4-d GUTs and their Experimental Signatures

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Outline

- Motivation/"Evidences" for GUTs
- Model building challenges
- Predictive $SO(10)$ model
- Experimental tests
- Conclusions

- KB, J. Pati, Z. Tavartkiladze, 2008
- KB, J. Pati, P. Rastogi, 2005
- KB, J. Pati, F. Wilczek, 2000
- S. Barr, S. Raby, 1997
- KB, S. Barr (1993)
- S. Dimopoulos, F. Wilczek, 1981
- H. Georgi, 1975
- H. Fritzsch, P. Minkowski, 1975
- H. Georgi, S. Glashow, 1974
- J. Pati, A. Salam, 1974

Motivations/Evidence favoring GUTs

- Electric charge quantization
  - $Q_p = -Q_e$ to better than 1 part in 10^{21}
- Miraculous cancellation of anomalies
- Quantum numbers of quarks and leptons
- Existence of ν_R and thus neutrino mass via seesaw
- Unification of gauge couplings with low energy SUSY
- $b - \tau$ unification
- Baryon asymmetry of the universe via leptogenesis

Structure of matter multiplets

$$Q = \begin{pmatrix} u_1 & u_2 & u_3 \\ d_1 & d_2 & d_3 \end{pmatrix} \sim (3, 2, \frac{1}{6})$$

$$u^c = (u_1^c \quad u_2^c \quad u_3^c) \sim (\overline{3}, 1, \frac{-2}{3})$$

$$d^c = (d_1^c \quad d_2^c \quad d_3^c) \sim (\overline{3}, 1, \frac{1}{3})$$

$$L = \begin{pmatrix} \nu \\ e^- \end{pmatrix} \sim (1, 2, \frac{-1}{2})$$

$$e^c \qquad \sim (1, 1, +1)$$

$$\nu^c \qquad \sim (1, 1, 0)$$

u_1	:	$\mid\uparrow\downarrow\uparrow\uparrow\downarrow>$
u_2	:	$\mid\uparrow\downarrow\uparrow\downarrow\uparrow>$
u_3	:	$\mid\uparrow\downarrow\downarrow\uparrow\uparrow>$
d_1	:	$\mid\downarrow\uparrow\uparrow\uparrow\downarrow>$
d_2	:	$\mid\downarrow\uparrow\uparrow\downarrow\uparrow>$
d_3	:	$\mid\downarrow\uparrow\downarrow\uparrow\uparrow>$
u_1^c	:	$\mid\downarrow\downarrow\uparrow\downarrow\downarrow>$
u_2^c	:	$\mid\downarrow\downarrow\downarrow\uparrow\downarrow>$
u_3^c	:	$\mid\downarrow\downarrow\downarrow\downarrow\uparrow>$
d_1^c	:	$\mid\uparrow\uparrow\uparrow\downarrow\downarrow>$
d_2^c	:	$\mid\uparrow\uparrow\downarrow\uparrow\downarrow>$
d_3^c	:	$\mid\uparrow\uparrow\downarrow\downarrow\uparrow>$
ν	:	$\mid\uparrow\downarrow\downarrow\downarrow\downarrow>$
e	:	$\mid\downarrow\uparrow\downarrow\downarrow\downarrow>$
e^c	:	$\mid\downarrow\downarrow\uparrow\uparrow\uparrow>$
ν^c	:	$\mid\uparrow\uparrow\uparrow\uparrow\uparrow>$

Standard Model

SO(10)

Miraculous cancellation of anomalies

- $SU(3)_C^2 \times U(1)_Y$: $\frac{1}{2}\left[2 \times (\frac{1}{6}) + 1 \times (\frac{-2}{3}) + 1 \times (\frac{1}{3})\right] = 0$

- $SU(2)_L^2 \times U(1)_Y$: $\frac{1}{2}\left[3 \times (\frac{1}{6}) + 1 \times (\frac{-1}{2})\right] = 0$

