A fibered category of virtual coverings
an object of $K$ over $X$ is a kind of cycle. It is a
formal sum $\sum my\delta_\gamma$ where $m \in \mathbb{Z}$ and $\delta_\gamma$
sums over the finite covering spaces of $X$. The concept of
isomorphism class of such things is clear. And now
an isomorphism is what the stable or cobordism should be
what

Difference between this and algebraic geometry is that a
cycle does not admit automorphisms so there is no isomorphism
question involved with the equality of two cycles.
Cobordism of cycles: you ask for one in the product
This is same as equality

**I.**

A fibered category in groupoids over $S$
thus if $f_0, f_1 : X \to Y$ are homotopic, then the homotopy
gives rise to an isomorphism $f_0^* y = f_1^* y$
check fiber axioms.

Suppose $\Theta : K \to E$ is a natural transformation
i.e. for each $x \in K(X)$ have $\Theta_x : X \to E$ and
for each $x \to y$ over $X \to Y$ have

```
X \downarrow \Theta_x \downarrow \Theta_y \downarrow \rightarrow E
```

satisfying evident compatibility conditions.
Now you should also notice that there is a natural map

\[ k(G \sqcup G) \times k(G \sqcup G) \rightarrow k(G \sqcup G) \]

Suppose you have an element of \( k(G) \) i.e. a integral linear combination of certain irreducible \( G \)-sets, write it as difference

\[ S_0 - S_1 \]

of positive disjoint \( G \)-sets. Now if this goes to zero in \( k(G) \), then \( S_0 \) and \( S_1 \) are isomorphic. Choosing an isomorphism one gets an element of \( k(G \sqcup G) \).

\[ d(S_0, S_1, w) \in A \]

and \( d(S_1, S_2, v) \)

I want \( d(S_0, S_1, w) = d(S_0, S_1, u) + d(S_1, S_2, v) \)
so you need an algebraic $K$-theory to associate to the algebraic general linear groups over a scheme $S$, i.e. $\text{BGL}_S$. What happens prime to the characteristic tends to be clear? Thus one tends to understand completely what happens prime to the characteristic, i.e. one gets that

$$\Pi_{2q}(\text{BGL}_S) = T \otimes \mathbb{Q}$$

$T$ is an inverse system of finite group schemes over $S$; hence this is the relative situation. Now you must understand what gives when you start taking sections of this animal and more generally "integrating it" over a map $f$.

This style intuition is that alg. $K$ generalizes cohomology ([Kummer theory].

$$0 \rightarrow \text{BGL}(F_q) \rightarrow \text{BGL}_S \rightarrow \text{BGL}_S \rightarrow 0$$

If $S$ is a topological space, then $\text{BGL}_S = S \times \text{GL}_S$ so the sections are space $\text{Hom}(S, \text{BU})$ whose homotopy groups are $K_*(S)$.

Idea is that $\text{GL}$ should be defined cat. of asked and gives something over any amelee topos. $\text{Den}$ non-commutative ring. Something like a generalization of a sheaf.
functor from simplicial sets to simplicial sets which hopefully will give \( k \) by truncation say.

Let us assume that we have a bisimplicial set \( A \) and we consider the functor from s-sets to s-sets represented by this:

\[
X \mapsto \text{Hom}(X, A) = Q(X)
\]

Then I can define

\[
k_i(X) = \pi_{i+1}(Q(X))
\]

and I can still ask for the required universal property.

So even if you can solve the category problem, it still remains to define \( k \). Thus if \( X \) is a space and if we are given a word

Thus to a space \( X \), associate the Picard category \( \mathbb{P}^8(X) \) defined by rigidifying BGL, etc. Then given an \( F_8 \)-vector bundle over \( X \), choose a classifying map \( f: X \to BGL_n(F_8) \) for \( E \) is a pair

\[
\begin{array}{c}
E \\
\downarrow \quad u \\
X \\
\downarrow \quad f \\
BGL_n(F_8) \\
\end{array} \quad \xrightarrow{\text{chosen}} \quad \mathbb{P}^8
\]

and associate to an exact sequence...
0 \to E' \to E \to E'' \to 0

better start with $BGL\mu(F_8) \xrightarrow{u_n} E^\Psi 8$

and for $n, m$

$B \Gamma_{mn}(F_8)$

$(p_1, p_2) \xrightarrow{(u)} BGL_m(F_8) \times BGL_m(F_8) \to E^\Psi \times E^\Psi$

