α) ~~Assume~~ Go over again and generalize.

$$\mathcal{A} = Ae \otimes_B eA \qquad E = eA \in \mathcal{P}(A^r)$$

$$B = \mathrm{Hom}_{A^r}(eA, eA)$$

$$\begin{cases} E^* \otimes_B E \longrightarrow A \\ B = E \otimes_A E^* \end{cases}$$

~~Thus~~ So $E = eA$ $\quad E^* = Ae$.

$B = eAe$

we do assume that ~~eAe~~ $E = eA$ is fp as B^l module At least projective

$$\mathcal{A} = Ae \otimes_B eA$$

~~$\mathcal{A}\otimes\mathcal{A}$~~

$$eA \otimes \mathcal{A} \longrightarrow eA$$

so I want a B^l linear section of this
We are assuming eA is fp B^l. Let then $t_i \in eA$ $\quad \phi_i \in \mathrm{Hom}_{B^l}(eA, B)$ be such that ~~$\sum$~~s.

$$\langle x, \phi_i \rangle t_i = x \quad \text{for} \quad x \in eA = eA.$$

$$\Downarrow$$

$$\langle \langle x, \phi_i \rangle t_i, \phi \rangle = \langle x, \phi \rangle$$

$$\langle x, \phi_i \rangle \langle t_i, \phi \rangle = \langle x, \phi_i \langle t_i, \phi \rangle \rangle$$

$$\Longrightarrow \quad \phi = \phi_i \langle t_i, \phi \rangle.$$

β) So exactly what happens?
I consider

$$eA \otimes (Ae \otimes_B eA) \longrightarrow eA$$

$$(ea_1, a_2e, ea_3) \longrightarrow ea_1a_2ea_3$$

B^e module map surjective. Have $t_i \in eA$
lift them to $(e, e, t_i) \in eA \otimes (Ae \otimes_B eA)$.
Then consider $x \in eA$ goes to
$$x = \langle x, \phi_i \rangle t_i$$

$$\langle x, \phi_i \rangle e, e, t_i$$

Finally if $\mathcal{A} \otimes \mathcal{A} \longrightarrow \mathcal{A} = Ae \otimes_B eA$
(s, t)

$$\Psi(s,t) = (s, \langle t, \phi_i \rangle e, e, t_i)$$
$$= (s\langle t, \phi_i \rangle, e, e, t_i)$$
$$(Ae \otimes_B eA) \otimes (Ae \otimes_B eA)$$

$\phi_i \in \mathrm{Hom}_{B^e}(eA, B)$ $\quad s \in Ae, \; t \in eA$.
Furthermore check linear.

$$(s_1, t_1)\Psi(s,t) = (s_1, t_1)(s\langle t, \phi_i \rangle, e, e, t_i)$$
$$= (s_1 t_1 s \langle t, \phi_i \rangle, e, e, t_i)$$

$$\Psi((s_1, t_1)(s,t)) = \Psi(s_1 t_1 s, t) = (s_1 t_1 s \langle t, \phi_i \rangle, e, t_i)$$

8) So it works fine, but I don't understand the significance. What would you like?

You are using something like (faithfully flat) descent. ~~Descent~~

Question: What $\mathcal{A} = Ae \otimes_B eA$ modules are of the form $Ae \otimes_B N$? ...

Is the category of $\mathcal{A}$-modules equivalent to the category of B-modules. Answer is no because one can make $\mathcal{A}$ act trivially.

T_x creation

T_y^* annihilation

In the V Jones situation you get even algebras from $T_B(A) = B \oplus A \oplus A^{\otimes 2} \oplus \cdots$ and the odd algs from $T_A(A \otimes_B A)$.

$$A = \begin{pmatrix} eAe & eAe^\perp \\ 0 & e^\perp Ae^\perp \end{pmatrix}$$

automatically satisfies condition that ~~...~~ Ae is fp as eAe^n module