- $(\text{gravity})^2 \times U(1)_Y$:
  $\left[3 \times 2 \times (\frac{1}{6}) + 3 \times (\frac{-2}{3}) + 3 \times (\frac{1}{3}) + 2 \times (\frac{-1}{2}) + 1 \times 1\right] = 0$

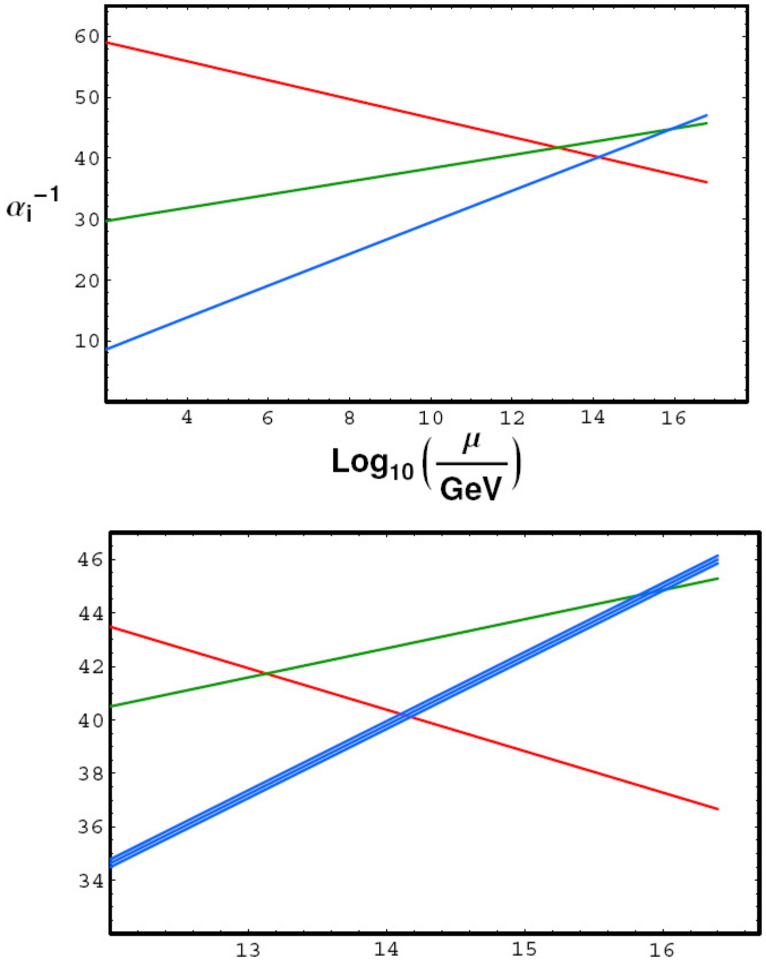
- $U(1)_Y^3$:

$$\left[3 \times 2 \times (\tfrac{1}{6})^3 + 3 \times (\tfrac{-2}{3})^3 + 3 \times (\tfrac{1}{3})^3 + 2 \times (\tfrac{-1}{2})^3 + 1 \times (1)^3\right] = 0$$