$h$ so choose such a homotopy. must check transitivity

hence have to worry about the uniqueness of the homotopy

$B \Gamma_{mn}(F_8)$

$\xrightarrow{u} E^\Psi \times E^\Psi$

$B \Gamma_{mn}(F_8)$

$\xrightarrow{u_{m+n}} E^\Psi$

$B \Gamma_{mn}(F_8)$

$\xrightarrow{u \circ \text{id}} E^\Psi$

$B \Gamma_{mn}(F_8)$

$\xrightarrow{u} E^\Psi$

but this should be $\text{OKAY}$ again by Atiyah's theorem.

Thus by universal property of the functor $k$ this part should be $\text{OKAY}$, hence if my universal property holds I get a map $B \to E^\Psi$

of spaces
mult. char. classes

Let $\mathbb{k}$ be a field and suppose $R = \bigoplus_{n=0}^{\infty} R_n$ is a graded anti-commutative $\mathbb{k}$-algebra. Then by a $R$-valued mult. char.

$$\Theta(E) = \sum \Theta_n(E) \quad \Theta_n(E) \in H^n(X) \otimes R_n$$

and in particular a DG ring. To each bundle $E \to X$ we want a coh. class

$$X \to \Gamma$$

and to each exact sequence I want a homotopy class of maps!!!!!!

If $R$ is a differential graded ring

$$R_2 \to R_1 \to R_0$$

not necessarily commutative, then by a multiplicative character class with values in $R$ I mean ultimately a hom. $\mathbb{Z}[GQ] \to R$

or $\mathbb{Q} \to \mathbb{GQ} \to R^\times$
Basic question which you would really like to solve is to produce something over $G(G)$, ideally a perfect complex, which would possess Chern classes.

Thus suppose you form cokernel of

$$\Gamma_{m,n} \rightarrow \Gamma_{m+n} \rightarrow C$$

or better you have $X$ with $E' E''$ over and an exact sequence $\cdots \rightarrow E' \rightarrow E \rightarrow E'' \rightarrow \cdots$ and you want to make $E = E' \oplus E''$.

So you want to map $C_m \times G_m \rightarrow \Gamma_{m,n}$ an equivalence.

$$0 \rightarrow E' \rightarrow E \rightarrow E'' \rightarrow 0$$

$G_m \times G_m \rightarrow \Gamma_{mn}$.

Form a 1 cocycle on $I \times X$.

In this setup what I get is

$$A - E + H \quad B - F + J$$

and a reason for these being the same.
Thus suppose $X$ etc.

The first obstruction to extending to $Z$ are

$$H^*(Z, X; \pi_*(W))$$

as

This works as long as $W$ simple.

$K(\pi_2 W, 3)$

$K(\pi_3 W, 3)$

$H^3(Z, X; \pi_2 W)$

$H^2(X, \pi_2 W)$

$H^2(Z, \pi_2 W)$

$H^2(Z, X; \pi_2 W)$

$K(\pi_3 W, 4)$
so $\mathbb{Z}/X$ is contractible because it has no cohomology and is 1-connected since it has only 2+3 cells.

Suppose $W$ is a 1-connected $H$-space, then $\pi_1(W) = 0$.

Then one knows that

$$[\mathbb{Z}, W] \to [X, W]$$

Hence

$$[\mathbb{Z}, \Omega W] = [X, \Omega W]$$

argument should be modifiable
killing all gp. without changing $H^*_f$.

Suppose $H^*_f(X) = 0$ \( X \) connected

attach 2 cells to kill \( \pi_1(X) \).

then get \( Y \) \( H_2^*(Y) = H_2^*(X) \)

except \( H_2^*(Y) = H_2^*(X) + \text{free gp. gen. by attached cells.} \)

but \( \pi_1(Y) = 0 \) so these new elements are spherical so attaching 3 cells can obtain space \( Z \) with same homology.

