Point: Suppose we have an even cocycle, i.e., a sequence of cochains

\[ f_1 = \tau(p) \]
\[ f_2 = \tau(\omega) \]

satisfying

\[ b'f_{2n} = (1-\lambda)f_{2n+1} \]
\[ b'f_{2n+1} = \frac{1}{N} N f_{2n} \]

This is not a trace on \( R \). But we have this \( K \) operator.

What does one know about \( \tau - K\tau \)?

If I fix \( I^m \), then I can always take \( f_{2m}, f_{2m+1} \) and say these come from a trace on \( I^m \). Then I can look at \( \tau - K\tau \). I know that \( \tau - K\tau \) is a trace on \( I^m/Im+1 \).

\[ \psi_{2m+1} \xrightarrow{b'} \psi_{2m} \]

\[ \psi_{2m+1} - K\psi_{2m+1} = b'\psi_{2m} \]
\[ \psi_{2m} - \psi_{2m}^K = (1-\lambda)\psi_{2m} \]

\[ Kf_{2m+1} = f_{2m+1} \]

\[ f - \]

So you can arrange...
Idea I have: I have certain linear functionals on $R$ and a basic operator $\tau \mapsto K\tau$. I know that $\tau - K\tau = \tau'd$ for some $\tau'$ determined by $\tau$.

$$R \xrightarrow{d} \Omega^1_R$$

This operator $d$ is clear. So what happens? Somehow you would like to do something like KMS. Want $\tau$. Suppose have $\tau$ on $R$.

$$\tau(\omega^n) = \varphi_{2n}$$

want $h_{2n} = \varphi_{2n}$ up to a scalar factor. For $2^{2n-2}t \varphi_{2n}$, as $g_{2n-1} = 0$. OK.

One thing you can look for is a map $$(2, R) \rightarrow R$$

You have $\tau(\omega^n) = \varphi_{2n}$, a priori.

$$\tau'(\omega^n, \omega^{-1}dp) = g_{2n}$$

so $h_{2n} = n g_{2n}$.

so you want $\varphi_{2n} \omega^{-1}dp \mapsto \omega^n$ up to scalars.
\[ b(\Theta^n)(a_0, \ldots, a_n) \]
\[ = [a_0^+ a_1^+, \ldots, a_{n-1}^+, a_n^-] + (-1)^{n+1}[a_0^-, a_1^-, \ldots, a_{n-1}^-, a_n^+] \]

\[ (b \varphi_{2n})(a_0, \ldots, a_{2n}) \]
\[ = \tau\left([a_0^+ a_1^- \ldots a_{2n-1}^-, a_{2n}^-]\right) \]

So define a trace on \( R = \mathbb{Q}^+ \) by means of the cochain

\[ f_{2n+1} \equiv b \varphi_{2n}, \quad f_{2n} = (-1)^n \varphi_{2n} \]

Call this trace \( \tau_1 \) so that

\[ \tau_1(a_0^+ a_1^- \ldots a_{2n}^-) = \tau\left([a_0^+ a_1^- \ldots a_{2n-1}^-, a_{2n}^-]\right) \]

\[ \tau_1(a_1^- \ldots a_{2n}^-) = \tau\left([a_1^- \ldots a_{2n-1}^-, a_{2n}^-]\right) \]

Apparently this defines a trace on \( R \).

Is it possible to describe this as

\[ \theta \circ \tau(D(a_0^+ a_1^- \ldots a_{2n}^-)) \]

where \( D \) is a derivation of \( R \), or maybe

\[ \tau' d' \text{ with } \tau' \text{ a trace on } \Omega^1 \Omega R. \]

Now

\[ \tau'(a_0^+ a_1^- \ldots a_{2n}^-) = \]
4 should work out easily.

\[ I' \delta (a_0^+ a_1^- a_2^-) \]
\[ = I' \delta (a_0^+ (a_1^+ a_2^-)^* - a_1^+ a_2^-) \] ?

This will see only the top.

\[ g_{2n} = I' (\sum \omega^{2n-1} \omega^k) \]
\[ h_{2n} = \left( \sum \omega^{2n-2} \right) g_{2n} = \varphi_{2n} \]

Thus I want to define \( I' \) by requiring

\[ I' \left( \sum -p \omega^{2n-1} \omega^k \right) = \varphi_{2n} \]

Think of having \( \tau \in (I^n/L, I^n) = (I^n/\theta_R)^n \)

\( \tau \cdot \theta_R^{(n)} \) is a trace on \( I^n/\theta_R^{(n)} \)

Okay.

Given \( \tau \in I^n/[I, I^n-1] = (I \theta_R)^n \)

\[ = (J \theta_R)^{2n} \]

Given \( \tau \in I/[I, J] = \theta_R = (J \theta_R)^2 \)

\( I < J^2 \)
Whatever this map is, it disconnects the different levels, so you therefore expect to see a trace on $\Omega$. 

\[ \mathbb{R} \subset \mathbb{Q} \]

There appears to be a map 
\[ R_q \rightarrow \mathbb{Q}_q \]
\[ a^+_0 q^{-1} \cdots a^-_{2n} \rightarrow \]

Amazing! You have

Given a trace on $R$ come from even cyclic cocycles: coboundaries?

\[ \psi_{2m} \overset{1-b}{\rightarrow} \psi_{2m+1} \rightarrow 0 \]

Fact that $2m$ cyclic 

subject $\psi_{2m}$ can take $\psi_{2m} = \psi_{2m}$

What about?

\[ f_{2m+1} \overset{1-b}{\rightarrow} \psi_{2m} \rightarrow 0 \]

$A\psi_{2m} = -\psi_{2m}$

So what about