$\langle p \rangle$ W I-torsion-free cocritical R-module

$\Rightarrow \underline{Hom_R(Q, W)}$ is J torsion-free cocritical S-module

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$IM = M \Longrightarrow J(P \otimes_R M) = P \otimes_R M$

${}_I M = 0 \Longrightarrow \underline{{}_J Hom_R(Q, M)} = 0$

$$Hom_S(S/J, Hom_R(Q, M))$$

$$\parallel$$

$$Hom_R(\underline{Q/QJ}, M)$$

$IQ = QPQ = QJ$

M τ-simple when exactly ~~only~~ one τ-pure submodule $< M$.

$T_\tau(M) \subset M$ ~~no other~~ $M/T_\tau(M)$ is torsion-free.

but any other submodule $T_\tau(M) < N < M$

$\ni$ M/N not torsion-free.

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left Ore set $S \subset R$ S closed under $\times$

$\forall s \in S, r \in R$ $Rs \cap Sr \neq \phi$

$\forall s \in S, r \in R$ $\exists$ $r_1 \in R$ $s_1 \in S$ $\ni$ $r_1 s = s_1 r$

Given S a mult. system in R

$\mu(S)$-torsion $\forall m \; \exists s \ni sm = 0$.

$$0 \to M' \to M \to M'' \to 0$$

$\langle 9 \rangle$ $S = \{1-a \mid a \in A\}$ assume A

S a left Ore set.

$R/\mathfrak{a}$ torsion $\iff$ $A + \mathfrak{a} = R$

Suppose A a left ideal $S = \{1-a \mid a \in A\}$

Simple module M. What about its localization $j_* j^* M$? M simple $\Rightarrow$ ${}_I M = 0$ or M, and ${}_I M = M$ means $IM = 0$ so $j^* M = 0$.

Suppose then ${}_I M = 0$ so that $M \hookrightarrow j_* j^* M$. The cokernel $j_* j^* M / M$ is torsion.

Formula $j_* j^* M =$ purification of M in $E(M)$. ~~Point is that~~ It's clear that $j^* M$ is a simple object in the

The question is whether. There's a problem but what I would like to do is to consider the different simple non null modules and show they are preserved under the Morita equivalence

I would like to know that if M is a simple R-module + $IM \neq 0$, then

$$\operatorname{Im}\left(P \otimes_R M \longrightarrow \operatorname{Hom}_R(Q, M)\right)$$

is a simple S-module.

First question is whether M is simple in $\mathcal{M}(R)$.

$\boxed{\begin{array}{l} M \text{ uniform} \iff E(M) \text{ indecomposable injective} \\ \qquad \iff M \text{ essential extension of any nonzero submodule} \end{array}}$