algebraic cycles, formulate good conjectures. The main problem is really to understand the deviation of étale coh. from K-theory. Thus I conjecture that there should be a descent spectral sequence for the étale topos of the form

\[ E_2^{p,q} = H^p(X_{\text{et}}, K_2^q(\mathcal{O}_X)) \Rightarrow K_{2p+q}(X), \]

but one knows this isn't the case.

**Example 1.** \( X = \text{Spec}(k) \), \( K_2^0 = F_2 \). Then \( X_{\text{et}} \) is equivalent to the category of sets with continuous \( \pi \) action, \( \pi = \text{Gal}(\overline{k}/k) \). Thus in the spectral sequence the \( E_2 \) term is

\[
\begin{array}{ccc}
\mathbb{Z} & H^1(\pi, \mathbb{Z}) & H^2(\pi, \mathbb{Z}) \\
K_1(k) & 0 & \\
K_2(k) & 0 & \\
K_3(k) & 0 & \\
0 & 0 & \\
\end{array}
\]

so we have the mysterious

\[ H^2(\hat{\mathbb{Q}}, \mathbb{Z}) \cong H^1(\mathbb{Z}, \mathbb{Q}/\mathbb{Z}) \cong \mathbb{Q}/\mathbb{Z} \]

term.
So the problem becomes how to formulate things. Idea might be as follows: On one hand we have the homotopy inverse limit

$$\text{holim} \left( \frac{B_k}{\pi} \right)$$

and on the other the theory $B_k$. There is a canonical map

$$B_k \rightarrow \text{holim} \left( \frac{B_k}{\pi} \right).$$

To be a bit more general I can suppose given a hypercovering $U$ of $X$ for the flat topo (fpqc). Then I get a cosimplicial spectrum

$$p \rightarrow B_{U_p},$$

and I take its holim, and there is a map

$$B_X \rightarrow \text{holim} \left( p \rightarrow B_{U_p} \right).$$

(Here $B_X$ denotes the spectrum one constructs from vector bundles on $X$. Possibly $X$ should be supposed regular so that there is no reason to suppose that $B_X$ should be disconnected.)
So we have this canonical map
\[ B_k \rightarrow \text{holim} B_k \]
and what we want to show is that it induces isomorphisms on homotopy groups \( \pi_q \) for \( q \geq \text{some constant depending on how big } k \text{ is.} \)

This is already sort of interesting from the homotopy viewpoint. It is slightly unusual to find a map \( X \rightarrow Y \) such that the fibre has vanishing homotopy groups above a certain point. Thus finitely many obstructions. So you might prove this simply by showing that if you had a \( k \)-connected space \( X \), then the maps of \( \text{holim} B_k \) to \( X \) and to \( Y \) are the same up to homotopy. \( k \)-connected? I know of something really like this in topology.

Suppose now that \( X \) is
Start with a curve $X$.

worry first about schemes finite type $\bar{k}$ over $\mathbb{F}_p$.

I understand $K$-theory of a finite field.

Let $X$ be a curve over $k = \overline{\mathbb{F}_p}$. Then we have

$$\rightarrow K_i F^2 \rightarrow K_i k \otimes D \rightarrow K_i X \rightarrow K_i F^2 \rightarrow$$

the localization exact sequence.

$$K_i X = K_i k \oplus K_i k \oplus \text{interesting part}$$

$$\rightarrow K_i F$$

basic conjecture should be set up carefully

$$K_i k \otimes F^* \rightarrow K_i k \otimes D \rightarrow K_i k \otimes \text{Pic}(X) \rightarrow 0$$

The point maybe is that $K_i X$ has a nat. filt.