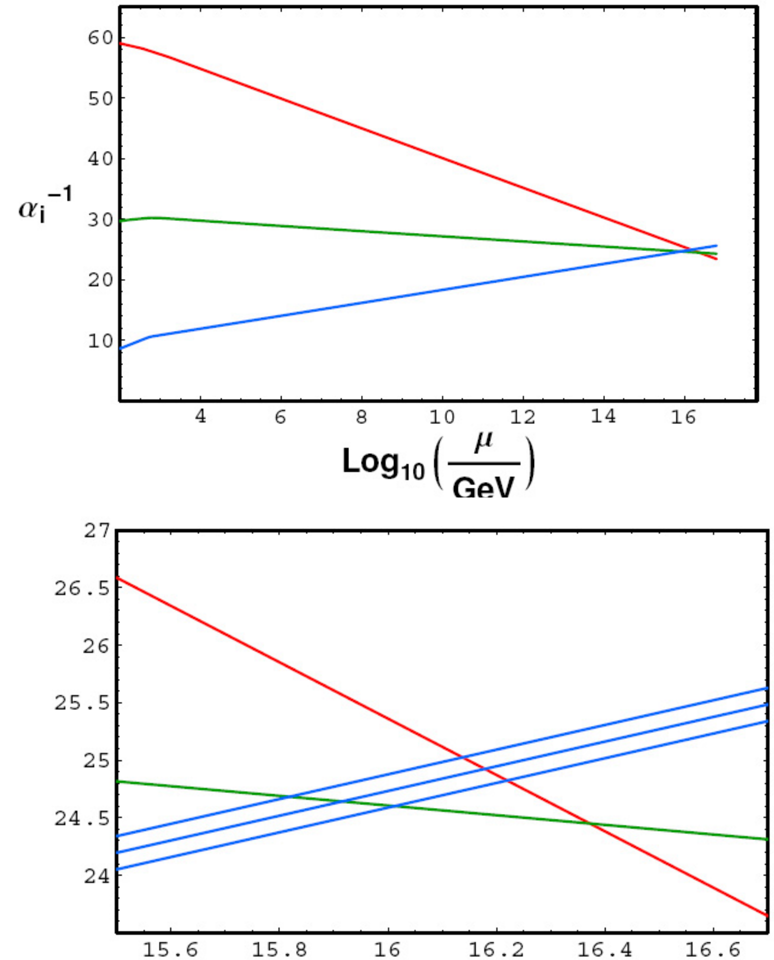
Suggests $SO(10)$ embedding

$SO(10)$ is automatically anomaly free

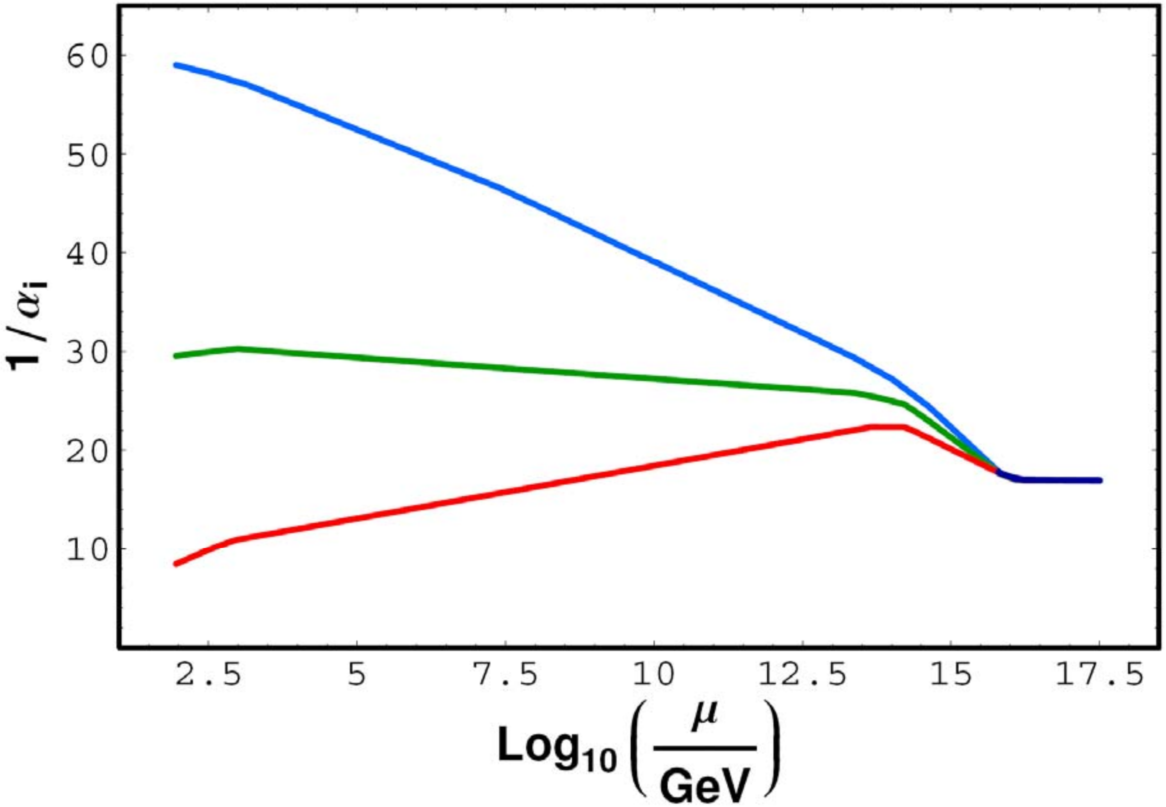
Evolution of Gauge Couplings



Standard Model



MSSM



Gauge coupling evolution in an explicit SO(10) model

Neutrino masses and the scale of new physics

$$\mathcal{L} = \frac{LLHH}{M_R}$$

$\langle H \rangle \sim 246 \text{ GeV}$ and $m_{\nu_3} \sim 0.05 \text{ eV}$

from atmospheric neutrino oscillation data

$$\Rightarrow \quad M_R \sim 10^{14} - 10^{15}\text{GeV}$$

Very Close to the GUT scale

$$M_R \sim \frac{M_{\text{GUT}}^2}{M_{\text{Pl}}}$$

Leptogenesis via ν_R decay explains cosmological baryon asymmetry

Finding order in fermion mass spectrum

$$\begin{aligned} m_t &= 1.0 \\ m_c &= 3.6 \times 10^{-3} \\ m_u &= 1.3 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} m_b &= 1.67 \times 10^{-2} \\ m_s &= 3.1 \times 10^{-4} \\ m_d &= 2.3 \times 10^{-5} \end{aligned}$$

$$\begin{aligned} m_\tau &= 1.0 \times 10^{-2} \\ m_\mu &= 6.2 \times 10^{-4} \\ m_e &= 3.0 \times 10^{-6} \end{aligned}$$

$$\begin{aligned} m_3 &= 2.9 \times 10^{-13} \\ m_2 &= 5.2 \times 10^{-14} \\ m_1 &= < m_2 \end{aligned}$$

$$V_q = \begin{pmatrix} 0.976 & 0.22 & 0.004 \\ -0.22 & 0.98 & 0.04 \\ 0.007 & -0.04 & 1 \end{pmatrix}$$

