In the present situation one has a lot of control of the situation. Say this better.

I feel that the K-theory attached to A is the same as the K-theory of $\mathcal{P}(B)$. I know that $\mathcal{P}(B)$ is equivalent to the full subcategory of $\mathcal{P}(\tilde{A})$ generated by $Ae = \tilde{A}e$. So what is the point? What puzzles me is how to link $\mathcal{P}(\tilde{A})$ and $\mathcal{P}(B)$.

What I want is a "good" $\mathcal{P}(A)$ which in the present case will consist of $Ae \otimes_B P$ for $P \in \mathcal{P}(B)$. These are A-modules $-$ H unital of finite presentation???

Go over things again. Start again. Consider the situation $A = Ae \otimes_B eA$, $B = eAe$, eA flat over B. Then I know that $A^2 = A$, A is A flat, hence A is H-unital, get excision in cyclic homology. Now the next question concerns the K-theory. Supposedly the K theory should be the same as that of B, and there should be some sort of excision result. There

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so we now understand why $K_0 B \to K_0 A$ is surjective. I recall that

$$K_0 A = K_0 \tilde{A} / K_0 \mathbb{C} = \mathrm{Ker}\{K_0 \tilde{A} \to K_0 \mathbb{C}\}$$

An element of $K_0 A$ is a difference $[P] - [Q]$