$$0 \rightarrow H^1(X, K_{i+1}) \rightarrow K_i X \rightarrow H^0(X, K_i) \rightarrow 0$$

cokernel.
and the conjecture is simply that

\[ H^i(X; \mathcal{K}_{i+1}) = 0 \]

the canonical maps

\[ K_i \otimes \mathcal{K}_i \to H^i(X; \mathcal{K}_i) \]

\[ \text{Tor}_1^Z(K_i; \mathcal{K}_i, \mathcal{K}_i) \to H^0(X; \mathcal{K}_i) \]

are isos. Roughly,

\[ 0 \to K_i \otimes \mathbb{Z} \tilde{\mathcal{K}}_i(X) \to \tilde{K}_i(X) \to \text{Tor}_1(K_i; \mathcal{K}_i, \mathcal{K}_0 X) \to 0 \]

so what we know is that

\[ K_i \otimes \mathbb{Z} \tilde{\mathcal{K}}_0(X) \to \tilde{K}_i(X) \to H^1(X; \tilde{K}_{i+1}) \to 0 \]

and

\[ \text{Tor}_1(K_{i-1}; \mathcal{K}_i, \mathcal{K}_0(X)) \to H^0(X; \mathcal{K}_i) \]
6. k alg. closure of a finite field, one conjectures that

\[ K_i k \otimes F/k^* = \tilde{K}_i(F) \]

free

i odd \( > 0 \).

true in general.

\[ K_{i+1}(F) = K_i k \oplus K_i k \otimes F^* \]

\[ 0 \rightarrow K_i k \rightarrow K_i F \rightarrow K_{i-1} k \otimes F/k^* \rightarrow 0 \]

Take direct limit as \( F \) goes to its alg.
closure. Then

\[ \tilde{K}_i F = K_i k \otimes F/k^* \quad i \geq 0 \]

\[ K_0 F = \mathbb{Z} \quad \mathbb{Q} \text{ vector space} \]

\[ K_1 F = F^* \quad \text{has lots of } \mathbb{Q} \text{-stuff} \]

\[ K_i k = K_i k \quad \text{for } i \geq 2. \]
k alg. of a finite field
$\text{Aut}_k$ over $k$, $F$ function field
$\bar{F}$ its alg. closure. Then I want to look at the Galois spectral sequence. So what happens is this.

$$\tilde{K}_i F = K_{i-1} k \otimes F^* \quad i \geq 2$$

$$K_1 F = F^*$$

$$K_0 F = \mathbb{Z}.$$  

$\Pi = \text{Gal}(F/F)$. $\Pi$ should be of coh. dim 1. (Tsen theory)

$$H^1(\Pi, K_{i-1} k \otimes F^*)$$

$$i = 0 \quad (2) \quad i > 0$$

$$K_{i-1} k \cong \mathbb{Q}/\mathbb{Z}.$$  

$$K_{2i-1} k = \bigoplus_{e \neq p} V_{l_1}^e / T_{l_1}^{e, i}$$

$$0 \rightarrow H^1(\Pi, K_{i-1} k \otimes F^*) \rightarrow H^2(\Pi, F^*) \rightarrow H^2(\Pi, \mathbb{Q} \otimes F^*)$$

$$\text{Br}(F) = 0$$

$$\left\{ \begin{array}{l}
H^1(\Pi, K_{i-1} k \otimes F^*) = 0 \quad i > 0 \\
H^2(\Pi, F^*) = 0 \\
H^1(\Pi, F^*) = 0
\end{array} \right.$$
Thus $E_2$-term

<table>
<thead>
<tr>
<th>$\mathbb{Z}$</th>
<th>0</th>
<th>$\text{Hom}(\pi_1, \mathbb{Q}/\mathbb{Z})$</th>
<th>0</th>
<th>0</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^*$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(k \otimes F^*)^T$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$(k_0 \otimes F^*)^T$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

And what works is the following.

$$(Q \otimes F^*)^T \rightarrow (\mathbb{Q}/\mathbb{Z} \otimes F^*)^T \rightarrow H^1(\pi_1, F^*) \rightarrow$$

