[b] (What about R such that I is an ideal in R.

$$\mathrm{mod}(R) \longrightarrow \mathcal{M}(R,I) \simeq \mathcal{M}(\tilde{I}, I)$$

$$R^{op} \longrightarrow \underbrace{\mathrm{Hom}_{\mathcal{M}}(I,I)}_{\text{plays role of multiplier alg.}} = \varinjlim_{\text{right}^n} \mathrm{Hom}_R(I^{\otimes n}, R)$$

~~So what~~? The question I have is whether I flat, ~~…~~ fin pres fin gen, proj over R has any intrinsic meaning. Thus I know for I fin pres that

$$j_* j^* M = \varinjlim \mathrm{Hom}_R(I^{\otimes n}, M)$$

Idea: Suppose $I = \sum_{i=1}^{n} a_i R$, Then for any module M, ~~such that~~ $M = IM \Longrightarrow M = \sum_i a_i M$

$$R \longrightarrow R^n \longrightarrow R^{n^2}$$

Specifically I have $P = R^n \overset{\eta}{\twoheadrightarrow} I < R$ P free right module $/R$

$$m = a_i m_i = a_i a_j m_{ij} = a_i a_j a_k m_{ijk}$$

$$M \xleftarrow{\eta \otimes 1} P \otimes_R M \xleftarrow{1 \otimes \eta} P \otimes_R P \otimes_R M \longleftarrow \quad ?$$

$$R \xleftarrow{\eta} P \xleftarrow{1 \otimes \eta} P \otimes_R P \xleftarrow{1 \otimes 1 \otimes \eta} \cdots\cdots$$

P not a bimodule

Try again $M = IM = \sum a_i M$

$$m = a_i m_i, \quad m_i = a_j m_{ij}, \quad m_{ij} = a_k m_{ijk}$$

[c]

$$\begin{tikzcd} R \arrow[r, "\cdot(a_i)"] \arrow[d, "\cdot m"] & R^n \arrow[r, "1\otimes(a_j)"] \arrow[d, "(\cdot m_i)"] & R^n \otimes_R R^n \arrow[r] \arrow[d, "(m_{ij})"] & {} \\ M \arrow[r, equal] & M \arrow[r, equal] & M \end{tikzcd}$$

$R \longrightarrow L$

$$\begin{tikzcd} R \arrow[r, "\cdot(a_i)"] \arrow[d, "\cdot m"] & R^n \arrow[r, "\cdot(a'_{ij})"] \arrow[d, "\cdot(m_i)"] & R^p \arrow[r, "\cdot(a''_{jk})"] \arrow[d, "(\cdot m'_j)"] & R^q \arrow[d, "\cdot(m''_k)"] \\ M \arrow[r, equal] & M \arrow[r, equal] & M \arrow[r, equal] & M \end{tikzcd}$$

Maybe it's $R \xrightarrow{\varphi} R \otimes V$ $\quad r \overset{\varphi}{\longmapsto} ra_i \otimes v_i$

Then

$$\begin{tikzcd} R \arrow[r, "\varphi"] \arrow[dd] & R \otimes V \arrow[r, "\varphi\otimes 1"] \arrow[dd] & R \otimes V \otimes V \arrow[r] \arrow[dd] & {} \\ & & & \\ M \arrow[r, equal] & M \arrow[r, equal] & M \end{tikzcd}$$

$r \longmapsto ra_j \otimes v_j$

$r_j \otimes v_j \longmapsto r_j a_i \otimes v_i \otimes v_j$

$r_j a_i \otimes v_i \otimes v_j$

$rm \qquad r_j m_j \qquad r_j a_i m_{ij}$

This gives a firm flat module which generates. If $R = T(V)$ then the V above should be V^*.

$r_{ij} \otimes v_i^* \otimes v_j^*$.

$R \otimes_{T(V)} \mathcal{O}$

$r_{ij}\, m$

[d] Idea. Suppose $V \otimes R \twoheadrightarrow I$

Then $V \otimes M \twoheadrightarrow IM$ so that when $M = IM$ we have

$$V \otimes M \twoheadrightarrow M$$

If we are over a field, then we can find a lifting as vector spaces. So if we choose $V = \bigoplus k a_i$, then we find

$$V \otimes M \hookleftarrow M$$
$$a_i \otimes \varphi_i(m)$$

such that

$$a_i \varphi_i(m) = m.$$

Thus M is a module over the ring generated by R and elements $\varphi_1, \dots, \varphi_n$ $\ni$ $\sum a_i \varphi_i = 1$.

Anyway

$k[x,y]/(xy-1)$
$=$ Laurent polys.

$\cancel{R \twoheadrightarrow R \otimes X}$

$$S(V) \hookrightarrow S(V) \otimes V^* \hookrightarrow S(V) \otimes V^* \otimes V^*$$

Apparently there's a limit

$$S(V) \hookrightarrow S(V) \otimes V^* \hookrightarrow S(V) \otimes S^2(V^*) \hookrightarrow$$

generated by x_i y_i such that the x_i commute y_i commute and $\sum x_i y_i = 1$.