\( \text{This construction functorial up to homotopy at least. Thus given} \)

\[ \begin{array}{ccc} X & \longrightarrow & W \\ \downarrow & & \downarrow \\ Y \end{array} \]

\[ [VS^2, W] \longrightarrow [Y, W] \longrightarrow [X, W] \longrightarrow [VS^1, W] \]

so unique if \( \pi_2(W) = 0 \), similarly need \( \pi_3(W) = 0 \) to get a unique extension to \( Z \).
If $k$ a field, and suppose one considers the maximal cyclotomic subfield $k_0$ say finite with $q$ elements. Then

$$
\mu_q \text{ is canonical.}
$$

**Example:** Galois group invariant because

$$
\text{Gal}(\overline{k}/k) \rightarrow \text{Gal}(\overline{k_0}/k_0)
$$

$$
(\mathbb{F}_q)^* \text{ is generated by Frobenius, } x \rightarrow x^q.
$$
Let \( R = S(P) \otimes S(P) \)

\[ \Delta : R \to S(P) \]

\[ S(P) \otimes S(P) \xrightarrow{\Delta} S(P) \xrightarrow{\pi} S(R_2) \]

\[ \Delta(x \otimes 1) = \Delta(1 \otimes x) = x \]

\[ \Delta(x \otimes 1) = x \]

\[ \Gamma(1 \otimes x) = 0. \]

\[ I = \ker \Delta \quad \text{gen. by} \quad x \otimes 1 - 1 \otimes x \]

So now I have computed the Chern classes of a character \( G \to \mathbb{F}_8(\mu_\ell)^* \).

Basic classes \( c_{ij}^r \in H^{2r}(\text{BGL}_n(\mathbb{F}_8) / \mu_\ell \otimes r) \)

Take values in some certain ring

\[ c_i(E) \in H^{2i}(\text{Gal}(\overline{\mathbb{F}_8}/\mathbb{F}_8) \times G ; \mu_\ell \otimes i) \]

This group is zero if \( i \neq 0 \)

\[ H^{2i}(G, \mu_\ell \otimes i) \oplus H^{2i-1}(G, \mu_\ell \otimes i) \]

\[ E_{p_i}^2 = H^p(G, H^q(\text{Gal}(\overline{\mathbb{F}_8}/\mathbb{F}_8), \mu_\ell \otimes i)) \to H^{p+q}(\text{Gal} \times G, \mu_\ell \otimes i) \]
question: How bad is $(i-1)!$ in comparison with $q^i-1$.

Answer: Is very bad since as $i$ goes to infinity $\nu_e((i-1)!) \sim (i-1)\left(\frac{1}{e} + \frac{1}{e^2} + \ldots\right)$

$$\frac{i-1}{e} \left(\frac{1}{1-\frac{1}{e}}\right) = \frac{i-1}{e-1}$$

It is linear in $i$ with slope $\frac{1}{e-1}$.

while $\nu_e(q^{i-1}) = i \nu_e(q-1)$

This is linear in $i$ with slope $\nu_e(q-1)$.  

Conclusion is that the map

$$K_{2i-1}(\mathbb{F}_q) \longrightarrow \lim_{n \to \infty} H^1(\text{Gal}(\mathbb{F}_q/\mathbb{F}_e), \mu_{q^{i-1}}) = \mu_{q^{i-1}}$$

is multiplication by $(i-1)!$.
I have to produce a proof of the existence of the $c'_i$ and $c''_i$ in part II.

$X$ polyhedron

Suppose you form inductive limit of various topoi associated to finer and finer triangulations then do you get all sheaves. First take the usual limit and

$U \subseteq X$ open then $Z_U = \lim \limits_\rightarrow V Z_V$

$V$ open subset of an open subpolyhedron so it seems clear that one must obtain all sheaves.

Existence of $c'_i$, $c''_i$ topologically

$BG \rightarrow E\mathbb{P}^8 \rightarrow BU \rightarrow BU \rightarrow BU$

So you produce elements $c'_i$, $c''_i \in \mathbb{H}^*(E\mathbb{P}^8)$ by your method.

So you must know how these behave under sums.

$E\mathbb{P}^8 \times E\mathbb{P}^8 \rightarrow E\mathbb{P}^8 \rightarrow BU \times BU \rightarrow BU$

$(\mathbb{P}^8 \times \mathbb{P}^8) \equiv \mathbb{P}^{16}$ id

$BU \times BU \rightarrow BU$
And the problem I ran into is uniqueness of the lifting.

Let us first suppose that the difference of these two maps from $B$ to $K(\pi_n X, n)$ is an $H$-space map, i.e.

and the problem is commutativity of

point is that we don't yet know that

\[
\text{Hom}(k, K(\pi_n X, n)) \to \text{Hom}(k, X) \to \text{Hom}(k, X_{n-1})
\]

is exact.