$e^* \rightarrow Q \otimes F^*$

So the spectral sequence seems to work. Anyhow, now suppose that $F^*$ is non-zero

Next want to take a curve over a finite field. It would appear reasonable that things would work. More or less have to

$$k_1 = \frac{k}{1 + [T]}$$

And we know that

$$K_i(k_1) = \begin{cases} 
\mathbb{Z} & i = 0 \\
 k_1 & i = 1 \\
 0 & i = 2, 3, 6, \ldots \\
 K_i(k_0) & i = 3, 5, 7, \ldots 
\end{cases}$$
Try taking a curve over $k_1$. Unless $k$, now $k'$ is non-torsion. And the same is true for the $\text{Pic}(X)$.

$$0 \to A \to B$$

$$0 \to \tilde{R}_i(X) / F_0 \to \tilde{K}_1 \to K_i \to D \to F_1 K_i X \to 0$$

$$0 \to \text{Tor}_1(K_i, \tilde{R}_0) \to K_i \otimes D \to K_i \otimes \tilde{R}_0 X \to 0$$

So what we do have is a map $
abla$

$$\text{Tor}_1(K_i, \tilde{R}_0) \to \tilde{R}_i(X) / F_0 K_i(X)$$

should be injective by the map class construction.

If you are in the good range where $K_i$ is torsion, then $K_i \otimes \tilde{R}_0 X = 0$, so $F_1 K_i(X) = 0$, and the conjecture is that you have an win.

General conjecture is that one has an exact sequence

$$0 \to \text{Tor}_1(K_i, \tilde{R}_0) \to \tilde{R}_i(X) / F_0 \to K_i \otimes \tilde{R}_0(X)$$
If $Q = \ker (K_i k \otimes \bar{K}_0 \to K_i(X))$
then I have
$$
\begin{array}{ccccccc}
0 & \to & K_i(X)/F_0 & \to & K_{i+1}(F) & \to & K_i(k \otimes D) & \to & F_1 K_i(X) & \to & 0 \\
& & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \to & \text{Tor}_1(K_i k, \bar{K}_0) & \to & K_i k \otimes (F/k) & \to & K_i(k \otimes D) & \to & K_i(k \otimes \bar{K}_0) & \to & 0 \\
& & \uparrow & & \uparrow & & \uparrow & & \uparrow & & \\
0 & \to & K_{i+1}(X)/F_0 & \to & K_{i+1}(F) & \to & \text{Im} \Theta & \to & 0 \\
& & \uparrow & & \uparrow & & \uparrow & & \\
0 & \to & \text{Tor}_1 & \to & K_i k \otimes F/k & \to & I & \to & 0 \\
& & \uparrow & & \uparrow & & \uparrow & & \\
0 & \to & \text{Im} \Theta & \to & K_i k \otimes D & \to & F_1 K_i & \to & 0 \\
& & \uparrow & & \uparrow & & \uparrow & & \\
0 & \to & I & \to & K_i k \otimes D & \to & K_i k \otimes \bar{K}_0 & \to & 0 \\
& & \uparrow & & \uparrow & & \uparrow & & \\
0 & \to & & & & & & & \\
\end{array}
$$
"Is it reasonable to conjecture the $A$ is zero?"

Conjecture (?): $\text{Tor}_1(K_i,k, R_0) \rightarrow \mathcal{H}^0(X, K_{i+1})$

Assume spectral sequence degenerates canonically $\Rightarrow$

might there be a chance that

$0 \rightarrow \text{Tor}_1(K_i,k, R_0) \rightarrow \tilde{K}_{i+1} F \rightarrow \text{D} \otimes K_i k$ $\rightarrow$ $\text{be exact? Probably not because for } K_2 \text{ one expects that}$ $K_i k \otimes (F/k) \rightarrow \tilde{K}_2 F \text{ certainly would be onto the torsion elements}$

\[ \text{Im} \bigg\{ K_{i+1}(X) \rightarrow \tilde{K}_{i+1}(F) \bigg\} \]

Is there a chance that this might consist of torsion elements?

