: Describing anomolies using cyclic cohomology.

December 11: Note that Witten-Alvarez obtain the $\hat{A}$-genus by means of a constant EM-filed. Fermion quantum mechanics.

December 13: Lot's problem.

Decmber 14: Fujikawa's approach to anomolies.

December 15: Return to Lot's problem. Witten's table and QCD.

December 16: Facts about QCD, Witten's observation about a physical interpretation of stable homotopy groups.

December 17: On the Connes S-operator.

December 18: The local index theorem using path integrals on Euclidean space with arbitrary gauge potential. Digression on spinor representations in 2n and almost complex manifolds.

December 19: The index of the Rarita-Schwinger operator.

December 21: Discussion of Clifford algebras.

December 22: Weyl quantization and its fermion version.

December 23: Developing a formal theory of path integral and fermion integration theory.

December 24: Quantizing the Toeplitz process. Fermionic analogues.

December 28: Review transgression process for constructing differential forms on $\mathscr{G}$.

December 29: Local index formula for a family of Dirac operators on M parametrized by a family of connections quotiented by a gauge group.

1983-15

December 30: Is the cohomology class of the form $\mathrm{tr}_E(\epsilon_E e^{L^2+[D,L]+D^2})$ independent of L?

December 31: Super-trace.

1983-Calculations relevant to cyclic theory

1983-Ideas

1983-Lecture Notes 0

Quillen: First lecture on local index theory

1983-Lecture Notes 1

Donaldson: Generalization of a theorem of Narasimhan-Seshadri

1983-Lecure Notes 2

Getzler: New proof close to Kotake.

1983-Lecture Notes 3

Kirwan: Theorems about convex bodies.

1983-Lecture Notes 4

Atiyah: Integrals over fixpoint submanifolds.

1983-Lecture Notes 5

Quillen: Review of local index theorem.

1983-Lectyre Notes 6

Ginsparg: Obtaining the Chern-Simons form.

1983-Lecture Notes 7

Witten: Equivariant index theorem.

1983-Lecture Notes 8

Witten: Some QCD inequalities.

1983-Lecture Notes 9:

Witten: Tables.

1983-Lecture Notes 10

Getzler: Heat kernel of $H = \frac{1}{2}[-\Delta + A^*A]$.

1983-Lecture Notes 11:

Kazhdan and Thomas Parker: On Hecke algebra $C_c^\infty(G)$. Super-symmetric σ-model.

1983-Lecture Notes 12

Atiyah: Newton Polyhedra and Algebraic Geometry.

1983-Lecture Notes 13

Cutz and Connes: Quasi-homomorphisms. Cyclic cohomology.

1983-Lecture Notes 14

Schroer: Super-symmetric theory

1983-Lecture Notes 15

Patterson: New results in ergodic theory.

1983-Notes

On Cyclic theory.

1983-Review

Of Getzler material.