Back to $R\tilde{A} \longrightarrow R_{\tilde{\mathbb{C}}}\tilde{A}$.

$$\rho(A)^3 = A \oplus \Omega^2 A$$

$$a_1 \circ a_2 \circ a_3 = a_1(a_2a_3 - da_2\, da_3) - da_1\, d(a_2a_3)$$

Consider a DG algebra of the form $R = R_0 \oplus R_1$ with $d: R_1 \longrightarrow R_0$. Then R_1 is a non-unital alg. ~~Why: Set $a = R_1$, and define~~

Use letters a_i for elts of R_1 and x's for R_0

Observe $a_1 a_2 = 0 \implies (da_1)a_2 - a_1 da_2 = 0$, so we obtain a product on a defined by

$$a_1 * a_2 = (da_1)a_2 = a_1 da_2 .$$

This is associative

$$a_1 * (a_2 * a_3) = (da_1)(a_2 da_3)$$

$$(a_1 * a_2) * a_3 = ((da_1)a_2)da_3$$

Next point is that each element $x \in R_0$ determine a multiplier on R_1 i.e. a pair of maps μ_l, μ_r satisfying some conditions.

~~$\mu_\ell(a) = xa$~~
~~$\mu_r(a) = ax$~~

$x_\ell(a) = xa$
$x_r(a) = ax$

$$(xa_1) * a_2 \stackrel{?}{=} x(a_1 * a_2)$$
$$\| \qquad\qquad \|$$
$$(xa_1)da_2 = x(a_1 da_2)$$

$$(a_1 x)a_2 \stackrel{?}{=} a_1(xa_2) \qquad \text{clear}$$

$$(a_1 * a_2)x \stackrel{?}{=} a_1 * (a_2 x)$$
$$\| \qquad\qquad \|$$
$$(da_1 a_2)x = (da_1)(a_2 x) .$$

Finally given $\mathcal{A}$ a nonunital algebra, let $M(\mathcal{A})$ be the space of multipliers on $\mathcal{A}$, i.e. a mult. μ is a pair of ~~map~~ operators on $\mathcal{A}$ denoted $a \longmapsto \mu(a)$ and $a \longmapsto (a)\mu$ such that

~~$\mu(a_1 a_2) = (\mu(a_1))a_2$~~
~~$((a_1)\mu)a_2 = a_1(\mu(a_2))$~~
~~$(a_1 a_2)\mu = a_1((a_2)\mu)$~~

$$\mu(a_1 * a_2) = \mu(a_1) * a_2$$
$$(a_1)\mu * a_2 = a_1 * \mu(a_2)$$
$$(a_1 * a_2)\mu = a_1 * (a_2)\mu$$

Check multipliers form an alg with identity

$$(\mu \cdot \nu)(a) = \mu(\nu(a))$$
$$(a)(\mu \cdot \nu) = ((a)\mu)\nu$$

Check

$$(\mu \cdot \nu)(a_1 * a_2) = \mu(\nu(a_1 * a_2))$$
$$= \mu(\nu(a_1) * a_2) = \mu(\nu(a_1)) * a_2$$
$$= ((\mu \cdot \nu)(a_1)) * a_2$$

$$((a_1)(\mu \cdot \nu)) * a_2 = (((a_1)\mu)\nu) * a_2 = ((a_1)\mu) * (\nu(a_2)) = a_1 * (\mu(\nu(a_2)))$$