$K_{i+1}(F) = \lim \rightarrow K_{i+1}(X-S)$

Thus I have an element of $K_{i+1}(X)$ which vanishes in $K_{i+1}(X-S)$ for $S$ finite, hence which is in the image of the transfer with respect to $S$.

$\mathbb{Z} S \otimes K_i k \rightarrow K_i$
i odd =

\[ 0 \rightarrow K_{i+1}^* \rightarrow \tilde{K}_{i+1}^* F \rightarrow K_{i}^* \otimes D_0 \rightarrow 0 \]

so what we want is consistent.

so what you want to prove maybe is that if all time symbols vanish then the element is torsion.

Thus one might try to see if \( \tilde{K}_{i+1}(F) \rightarrow K_{i}^* \otimes D \) has to be injective modulo torsion in the complete case.

\[ K_{i+1}^* \otimes D \rightarrow \tilde{K}_{i+1}^*(X) \rightarrow \tilde{K}_{i+1}^*(F) \]

affine

and you want something about ch. classes of bundles. Anyway.

Deligne's construction. L line bundle trivialized at a set of points S. View as divisors \( D \) strange to S modulo \( k_i \) equivalence by \( f = 1 \) on S.

\[ \sum f(P) \]

\[ \sum f(P) \cdot \text{ord}_P(D), \quad P \in K_i^* \otimes D \]
13. \( \Sigma \ f(P) \otimes P \) if \( D = \Sigma \text{ord}_p(D) \cdot P \) then to check that \( \Sigma \ f(P) \otimes P = 0 \) if \( g = 1 \) at \( S \)

Therefore \( \mathcal{O} \{ f, g \} = \Sigma (-1)^{v_p(f)} v_p(g) \cdot f \frac{v_p(g)}{v_p(f)} (P) \otimes P \)

and if you are at \( Q \in S \) then \( v_Q(g) = 0 \)

so we have \( \frac{f^0}{g^{v_p(f)}(P)} = 1 \).

The significance is clear

\[
\begin{align*}
H^0(X - S, \mathcal{O}_X^*) \otimes \{ \text{line bundles trivialized at points of } S \} & \longrightarrow H^1(X, K_2) \\
0 & \longrightarrow H^0(X - S, \mathcal{O}_X^*) \longrightarrow K_1F \longrightarrow \bigoplus_{P \in S} \mathbb{Z} \\
& \quad \quad \quad \downarrow K_1(F) \\
& \quad \quad K_1F \longrightarrow \bigoplus_{P \in S} \mathbb{Z} \quad \longrightarrow
\end{align*}
\]
Generalized Jacobians:

1) Rational map of a curve to an alg. group comm.
2) Picard scheme of a singular curve.

First point is this: If \( f : X \to S \to G \) is given, one can extend \( f \) to divisors prime to \( S \) and then show the existence of a module, i.e. an \( m \) such that 
\[ g \equiv 1 \pmod{m} \Rightarrow f(g) = 0. \]

Then we have universal such thing \( J_m \).

Somehow the singular curve arises from the equivalence relation defined by the module.

Better approach would be to start stability this way:
\[ X \]
\[ P(X) \]
\[ \text{set of v.b. on } X \]
\[ \text{have to filter by rank} \]
\[ O = F_0 P(X) \subset F_1 P(X) \subset \ldots \]

and I consider corresponding filt. of \( Q \)
\[ \text{filt. } P_0(QP(X)) \subset F_1 (QP(X)) \subset \ldots \]
\[ \text{filt. } \subset \bigvee L_{\text{Aut}(L)} \subset \]

and I consider the assoc. filtration.
To compute \( L_0 \beta_j \) homology, we have a spectral sequence

\[
E^2_{p,q} = H_p(\pi_1(\mathcal{C}^\prime, Y \rightarrow H_q(f/Y)) \Rightarrow H_{p+q}(\mathcal{C}).
\]

So I want to consider the setup, which goes as follows: the pullback of the map \( f/Y \) is the building of \( \mathcal{C}^\prime \).