$$U_\ell = \begin{pmatrix} 0.85 & -0.54 & < 0.2 \\ 0.33 & 0.62 & -0.72 \\ -0.40 & -0.59 & -0.70 \end{pmatrix}$$

$$\mathrm{Im}\left(\frac{V_{ub}V_{cs}}{V_{us}V_{cb}}\right) = 0.33$$

Minimal SU(5) GUT

Georgi, Glashow (1974)

Matter multiplets: $\{\mathbf{10}+\bar{\mathbf{5}}+\mathbf{1}\}$

$$\mathbf{10}:\begin{pmatrix} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ -u_1 & -u_2 & -u_3 & 0 & e^c \\ -d_1 & -d_2 & -d_3 & -e^c & 0 \end{pmatrix}$$

$$\bar{\mathbf{5}}:(d_1^c, d_2^c, d_3^c, e, -\nu_e)$$

$$\mathbf{1}:\nu^c$$

Higgs: $\mathbf{24}_H, \quad \{\mathbf{5}_H, \quad \bar{\mathbf{5}}_H\}$ ⇨ Contain color triplets $\{H_C, \ \bar{H}_C\}$

Yukawa Couplings $Y_u^{ij}\mathbf{10}_i\mathbf{10}_j\mathbf{5}_H + Y_d^{ij}\mathbf{10}_i\bar{\mathbf{5}}_j\bar{\mathbf{5}}_H$

$$M_\ell = M_d^T \Rightarrow m_b = m_\tau, m_s = m_\mu, m_d = m_e$$

MSSM Higgs doublets have color triplet partners in GUTs.

$$H(1,2,1/2) \oplus H_c(3,1,-1/3) = \mathbf{5} \text{ of } SU(5)$$

$$\bar{H}(1,2,-1/2) \oplus \bar{H}_c(\bar{3},1,1/3) = \bar{\mathbf{5}}$$

$H, \ \bar{H}$ **must remain light**

$H_c, \bar{H}_c$ **must have GUT scale mass to prevent rapid proton decay**

Doublet-triplet splitting

Even if color triplets have GUT scale mass, d=5 proton decay is problematic.

