(E1) ~~Consider~~ $E = A * S \qquad S = k[F]$

$$\Omega^1 E = E\,dA\,E \oplus E\,dF\,E$$

$$(E\,dA\,E)_{\natural s} = E \otimes_A \Omega^1 A \otimes_A \simeq \underbrace{(Q \otimes_A \Omega^1 A \otimes_A)}_{\varepsilon\text{ even}} \oplus \underbrace{(FQ \otimes_A \Omega^1 A \otimes_A)}_{\varepsilon\text{ odd}}$$

So a supertrace on $E\,dA\,E$ is equiv to

$$T^+(\theta q^{n-1} d\theta) \qquad n = 1, 2, 4, 6, \dots$$

$$T^-(F\theta^\gamma q^{n-1} d\theta) \qquad n = 1, 3, 5, \dots \qquad Q^+[F,dF]_{s+} + Q^-[F,dF]_-$$

$$(E\,dF\,E)_{\natural s} = E\,dF/[F, E\,dF]_s = Q^F dF/[F, Q^- dF]_s \oplus Q\,dF/[F, QF\,dF]_s$$

$$= Q^+ F\,dF \oplus Q^- dF$$

A supertrace T' on $E\,dF\,E$ is equiv to

$$T'^+(\rho q^{2n} F\,dF) \qquad n = 0, 1, 2, \dots$$

$$T'^-(\rho q^{2n+1} dF)$$

$$\boxed{\text{Step 1: Description of s.traces on } \Omega^1 E \text{ by cochains}}$$

Next step is to compute b^t, d^t

$$(\tau^+ b_s)(\rho q^{2n} F\,dF) = 2\tau^+(\rho q^{2n}) \qquad \tau^+ \text{ any lin. fn. on } Q.$$

$$(\tau^- b_s)(\rho q^{2n+1} dF) = \tau^-(\rho q^{2n+1} F - F\rho q^{2n+1})$$

$$= -2\tau^-(F\rho q^{2n+1})$$

These are the easy pieces. Harder are

$$(\tau^+ b_s)(\theta q^{2n-1} d\theta) = -b\tau^+(\rho q^{2n-1}) + 2s\tau^+(\rho q^{2n+1})$$

$$(\tau^- b_s)(F\theta^\gamma q^{2n} d\theta) = b\tau^-(F\rho q^{2n}) - 2s\tau^-(F\rho q^{2n+2})$$

These formulas are missing the contributions from τ^+ on Q^+, τ^- on FQ^*

E2 Let's work out the correct fmlas.

First we have

$$b\tau^+(\rho q^{2n-1}) = \tau^+(-\rho q^{2n-1}\rho) + \lambda\tau^+(\rho^2 q^{2n-1}) + \tau^+(q^{2n+1}) + \lambda\tau^+(q^{2n+1})$$

$$b\tau^-(F\rho q^{2n}) = \tau^-(F\rho q^{2n}\rho) + \lambda\tau(F\rho^2 q^{2n}) + \tau^-(Fq^{2n+2}) + \lambda\tau(Fq^{2n+2})$$

Next $\tau^+ b(\theta q^{2n-1}d\theta) = \tau^+(\theta q^{2n-1}\theta) - \lambda\tau^+(\theta^2 q^{2n-1})$

Part not handled before is

$$Z = \tau^+(\rho q^{2n} + q^{2n}\rho) - \lambda\tau^+(\rho q^{2n} + q\rho q^{2n-1})$$

and this should be expressible using

$$(\tau^+ b_s)(\rho q^{2n}) = 2\tau^+(\rho q^{2n})$$

$$b\tau^+(q^{2n}) = \tau^+(\rho q^{2n} - q^{2n}\rho) + \lambda\tau((\rho q + q\rho)q^{2n-1})$$

$$Z + b\tau^+(q^{2n}) = 2\tau^+(\rho q^{2n})$$

$$\therefore \boxed{Z = (2 - bs)\,\tau^+(\rho q^{2n})}$$

Next $\tau^- b(F\theta^{\gamma} q^{2n}d\theta) = \tau^-(F\theta^{\gamma} q^{2n}\theta) + \lambda\tau^-(F\theta^{\gamma 2} q^{2n})$

Part not handled before is

$$Z = \tau^-(F\rho q^{2n+1} - Fq^{2n+1}\rho) - \lambda\tau^-(F(\rho q^{2n+1} + q^{2n+1}\rho))$$

$$b\tau^-(Fq^{2n+1}) = \tau^- F(\rho q^{2n+1}) + \tau^- F(q^{2n+1}\rho) + \lambda\tau^- F((\rho q + q\rho)q^{2n})$$

$$\therefore Z + b\tau^-(Fq^{2n+1}) = 2\tau^-(F\rho q^{2n+1})$$

$$\boxed{Z = (2 - bs)\,\tau^-(F\rho q^{2n+1})}$$