

Back to  $R\tilde{A} \rightarrow R\tilde{A}$ .

$$\rho(A)^3 = A \oplus \Omega^2 A$$

$$a_1 \circ a_2 \circ a_3 = a_1(a_2 a_3 - da_2 da_3) - da_1 d(a_2 a_3)$$

~~Given  $R$  unital~~

Consider a DG algebra of the form  $R = R_0 \oplus R_1$  with  $d: R_1 \rightarrow R_0$ . Then  $R_1$  is ~~is~~ a non-unital alg. ~~unital~~ Why: let  $A = R_1$ , and define

~~$a_1 a_2 = (da_1) a_2$~~

Use letters  $a_i$  for elts of  $R_1$  and  $x$ 's for  $R_0$

Observe  $a_1 a_2 = 0 \implies (da_1) a_2 - a_1 da_2 = 0$ ,

so we obtain a product on  $A$  defined by

$$a_1 * a_2 = (da_1) a_2 = a_1 da_2.$$

This is associative

$$a_1 * (a_2 * a_3) = (da_1)(a_2 da_3)$$

$$(a_1 * a_2) * a_3 = ((da_1) a_2) da_3$$

Next point is that each element  $x \in R_0$  determine a multiplier on  $R_1$  i.e. a pair of maps  $\mu_L, \mu_R$  satisfying some conditions.

~~March 6~~

~~So let's review something before we come far  
 One more it appears that some properties  
 has been made.~~

~~$T(A) = \mathbb{C} \oplus A \oplus A \otimes^2 \mathbb{C} \dots$~~

~~$0 \neq \xi \in A$  given~~

~~$d_p, d_r$  induces~~

~~$d$  on  $T(A)$~~

~~tensor algebra~~

~~Ex~~

~~$$\mu_2(a) = xa$$

$$\mu_1(a) = ax$$~~

$$x_2(a) = xa$$

$$x_1(a) = ax$$

$$(xa_1) * a_2 \stackrel{?}{=} x(a_1 * a_2)$$

$$\parallel$$

$$(xa_1) da_2 = x(a_1 da_2)$$

~~$$(a_1 x) a_2 \stackrel{?}{=} a_1 (x a_2)$$~~

clear

~~$$(a_1 * a_2) x \stackrel{?}{=} a_1 * (a_2 x)$$~~

$$\parallel$$

$$(da_1 a_2) x = (da_1)(a_2 x)$$

Finally given  $A$  a nonunital algebra, let  $M(A)$  be the space of multipliers on  $A$ , i.e. a mult.  $\mu$  is a pair of ~~mult~~ operators on  $A$  denoted  $a \mapsto \mu(a)$  and  $a \mapsto (a)\mu$  such that

~~$$\mu(a_1 a_2) = (\mu(a_1)) a_2$$

$$((a_1)\mu) a_2 = a_1 (\mu(a_2))$$

$$(a_1 a_2) \mu = a_1 ((a_2)\mu)$$~~

$$\mu(a_1 * a_2) = \mu(a_1) * a_2$$

$$(a_1)\mu * a_2 = a_1 * \mu(a_2)$$

$$(a_1 * a_2)\mu = a_1 * (a_2)\mu$$

Check multipliers form an alg with identity

$$(\mu \cdot \nu)(a) = \mu(\nu(a))$$

$$(a)(\mu \cdot \nu) = ((a)\mu)\nu$$

Check

$$(\mu \cdot \nu)(a_1 * a_2) = \mu(\nu(a_1 * a_2))$$

$$= \mu(\nu(a_1) * a_2) = \mu(\nu(a_1)) * a_2$$

$$= ((\mu \cdot \nu)(a_1)) * a_2$$

$$((a_1)(\mu \cdot \nu)) * a_2 = (((a_1)\mu)\nu) * a_2 = ((a_1)\mu) * (\nu(a_2)) = a_1 * (\mu(\nu(a_2)))$$