Symmetry breaking in SUSY SU(5)

Doublet-triplet splitting:

$$W_{D-T} = \bar{\mathbf{5}}_H(\lambda \mathbf{24}_H + M)\mathbf{5}_H$$

$$< 24_H >= \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -3/2 & 0 \\ 0 & 0 & 0 & 0 & -3/2 \end{pmatrix} V$$

FINE-TUNED TO $O(M_W)$

$$M_{H_c} = \lambda V + M \sim O(M_{GUT}) \quad M_H = -\frac{3}{2}\lambda V + M$$

The GOOD

(1) Predicts unification of couplings

(2) Uses economic Higgs sector

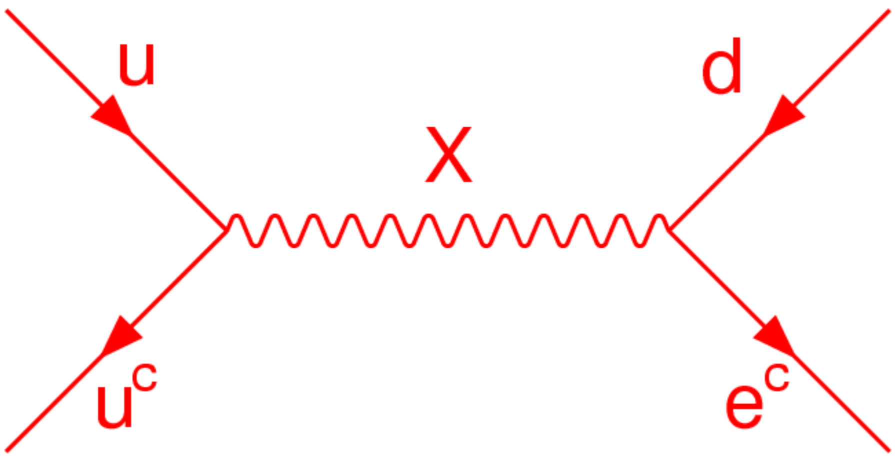
The BAD

(1) Unnatural fine tuning

(2) Large proton decay rate

Nucleon decay in SUSY GUTs

Gauge boson exchange:



$$p \to e^+\pi^0,\ \tau_p^{-1} \approx \left[\frac{g^2}{M_X^2}\right]^2 m_p^5 \approx [10^{35\pm1} yr]^{-1}$$

SUSY SU(5) Prediction for proton decay

$$\frac{1}{\Gamma(p \to e^+\pi^0)} = (2.0 \times 10^{35}\ \mathrm{yr}) \times \left(\frac{\alpha_H}{0.01\ \mathrm{GeV}^3}\right)^{-2} \left(\frac{\alpha_G}{1/25}\right)^{-2} \left(\frac{A_R}{2.5}\right)^{-2} \left(\frac{M_X}{10^{16}\ \mathrm{GeV}}\right)^4$$

$$(-2\alpha_3^{-1} - 3\alpha_2^{-1} + 3\alpha_Y^{-1})(M_Z) = \frac{1}{2\pi}\left\{36\ \ln\left(\frac{M_X}{M_Z}\left(\frac{M_\Sigma}{M_X}\right)^{1/3}\right) + 8\ \ln\left(\frac{M_{\mathrm{SUSY}}}{M_Z}\right)\right\}$$

- Color octect Higgs boson mass is near GUT scale, but it is not precisely known.

