Schmidt's game, linear forms and fractals

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January 2, 2008

Abstract

In his 1966 paper entitled *On badly approximable numbers and certain games*, W. M. Schmidt first introduced what is now referred to as "Schmidt's game". The game was used to prove in the above mentioned paper that the set of badly approximable vectors in the N dimensional Euclidean space is winning in this game's sense, and as a direct consequence of this fact has a full Hausdorff dimension, i.e., N.

Schmidt was able later to generalize this result to badly approximable linear forms, proving that the set of badly approximable $M \times N$ matrices is winning in $\mathbb{R}^{MN}$ and thus having a full dimension.

Using definitions and results obtained by D. Kleinbock, E. Lindenstrauss and B. Weiss, in the paper entitled *On fractal measures and Diophantine approximation*, we generalize Schmidt's results to self similar fractals. Specifically, if $\{\phi_1, ..., \phi_k)$ is a finite irreducible family of contracting similarity maps of $\mathbb{R}^{MN}$ satisfying the open set condition with $\mathcal{K}$ its attractor, then

$$\dim(\mathbf{BA} \cap \mathcal{K}) = \dim \mathcal{K}.$$

where **BA** the set of badly approximable $M \times N$ matrices.