Thus suppose for each $E \to Y$ you give a homotopy class.

Maybe you have to formulate the desired condition 2-universally, e.g.
The idea is that a cartesian functor

\[ \begin{array}{ccc} k & \rightarrow & \mathcal{E} \{ \text{fibred cats} \} \\
& | & \\
& \downarrow \text{uni-} & \\
& & B \end{array} \]

what you have already is a functor, but you need a one of fibred cats.

The other thing you want to be able to handle is cohomology, and for this you must know that

\[ \Pi_0 \text{Hom}_{\text{cart}} (k, K(A, g)) = \text{Hom} (k, H^g(A)). \]

Modulo these two unknowns the theory should work.

Back to simplicial setup. Thus we study

\[ Q(X)_1 = \text{set of } R \text{-v. bundles } \mathcal{V} \text{ over } X. \]

but instead we take

\[ B_0 = B \{ 0 \} \]

\[ = \text{Hom} (X, B) \]

\[ B_1 = B \{ \text{Cat}(V) \} \]

\[ Q(X)_2 = \text{set of exact sequences} \]

instead take

\[ = \text{Hom} (X, B_2) \]

\[ B_2 = B \{ \text{Cat (exact sequences)} \} \]

and similarly for all \( Q(X)_n \). Thus get a
scratch:
terrifically confusing.

I have been looking at category of \( R \)-vector bundles over \( X \), which is not a functor of \( X \) in the strict sense; make it so, at least the objects. Choose representatives for the proj. f.t. \( R \)-modules; form a set \( \mathcal{V} \); then consider

\[ \coprod_{\text{P} \in \mathcal{V}} \text{Aut}(\text{P}) \]

and form category of \( \mathcal{E} \) exact sequences.

Classifying space of the category of exact sequences

analogy is this: think of \( k \) as \( H_0(\mathcal{L}) \)

where \( \mathcal{L} \) is a \( \text{chain complex} \) in an abelian category. Think of \( k \) as \( \mathcal{L} \) itself and \( k_1 \) as \( H_1(\mathcal{L}) \). Then I want to think of \( B \) as a complex \( B \) and what I think I should gain by working with 2-cats etc. is embodied by the difference between

\[ \text{Hom}_D(\mathcal{L}, B) \]

and \( \text{Hom}(H_0(\mathcal{L}), H_0(B)) \).

keep up the analogy:

Go on to what you need to finish of your theorem. Thus you have constructed a natural transformation \( k \rightarrow \mathfrak{O}[1, E^\oplus] \) but you need more, namely a functor

\[ k \rightarrow E^\oplus \]
Suppose I have two cartesian functors

\[ \Theta_i : \mathbb{k} \longrightarrow \mathcal{X}, \]

such that \( f \Theta_1 \) is isomorphic to \( f \Theta_2 \). What \( \Theta_1 \) does is to for each space \( S \) associate a functor

\[ \mathbb{k}(S) \longrightarrow \text{Hom}(S, \mathcal{X}) \]

plus functoriality for all maps in \( \mathbb{k} \). Thus if I am given a map \( u : S' \rightarrow S \) and a map of virtual bundles \( v : X' \rightarrow X \) over \( u \), then I want to have a homotopy \( \Theta_i(v) \) making

\[ S' \xrightarrow{u} S \]

\[ \Theta_i(X') \Rightarrow \Theta_i(X) \]

commute, plus transitivity, etc. Now suppose \( \beta \) is an isomorphism from \( f \Theta_1 \) to \( f \Theta_2 \), so for each pair \((S, \sigma)\) I have a homotopy \( \beta(S, \sigma) \)

\[ f \Theta_1(S, \sigma) \frac{\sim}{\sim} f \Theta_2(S, \sigma) \]

Then let \( \beta(S, \sigma) = (\Theta_1(S, \sigma), \Theta_2(S, \sigma), \Theta_2(S, \sigma)) : S \rightarrow X \wedge Y \wedge X \)
I form a fibred category over the homotopy category, associating to each $X$ the Picard category $\mathcal{K}(X)$ generated by $R$-vector bundles over $X$. An object of $\mathcal{K}(X)$ might be defined to be an element of the free group generated by the set of $R$-vector bundles over $X$. On the other hand, given a space $B$, I can form the fibred category $\mathcal{B}$ over the homotopy category $\mathcal{H}_0$ where $\mathcal{B}(X) = \text{Hom}(X,B)$ and maps are homotopy classes of homotopies. Note that a map $B_1 \to B_2$ induces a functor $\mathcal{B}_1 \to \mathcal{B}_2$ over $\mathcal{H}_0$ and that homotopic maps give rise to isomorphic functors.