So

\[
\begin{align*}
F_{p-1}Q & \xrightarrow{j} F_pQ \\
& \xrightarrow{\beta} \prod_{E \in \text{Vect}_p(X)} \text{Aut}(E)
\end{align*}
\]

\[
H_b(f|/E) = H_b(\text{sum, of } X(E_i))
\]

\[
= \begin{cases} 
\mathbb{Z} & b = 0 \\
0 & b \neq 0, p-1 \\
\text{St}(E_i) & b = p-1
\end{cases}
\]

if rank \( E = p \)

and \( = \mathbb{Z} \) in deg. 0 if rank \( E < p \).

Thus we have an exact seq.

\[
\begin{align*}
0 \rightarrow \prod_{E \in \text{Vect}_p(X)} \text{St}(E) & \xrightarrow{f_p} L_0 \beta_j \pi_1(\mathcal{C}) \rightarrow \mathbb{Z} \\
& \rightarrow \mathbb{Z} \rightarrow 0
\end{align*}
\]

should say triangle
\[(i^*F)(\mathcal{U}) = H_0(i^!i_\mathcal{U}, F)\]

\[(i^!F)(\mathcal{U}) = H_0(i^!i_\mathcal{U}, F)\]

Now if \(\mathcal{U}, i^!i_\mathcal{U} \to Y\) then it is empty if \(Y \in F_{p-1}Q\). And if \(Y \in \mathcal{U}_p\)

\[i^!\] is exact and extends by zero.

So get long exact sequence

\[\cdots \to H_{b+1}(F_pQ) \xrightarrow{\partial} \left[ \frac{\text{Aut}_E}{\mathcal{U}_p} \right] \xrightarrow{\mathcal{E}} H_b(F_{p-1}Q) \to H_b(F_pQ) \to H_{b-p}(Q) \to \cdots\]

\[H_{b-p}(Q)\]

\[\cdots \to \prod_{E \in \mathcal{G}, \mathcal{G}_p(E) = p} H_b(\text{Aut}_E, \mathcal{E}(E)) \to H_b(F_{n-1}Q) \to H_b(F_nQ) \to H_b(F_{n+1}Q) \to \cdots\]

So now I want to consider the fibre of the map

\[\cdots \to F_{n-1}Q \to F_nQ \to F_{n+1}Q \to \cdots\]

I am interested in \(\prod Q = H_1Q\).

\[H_2(F_1) \xrightarrow{\text{tr}(\cdot)} \prod_{i} H_1(\text{Aut}_i, \mathcal{E}(E)) \xrightarrow{\mathcal{E}} H_1(F_0) \to H_1(F_1) \to 0\]

\[0 \to H_1(F_1)\]
\[ 0 \rightarrow H_1(F_1) \overset{1}{\rightarrow} \frac{H_0(\text{Aut}(L), \text{St}(L))}{\text{LePic}} \rightarrow H_0(F_0) \cong H_0(F_1) \]

\[ H_i(F_1) \cong \frac{H_{i-1}(\text{Aut}(L))}{\text{LePic}} \quad i > 0 \]

\[ H_2(F_1) \cong \frac{H_1(\text{Aut}(L))}{\text{LePic}} \]

\[ \frac{(H_1(\text{Aut}(E), \text{St}))}{E \in V_2} \]

\[ H_2(F_1) \rightarrow H_2(F_2) \rightarrow \frac{H_0(\text{Aut}(E), \text{St}(E))}{E \in V_2} \]

\[ H_1(F_1) \rightarrow H_1(F_2) \rightarrow \frac{H_1(F_2)}{E \in V_2} \]

\[ \mathbb{Z} \overset{\oplus \mathbb{Z}[-1]}{\rightarrow} \mathbb{Z}[\text{Pic}] \]

\[ \mathbb{Z} \overset{\oplus \mathbb{Z}[-1]}{\rightarrow} \mathbb{Z}[\text{Pic}] \]

\[ H_2 \rightarrow H_1(F_2) \cong H_1(F_2) \rightarrow \frac{H_1(F_2)}{H_1-3} \]