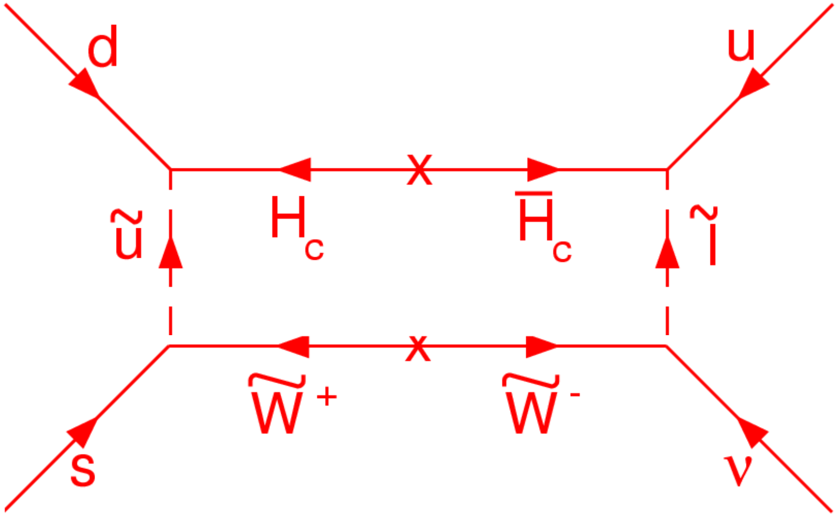
$$\frac{M_\Sigma}{M_X} \leq 1.8$$ (Perturbation theory)

Hisano, Murayama, Yanagida (1993)

Higgsino Exchange:

Sakai, Yanagida (1982)

Weinberg (1982)



$$p \to \bar{\nu} K^{+}$$

$$\tau_p^{-1} \approx [\frac{f^2}{M_{H_c} M_{SUSY}}]^2 (\frac{\alpha}{4\pi})^2 m_p^5 \approx [10^{28} - 10^{32} yr]^{-1}$$

SUSY SO(10) Grand Unification

- ☆ **Quarks and leptons ~ {*16$_i$*}**
- ☆ **Contains ν_R and Seesaw mechanism**

Higgs: $\{45_H + 10_H + 16_H + \overline{16}_H\}$

$$\mathcal{L}_{\text{Yukawa}} = f_{ij}\mathbf{16}_i\mathbf{16}_j\mathbf{10}_H + h_{ij}16_i16_j\overline{16}_H\overline{16}_H/M_{Pl}$$

$$\Rightarrow \quad m^D_{\nu_\tau} \simeq m_t;\ m^M_{\nu_{\tau_R}} \simeq h_{33}\frac{M^2_{GUT}}{M_{Pl}}$$

$$m_{\nu_\tau} = \frac{m_t^2}{m_{\nu_{\tau_R}}} \simeq 0.05\ \text{eV},\ h_{33} \sim 1$$

Fits the atmospheric neutrino data well

- Small Higgs rep ⇨ small threshold corrections for gauge couplings
- R-parity not automatic (needs a Z_2 symmetry)

Symmetry breaking in SUSY SO(10)

$$W_{D-T} = \lambda(\bar{10}_H 45_H 10'_H) + \ldots$$

$$< 45_H >= \begin{pmatrix} a & 0 & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \otimes i\tau_2 \quad \propto B-L$$

⇨ B-L VEV gives mass to triplets only (**DIMOPOULOS-WILCZEK**)

⇨ If 10_H only couples to fermions, no d=5 proton decay

⇨ Doublets from 10_H and $10'_H$ light

4 doublets, unification upset

⬇

Add mass term for *10'*$_H$

$$W_{D-T} = \lambda(\bar{10}_H 45_H 10'_H) + M 10'_H 10'_H$$

Fermion masses and mixings

KB, Pati, Tavartkiladze

$$U = \begin{pmatrix} 0 & \epsilon' & 0 \\ -\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{pmatrix} m_U, \quad D = \begin{pmatrix} 0 & \epsilon' + \eta' & 0 \\ -\epsilon' - \eta' & 0 & \sigma + \sigma \\ 0 & \bar{\epsilon} + \sigma & 1 \end{pmatrix} m_D,$$

