Ideas: 1) cohomology of gauge group is formally given by the forms $\mathrm{Tr}\,(g^{-1}dg)^k$, but to make sense of the trace one must regularize the trace. This destroys invariance or closedness, so the Lie algebra cohomology is smaller than that of the group.

2) For a general gauge group Lie algebra cohom. = cohomology of the group in high dimensions.

3) Can one make the Lie algebra of vector fields act on the infinitesimal gauge transformations? Bring in Gelfand-Fuks. This is what happens with the Virasoro algebra + loop groups.

4) One can produce invariant forms on the gauge group $\mathcal{G}$ by mapping $\mathcal{G}$ to the Grassmannian $g \longmapsto g\mathcal{e}$ of the Hilbert space which $\mathcal{G}$ acts on, then pulling back natural forms on the Grassmannian. What are these forms for loop groups?

5) The subbundle S over the Grassmanian has a natural invariant connection. What is the moment map?

6) Explain the apparent similarity between equivariant cohomology of the (free) loop space ΛM for the natural S^1 action and the cyclic homology.

Analogy:

$$H_S(\Lambda M) \longleftrightarrow HC(M)$$

$$H(\Lambda M) \longleftrightarrow \mathrm{Hoch}(M) = \Omega_M$$

$$H(\mathrm{Fixpts}) \longleftrightarrow H^{\pm}(M)$$

Problem: Localization Thm. not valid for ΛM.

The preceding is related to Witten part of the Bott-Atiyah paper.

Formula for equivariant cohomology of a circle action: Take complex of invariant forms with differential $d + i(X)u$. <u>Better</u>: $\Omega(M)^S[u]$ with this diffl.

8) Now that Connes has given an interpretation of cyclic homology in terms of Λ, can you work in Bott's model for loops?

9) What is the cohomology of the complex of equivariant forms for $\mathcal{G}$ acting on $\mathcal{A}$?

10) Cohomology of loop groups: $\Omega G = \Lambda G / G$. Think of as similar to $G/B = K/T$. To get harmonic forms you need a Hodge theory (as Kostant does) or a Casimir operator (as is done now.)

11) To get diffl. forms on $\mathcal{G}\backslash\mathcal{A}$ one chooses a connection on the principal bundle $\mathcal{A} \longrightarrow \mathcal{G}\backslash\mathcal{A}$. Then equivariant forms on $(\mathcal{G}, \mathcal{A})$ should descend canonically? This must be related to what Singer said.

12) Diffl forms on ΛM and Chen's loop integrals. 1-forms on M give rise to functions on ΛM which are time-ordered products.

13) Two ways to think of the anomaly: Failure of $\mathcal{G}_c$ to preserve the connection on L or failure of $\mathcal{G}$ action to be horizontal. In the

Riemann surface case, where we can think of $\mathcal{A}$ as holom. structures, the line bundle descends to $\mathcal{G}_c \backslash \mathcal{A}_{stable} = \mathcal{G} \backslash YM$. Former gives a holom. structure, the latter a metric to $\mathcal{L}$.

14) Diff'l forms on $\mathcal{G} = \Lambda G$ left invariant under $\mathcal{G}$ and right inv. under G. Given by Connes complex when $G = U_n$ n large. Can you exploit fact that $\Omega\mathcal{G} \to \Lambda G \to G$ is principal or $\Omega\mathcal{G} \backslash \mathcal{A} = G$ to get cohomology, or explicit forms?

15) Although $\mathcal{L}$ on $\mathcal{A}$ descends to $\mathcal{G} \backslash \mathcal{A}$ the connection on $\mathcal{L}$ does not because there is a non-trivial moment map, the anomaly.

16) Connes uses $F^2 = 1$, not $F =$ Dirac operator. Is it related to the unbounded version of Kasparov, the thing which introduces 2π as Connes says?

17) Question:

Could there exist a K-theory of holomorphic bundles over complex manifolds with compact supports.? I should want it to explain transcendence results. What are the possible obstructions? One idea is that for algebraic varieties we have the Deligne theory of compactifying. So a first step would be to know that the ~~compact~~ compactification in the algebraic sense is really fixed by the holomorphic structure. Let us consider the general question of proving that for a schem of finite typ over the integers, the K'-groups are finitely generated. How am I going to prove a theorem like this? The only known models ~~xxx~~ come from the standard limit formula for evaluating the value of zeta at zero or one. So what are the essential ingredients of this proof?

18) Modular function Dedekind's η function is essentially $\prod_{n\geq 1}(1-z^n)$. The reason why it is a modular fn. is that it is $\sim \prod'(m+n\tau)$ and the factors are permuted under $SL_2(\mathbb{Z})$

19) Topos associated to a foliation; action of holonomy on a stalk.

20) The notion of vector bundle on the orbit topos of a foliation is too restrictive. Connes C^*-algebra in the Kronecker foliation has projective modules not of this type

21) Determ. of $\bar\partial$ ops over surface when the complex structure changes. Flat parametrix might not be good enough to regularize a first order δD.

22) Quantum field theory of a Riemann surface. Take the fields ψ, A, g and form the functional integrals. This is a pure imaginary time situation. The Green's fns. are there but the Hilbert space isn't clear.

23) I have tended to use symplectic forms which come from line bundles, hence are integral. But the one's occurring with moment maps seem to be real, e.g. differ orbits in the adjoint repn. of a compact group.

24) dilogarithm, Atiyah's thm on T_c orbits etc.

25) If M is a sphere, then $\mathcal{G} = \mathrm{Maps}_{pt}(S^n, G)$ is $\Omega^n G$, and so $B(\Omega^n G) = \Omega^{n-1} G$. For example $B(\Omega^2 G) = \Omega G$. This is closely related to the fact that in physics one can work either in space-time with Lagrangians, or space with Hamiltonians.

26) Gelfand-Fuks coh, Haefliger's classifying space, are related; also the Thurston-Mather theorem. Does Connes foliation ideas shed any light on Haefliger's space? Does Connes give new techniques to treat the old unsolved problems?

27) After you understand descending $\mathcal{L}$ to $\mathcal{G}\backslash\mathcal{A}$ and computing curvature, you should look carefully at R.S. case where the holom. structure comes in. Why can one descend the connection after restricting to YM?

28) Michael's idea that it is not the connection on $\mathcal{L}$ you construct involving the anomaly as part of its moment map, which is important, but rather the cohomology class of $\tilde{\mathcal{L}}$ on $\mathcal{G}\backslash\mathcal{A}$. Thus you need to ask when the anomaly and curvature can be killed.

29) There is an accessible part of $H^*(\mathcal{G})$ and $H^*(B\mathcal{G})$ which comes from the Lie algebra $\tilde{\mathfrak{g}}$. This fits with the Borel idea that stable cohomology of arithmetic groups comes from invariant forms on the symmetric space.