**In any case what happens?**

The point unfortunately is that this leads nowhere because you have no way to get at the Steinberg homology on a p.i.d. e.g. take a field. Then \( H_1(F_1) = \mathbb{Z} = H_1(F_2) \).

\[ H_0(\text{Aut}(E), \text{St}(E)) = 0 \quad \text{since } \text{Aut}(E) \text{ transitive on lines} \]

\[ \text{St}(E) = \mathbb{Z}[\rho]/\mathbb{Z} \]
This approach doesn't lead anywhere even in the case of a field. Except it must give some stability result. Suppose I want to know about \( \pi_2(F_n) \).

\[
\begin{align*}
H_2(F_1) \to H_2(F_2) &\to H_2(F_3) \to \lim_{\eta} H_0(\text{Aut } E_\eta, \text{St}(E_{\eta})) \\
\text{thus} \quad H_2(F_2) \to H_2(F_3) &\to H_2(F_4) \to \cdots
\end{align*}
\]

Thus this tells me that the K_1 of a curve is determined by bundles of rank \( \leq 3 \). I would like it to be determined by bundles of rank \( \leq 2 \).
Basic stability yoga should demand that on something of \( \dim d \) \( K_i \) should be gen. by bundles of rank \( \leq i + d \) and inj for bundles of rank \( \leq i + d + 1 \).

\[
\text{rank } E > d \implies E = E' \oplus 1.
\]

A local ring. Can do the same thing, namely, can consider building.

So consider a free \( \mathbb{A} \)-module \( E \) and flags inside of it.

A local ring \( E \) is free f.g. \( \mathbb{A} \)-module rank \( n \) to consider \( X(E) \) ordered set of subquotients of rank \( \leq n \). To show \( X(E) \) begins in \( \dim n - 1 \). Suspension of building in some sense.

\[
X(E) \to X(E) \quad E = \mathbb{A}_{\mathbb{K}} \otimes_{\mathbb{A}} E
\]

I want \( \text{Aut}(E), X(E) \) to have a certain connectivity. Not clear.
20. Suppose \( A \) is a local ring and I filter \( \mathbb{Z}/r \) according to the rank. What gives? So I have to worry about \( \text{Aut}(E) \) acting on the reduced chains of \( X(E) \).

The point should have something to do with the way \( X(E) \) the map

\[
X(E) \longrightarrow X(E)
\]

identifies things together. So consider the situation in The situation

\[
X(E) \longrightarrow X(E) \quad \text{E}/mE.
\]

anyway. So suppose now that things vary.

let I be compact complete etc.

\( E \) being given consider all possible ways of extending \( F \) to \( F \). Thus I have

for each \( E \) is given
Question: The problem has to do with the discrepancy between the K-theory of X and the integral of the local K-theory.

\[
K \rightarrow \text{ho-} \Gamma(K)
\]

Now from a spatial viewpoint one has a map of spectra

\[
K \rightarrow \text{Pic}
\]

given by the determinant. As spaces this map splits so that on the spatial level one has

\[
K = SK \times \text{Pic}
\]

\[
\text{ho-} \Gamma(K) = \text{ho-} \Gamma(SK) \times \text{ho-} \Gamma(\text{Br})
\]

Thus the discrepancy on the K should be at least as big as the discrepancy on Br.

\[
\hat{\text{Br}}(F)
\]

\[
F^\times
\]

\[
H^X(\hat{\mathcal{M}}, F^\times)
\]

\[
\text{Br}(F)
\]

\[
\hat{\text{Br}}(F)
\]

\[
\text{Br}^\infty
\]