The basic question is now whether there exists a morphism of fibred categories $\mathcal{K} \to \mathcal{B}$ which is universal, i.e., given $\mathcal{K} \to \mathcal{B}'$, there is a unique map $f: B \to B'$ and an isomorphism $\theta$ rendering $\theta \circ f = \text{id}_{B'}$ and the pair $(\theta, f)$ is unique in some sense.
The semi-simplicial approach is wrong! Only a universal approach is categorically acceptable.

So before giving up the simplicial stuff observe that if $Q(X)$ is the simplicial set constructed out of $R$-vector bundles over $X$, then there is a map

$$Q(X) \longrightarrow \text{Hom}(X, Q)$$

or equivalently a map

$$X \times Q(X) \longrightarrow Q.$$

Indeed a map $Y \longrightarrow X \times Q(X)$ is the same thing as a pair consisting of a map $u: Y \longrightarrow X$ and a 1-cocycle on $Y$ with values in $R$-vector bundles over $X$. So if $y$ is a 1-simplex in $Y$, then associate to $y$ the $R$-module $f'(y) = f(y)_{u(y)}$, i.e. the fiber of $f(y)$ over the 1-simplex $u(y)$. Then $f'$ is a 1-cocycle on $Y$ with values in $R$-modules, so we have a map $Y \longrightarrow Q$.

Ideas: to $Q(X)$ I associate the Picard category $\mathcal{P}(X)$, say defined as the Postnikov part of $\mathcal{G}(Q(X))$. Then what I want to produce is a map of Picard categories from

$$\mathcal{P}(X) \longrightarrow \Pi(X, E[5])$$

when I have a finite field.
Perhaps it is slightly better to use the suspension
\[ \Sigma X = \Delta(1) \times X / \Delta(1) \times X \]
and to associate the 1-cocyle which over the product of \( \Delta(1) \) and a 1-simplex \( u \) looks like

\[ \begin{array}{c}
0 \\
E_x \\
E_y \\
E_z \\
\end{array} \]

The reason is that if \( 0 \to E' \to E \to E'' \to 0 \) is an exact sequence of \( R \)-bundles over \( X \), then we get a map

\[ \Delta(2) \times X / \Delta(2) \times X \to Q \]

whose faces are the maps associated to \( E', E, \) and \( E'' \). This shows that we get a map

\[ k(X) \to \pi_1 \text{Hom}(X, Q) \]

for any "space" \( X \) (note that the Grothendieck group as a group defined by generators and relations is necessarily abelian since \( E' \oplus E'' \) fits into exact sequences).
Situation:

For each \( n \geq 0 \) I have a simplicial set \((n\text{-}reduced)\)
\(Q(n)\) and probably maps \(\Sigma Q(n) \to Q(n+1)\) which come from the fact an exact sequence

\[
0 \to V_0 \to V_1 \to \cdots \to V_{n+1} \to 0
\]

can be extended one more step by adding 0 at the end (or beginning depending on how \(\Sigma Q(n)\) is defined).

Problem 1: Do induced maps \(\prod_i Q(n) \to \prod_{i+1} Q(n+1)\)
are isomorphism?

If so then the cup product maps

\[
\mathbb{Q}_R(p) \times \mathbb{Q}_S(q) \to \mathbb{Q}_{R \oplus S}(p+q)
\]

will give the appropriate product structures on \(K\)-groups.

(How about \(\lambda\)-operations? seem okay in char 0!)

Suppose \(E\) is a \((R-)\) vector bundle over \(X\). Then over each vertex you get a vertex so over \(\Sigma X\) you get for each 1-simplex a vector space

\[
\text{this is a typical 2-simplex in the cone on } X.
\]

This shows that there is a map

\[
\Sigma X \to Q = Q(1)
\]

associated to an \(R\)-vector bundle over \(X\).
part of paper on algebraic $K$-theory theorems:

I. $BGL^+ = \Omega B(\bigcup B\mathbb{G}_m)$.

II. $K_*A \cong K_*A$.

Stability: manifold $X$ compact with basepoint $x$, then get a framed codimension 2 variety $Z$; $t$ must be done in a certain order.