$$N = \begin{pmatrix} 0 & -3\epsilon' & 0 \\ 3\epsilon' & 0 & \sigma \\ 0 & \sigma & 1 \end{pmatrix} m_U, \quad L = \begin{pmatrix} 0 & -3\epsilon' - \eta' & 0 \\ 3\epsilon' + \eta' & 0 & 3\bar{\epsilon} + \sigma \\ 0 & 3\epsilon + \sigma & 1 \end{pmatrix} m_D$$

$$M_\nu^R = \begin{pmatrix} x & 0 & z \\ 0 & x & y \\ z & y & 1 \end{pmatrix} M_R$$

$$\begin{aligned} "1" &: \ 16_3 16_3 10_H \\ "\sigma" &: \ \vec{16}.\vec{X} 16_3 10_H / M \\ "\epsilon" &: \ \vec{16}.\vec{Y} 16_3 C.C' 45_H / M^3 \\ "\eta'" &: \ \vec{16}.\vec{16} 10_H 45_H S / M^2 \end{aligned}$$

$\vec{16} \sim 2$ of Q_4

$\vec{X}, \vec{Y} \sim$ flavons

$$\langle 45_H \rangle \propto (B - L)$$

Postdictions

$$\begin{aligned} m_b^0 &\approx m_\tau^0 \\ m_d^0 m_s^0 m_b^0 &= m_e^0 m_\mu^0 m_\tau^0 \\ m_s(1\ GeV) &\approx 116\ MeV \\ V_{cb} &\approx 0.043 \end{aligned}$$

$$\begin{aligned} \sin^2 2\theta_{\mu\tau} &= (0.99, 0.96,\ 0.91,\ 0.86,\ 0.83\ , 0.81) \\ \frac{m_{\nu_\mu}}{m_{\nu_\tau}} &= (1/6, 1/10,\ 1/15,\ 1/20,\ 1/25,\ 1/30) \end{aligned}$$

$$\begin{aligned} m_d(1\ GeV) &\approx 8\ MeV \\ \theta_c &\approx |\sqrt{m_d/m_s} - e^{i\phi}\sqrt{m_u/m_c}| \\ \left|\frac{V_{us}}{V_{cs}}\right| &\approx \sqrt{\frac{m_u}{m_c}} \approx 0.07 \end{aligned}$$

Realistic SO(10) model without fine-tuning

Quarks and leptons: $\{16_i\}$

Economic Higgs system

$\{45_H + 16_H + \overline{16}_H\} \Rightarrow$ breaks symmetry to SM

$\{10_H + 10'_H\} \Rightarrow$ EW symmetry breaking

$\{16'_H + \overline{16}'_H\} \Rightarrow$ avoids pseudoGoldstones

$\{2 \text{ singlets} : S, Z\} \Rightarrow$ Fixes VEVs

Small Higgs representation $\Rightarrow$ Small threshold effects for gauge couplings

A Z_2-assisted anomalous $\mathcal{U}(1)$ symmetry guarantees stability

Issues to be addressed with DW mechanism

- Can the VEV pattern for 45_H be realized?
- Is the VEV structure stable?
- Are there flat directions?
- Are there pseudo-Goldstone bosons?
- Are threshold corrections small?
- Is $d = 5$ proton decay consistent with data?

We now have a complete model where all issues are successfully addressed

Complete SO(10) Model

Z_2-assisted anomalous $\mathcal{U}(1)$

	$A(45)$	$H(10)$	$H'(10)$	$C(16)$	$\bar{C}(\overline{16})$	Z	S	$C'(16)$	$\bar{C}'(\overline{16})$	16_i
$\mathcal{U}(1)$	0	1	-1	$\frac{k+4}{2k}$	$-\frac{1}{2}$	$\frac{2}{k}$	$\frac{2}{k}$	$\frac{k-4}{2k}$	$-\frac{k+8}{2k}$	$-\frac{1}{2}+a_i$
Z_2	$-$	$+$	$-$	$+$	$+$	$-$	$+$	$+$	$+$	P_i

