~~Thus it seems that one has determined!!!!!!~~

But we have a monoidal functor

$$\mathcal{S} \longrightarrow \mathcal{S}'$$

$$S \longmapsto (\gamma S,\ f_0 S,\ f_0 \mathcal{S}(\gamma S) \simeq f_0 S)$$

which is an equivalence of mon. cats. So we get as above a strict

$$\tilde{\mathcal{S}} \longrightarrow \mathcal{S}'$$

Uniqueness Lemma: Let $\mathcal{S} \xrightarrow{\gamma} \mathcal{S}'$ be an equiv. of monoidal cats with $\mathcal{S}'$ strict, let

$f_1, f_2 : \mathcal{S}' \longrightarrow \mathcal{U}$ be strict mon. functors

with $\mathcal{U}$ strict and suppose $f_1\gamma = f_2\gamma$. Then $f_1 = f_2$, provided $Ob(\mathcal{S})$ generates the mon. $Ob(\mathcal{S}')$.

Proof: Because γ is an equivalence, $\exists!$ natural isom $\theta : f_1 \simeq f_2$ of monoidal functors $\ni$

$$\theta\cdot\gamma : f_1\gamma \simeq f_2\gamma$$

is the identity. Consider $\{X \in Ob(\mathcal{S}') \mid \theta_X = id\}$. Since

$$\begin{array}{ccc} \theta_{X_1X_2} : f_1(X_1X_2) & \simeq & f_2(X_1X_2) \\ \| & & \| \\ f_1(X_1)f_1(X_2) & \underset{\theta_{X_1}\theta_{X_2}}{\simeq} & f_1(X_1)f_2(X) \end{array}$$

follows this set is closed under product, so done.