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\[ 0 \to E' \to E \to E'' \to 0 \]
```

and a representative of $\pi_1(X - Z)$ except that certain ramification behavior is to take place.

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0 \to A \otimes E' \to E \to E'' \to 0
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```
0 \to A \otimes E' \to A \otimes E \to A \otimes E'' \to 0
```

don't

so over the 2-sphere $I$ get

so the fundamental group is free

group on generators

cohomology is a bit amusing for then singularities come in.
Problems: Suppose \( K(X) \) is a representable contravariant functor from finite complexes to \( \text{Ab} \) endowed with traces for finite coverings. Does \( K \) extend to a generalized cohomology theory? (Legal needs \( K(X) \) defined for all CW complexes \( X \), I think.)

Suppose \( K(X) \) has a natural ring structure such that the projection formula holds. Then does the cohomology theory \( K^*(X) \) admit products?

In the case of the connected theory \( K^*_c(X, A) \) does \( \Omega \) extend to a stable cohomology operation when denominators are introduced? And how about the filtration properties signalled by Milgram as a possible method of attack on the stability problem? Any relation between the two kinds of stability?

If \( A \) is a small abelian category and we form \( \Omega \ l C . \) is its homology the same as what one gets from characteristic classes?
Connection of algebraic $K$-theory and the theory of motives: Fix some groundfield $k$ and consider algebraic schemes over $k$. Assume that $k$ is small, i.e. of finite type absolute
whence one expects that étale cohomology is a good functor in the sense that it detects lots of stuff. To each scheme $X$, let $K_*(X)$ be the rational $K$-groups.
Then $K_*(X)$ should satisfy the projective bundle theorem, and this suggests strongly
that it has a Gysin homomorphism; assume so. Wait. Let's review the proj. bundle thus.
Thus if $E$ is a vector bundle over $X$, one knows that $R(G,FE)$ is a free module over
$R(G,X)$ with standard generators, and this isomorphism is compatible with $\otimes$ changes in $G$. Thus the universal map

$$o(1) \quad \otimes \quad R(G,FE) \quad H^0(G, K_*(FE))$$

Thus $o(1)$ defines an element $K_0^e(FE)$ and $K_*(FE)$ is a free module over $K_*(X)$
with usual basis and relation. So what we're getting is the various difference in
weights already in $K_0$. Thus if there is a Gysin morphism it is of degree 0
for the grading but changes the weights in the appropriate way.

Conjecture which should be answerable: Show that the Adams operations on $K_*$
admit eigenspaces of the standard sort.

What's missing is a type of periodicity result which would connect up $K$-groups of different dimensions.

Grand conjecture would run like this: Take the groups $K_*$ and rearrange the grading
so as to be by weights. Vaguely the $K$-groups should measure the deficiency between motives and cohomology. Nonsense.
Connection with the theory of motives is becoming more clear now. We should be the projection on the part of weight 1.

\[ \text{ch}: K^* (\mathbb{A}^1_{/\mathbb{Q}}) \to \mathbb{Q}^* \text{ (Spec } \mathbb{A}^1_{/\mathbb{Q}}) \]

Is something like an isomorphism, then one knows the decomposition of the eigenspaces for the various characters of \( \mathbb{Q}^* \) and the \( \mathbb{Q} \) on the ch. Thus if the hom, above and below, gives a weight decomposition of this space, \( \text{ch} \), i.e., does it split up as a direct sum of eigenspaces for the various characters of \( \mathbb{Q}^* \)?

What about the \( \mathbb{Q} \) ? They must act on \( K^* (\mathbb{A}^1_{/\mathbb{Q}}) \); it is reasonable to expect that they give a weight decomposition of this space, \( \text{ch} \), and that the above is a ring homomorphism.

**Corollary:** If \( A \) is commutative, then there is a unique ring structure on \( K^* (\mathbb{A}^1_{/\mathbb{Q}}) \).

What is \( K^* (\mathbb{A}^1_{/\mathbb{Q}}) \) over \( \mathbb{Q} \)?

**Universal property:** Suppose that \( \mathbb{Q}^* \) is an abelian group, then a map \( \text{ch} \) should be first of all a homomorphism of functors from \( \mathbb{Q}^* \) to \( \text{Ab} \).

**Program:** Characteristic classes with field coefficients. Formula for the rational K-groups.