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\[
0 \rightarrow \mu \rightarrow \tilde{F}^* \rightarrow V \rightarrow 0
\]

\[H^8(\mu) = H^8(\tilde{F}^*), \quad g \geq 2\]

\[\rightarrow H^1(\mu) \rightarrow H^1(\tilde{F}^*) \rightarrow 0\]

Know \[H^i(X, \mathcal{O}_X^*) = \# \Gamma(X, \mathcal{O}_X^*) \text{ Pic}(X)\]

so that in fact there is no Zariski discrepancy.

Situation: If I look at the $\mathbb{G}_m$ situation, then I do not have a cohomological descent setup. That is the stack of line bundles is not trivial so there is no homotopy integral since $\mathbb{G}_m$ has higher cohomology.

And I know that the situation is no good from the spectral viewpoint, because $X$ has higher cohomology, not to mention the Brauer group.

GL situation is as follows

You have a mechanism for understanding rational $K$-groups at least. Thus $X$ scheme has

\[
0 \rightarrow \text{K}_n(O_X) \rightarrow \bigoplus_{x \in X_0} \text{K}_n^*(k(x)) \rightarrow \cdots
\]

this is a resolution of sorts. But perhaps it is a resolution in the etale topology provided you forget torsion.
23. $X$ regular scheme

for any $U \to X$ etale I want to consider

$$\prod_{x \in U_p} \mathbb{K}_0(k(x)) \otimes \mathbb{Q}$$

which is a contravariant functor in $U$.

Conjecture: This is a sheaf for the etale topology.

and it is flasque.

And what can one hope to understand.

$$U \times_X U \Rightarrow U \to X$$

Hx

Probably true, so what tells me that

$$H^*_\text{et}(X, \mathbb{K}_0 \otimes \mathbb{Q}) = H^*_\text{zar}(X, \mathbb{K}_0 \otimes \mathbb{Q}).$$

$$i : \text{Spec } K \to X$$

$$0 \to \mathbb{G}_m, X \to \bigoplus_{x \in X_0} i'_*(\mathbb{G}_m, K) \to \prod_{x \in X_0} (i_x)_*(\mathbb{Z}) \to 0$$

$$H^1(X, \mathbb{G}_m) \to H^1(X, i'_*(\mathbb{G}_m, K)) \to \prod_{x \in X_0} H^1((\mathbb{G}_m, K), (i_x)_*(\mathbb{Z}))$$

$$H^2(X, \mathbb{G}_m) \to H^2(X, i'_*(\mathbb{G}_m, K))$$

$$R^0 i'_*(\mathbb{G}_m, K)$$

$$H^8(\mathbb{K}_x U, \mathbb{G}_m) = 0 \quad \text{if } \theta = 1$$

$$R^1 \mathcal{H}(\ldots) \quad \text{if } \theta = 2.$$
24. $X_0, X_1, X_2, \ldots$

next point might be to consider

$$\prod_{x \in X_0} K_2 k(x) \rightarrow \prod_{x \in X_1} k(x) \rightarrow \prod_{x \in X_2} \overline{k(x)}$$

problem: under what conditions might I get an étale sheaf?

Is there any arrangement?

$$\prod_{x \in U_0} K_2 k(x)$$

presheaf?

what is the associated sheaf? Conj. would be that it is the set of $$\text{alg. closure of } k(x)^\text{sep} \otimes K$$

Choose $$\text{alg. closure of } x \in X_0$$

then have $$\text{Spec } \Omega$$

$$\text{sep closure of } k(x)^\text{sep}$$

so have Galois module

$$K_2 \Omega$$

certainly the stalk ought to be $$K_2 \otimes \overline{k(x)}$$
elliptic curves
basic problems
you must understand buildings
need upper bound methods. The place to start
must be with curves over finite fields.
Milgram theory
[F.S.S] review E9 theory