Witten (in Bott's office 12/15/83)

color	F flavors of quarks a flavor by defn is	flavor symm gp
$SU(N)$	$N + \bar{N}$	$SU(F) \times SU(F)$
$O(N)$	N	$SU(F)$
$Sp(N)$	$(2N) \oplus (2N)$	$SU(2F)$ 2 copies to kill $\mathbb{Z}_2$ anomaly
$N \to \infty$		

$$\pi_4 Sp(N) = \mathbb{Z}_2 \quad \Rightarrow \quad \pi_1(\mathcal{A}/\mathcal{G}) \neq 0$$

unbroken symm gp. = flavor symm of generic vacuum	space of vac. states
$SU(F)_{diag}$	$SU(F) \times SU(F) / \text{diag } SU(F) \cong SU(F)$
$O(F)$	$SU(F)/O(F)$
$Sp(F)$	$SU(2F)/Sp(F)$

stably homotopy

π_2	π_3	π_4	π_5
0	$\mathbb{Z}$	0	$\mathbb{Z}$
$\mathbb{Z}_2$	$\mathbb{Z}_2$	0	$\mathbb{Z}$
0	0	0	$\mathbb{Z}$
confinement of certain color charges	existence of baryons	ambiguity in quantization in σ-model	number of colors in underlying theory is an integer

G , $\hat{G}$ = ~~univ.~~ cover

$$\frac{\text{Rep of } \hat{G}}{\text{Rep of } G} = \mathbb{Z}_2 \text{ for } O(N)$$

Why it only works for large F

Why

[illegible] t'Hooft (1974) conjecture — For $N \to \infty$ QCD becomes an effective theory of mesons with complicated config. space P

Witten believe that P has the homotopy type of Maps $(S^4, G/H)$ in the stable range (large F).

~~[illegible] What~~ Can we guess P in general

~~[illegible]~~

For fixed N P exists but there is no reason to ~~[illegible]~~ expect topology of P to be interesting physically.

$$\int_{\text{Maps}} e^{-\text{Action}} \cdot \alpha$$

Given $\alpha : \pi_4(G/H) \to U(1)$

In QCD ~~[illegible]~~ one integrates over all $SU(N)$-bundles and connections

$$\pi_3(\mathcal{G}) = \pi_4(B\mathcal{G}) = \mathbb{Z}$$

leads to Θ in QCD

two ways to quantize rigid rotator because
$\pi_1(O(3)) = \mathbb{Z}/2$ as either fermion + boson.

L + L molecule

π_5 now.

$\int_{\text{Maps}} e^{-A}$ A isn't real necessarily

physical requires | $A \longrightarrow A^*$ under reversal of orientation of S^4. | CPT thm.

so parity preserving parts of A are real
parity reversing " " A are imag.

Only e^{-A} is needed, so A may be defined only modulo $2\pi i$

monopole action $(2\pi i)(2eg) \Rightarrow 2eg \in \mathbb{Z}$

So $\pi_5(G/H) = \mathbb{Z}$ means we can

π_3 + baryons

should go to $M = \underset{\text{time}}{R^1} \times S^3$

Ask about classes of phys. states What could have existed at $t=0$?

The top. classification of initial conditions is $\pi_3(G/H)$.

(Answer $\mathbb{Z}$, $\mathbb{Z}_2$, 0) $\mathbb{Z}$ baryon no.

Meson $q^i \bar{q}_i$ } SU(N)
Baryon $\epsilon_{i_1 \cdots i_N} q^{i_1} \cdots q^{i_N}$

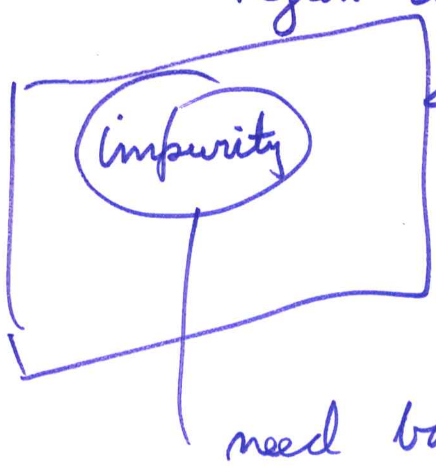
Meson $q^i q^i$ } O(N)
Baryon $\epsilon_{i_1 \cdots i_N} q^{i_1} \cdots q^{i_N}$

group theoretically

$$\epsilon_{i_1 \cdots i_N} \epsilon_{j_1 \cdots j_N} = \sum_{\text{perm}} \delta_{i_1 j_{\pi 1}} \cdots \delta$$

So 2 baryons = N mesons.

π_2 region where equations break down



$t=0$ to begin with we are in 3 dims.

need bdry conditions classified by $\pi_2(G/H)$.

If O(n) there is one impurity

Only interesting impurity should transform under a representation of the gauge gp. not already included in repns. generated by quarks + gluons. For O(n) take Spin repn.