Superpotential:

$$W(A) = M_A \mathrm{tr} A^2 + \frac{\lambda_A}{M_*}\left(\mathrm{tr} A^2\right)^2 + \frac{\lambda'_A}{M_*}\mathrm{tr} A^4$$

$$W(A,C,C') = C\left(\frac{a_1}{M_*}ZA + \frac{b_1}{M_*}C\bar{C} + c_1 S\right)\bar{C}' + C'\left(\frac{a_2}{M_*}ZA + \frac{b_2}{M_*}C\bar{C} + c_2 S\right)\bar{C}$$

$$W(DT) = \lambda_1 HAH' + \lambda_{H'}\frac{S^k}{M_*^{k-1}}(H')^2 + \lambda_2 H\bar{C}\bar{C}$$

Fixing the VEVs

$$\langle A\rangle = i\sigma_2 \otimes \mathrm{Diag}\,(a,\ a,\ a,\ 0,\ 0)$$
$$\langle C\rangle = \left\langle \overline{C}\right\rangle = c, \quad \left\langle C'\right\rangle = \left\langle \overline{C'}\right\rangle = 0$$
$$\langle S\rangle = s, \quad \langle Z\rangle = z$$

$$F_A = 0 \Rightarrow a^2 = \frac{M_A M_*}{2(6\lambda_A + \lambda'_A)}$$

$$\mathcal{U}(1)\ \mathrm{Dterm} \Rightarrow c^2 + z^2 + s^2 = \frac{k}{2}\xi \quad \left(\xi = \frac{g_A^2 M_P^2}{192\pi^2}\mathrm{Tr}Q\right)$$

$$F_{C'} = 0 \Rightarrow -\frac{3a_1}{M_*}za + \frac{b_1}{M_*}c^2 + c_1 s = 0$$

$$F_{\overline{C'}} = 0 \Rightarrow \frac{3a_2}{M_*}za + \frac{b_2}{M_*}c^2 + c_2 s = 0\ .$$

$$F_S = 0 \Rightarrow \left\langle C'\right\rangle = 0$$

$$F_Z = 0 \Rightarrow \left\langle \overline{C'}\right\rangle = 0$$

$$\Rightarrow s = \frac{c^2}{M_*}\frac{b_1 a_2 - b_2 a_1}{a_1 c_2 - a_2 c_1}\ , \qquad z = \frac{c^2}{3a}\frac{b_1 c_2 - b_2 c_1}{a_1 c_2 - a_2 c_1}$$

All the VEVs are fixed, in desired directions

Doublet-Triplet Mass Matrix

$$M_{D,T} = \begin{array}{c} \\ \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{array} \begin{array}{c} \begin{array}{cccc} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \end{array} \\ \left(\begin{array}{cccc} 0 & \eta_{D,T} a & \lambda_2 c & 0 \\ -\eta_{D,T} a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T} Y_1 \\ 0 & 0 & \kappa_{D,T} Y_2 & M' \eta \end{array} \right) \end{array}$$

with $\eta_D = 0\ ,\quad \eta_T = \lambda_1\ ,\quad \kappa_D = 3\ ,\quad \kappa_T = 2$

$M_{H'} = \lambda_{H'} s^k / M_*^{k-1}$, $Y_{1,2} = 2 a_{1,2} z a / (M_*) \sim M_{\mathrm{GUT}}^2 / M_*$

$M' \subset Z^2 S C' \bar{C}' / M^2$, $\eta = 1$ or 0

All color triplets heavy, one pair of Higgs doublets light

H_u is in 5_H, H_d in $\bar{5}_H + 16'_H \Rightarrow \tan\beta \neq \frac{m_t}{m_b}$

Stability of doublet mass

$$M_{D,T} = \begin{array}{c} \\ \bar{5}_H \\ \bar{5}_{H'} \\ \bar{5}_C \\ \bar{5}_{C'} \end{array} \begin{array}{c} \begin{array}{cccc} 5_H & 5_{H'} & 5_{\bar{C}} & 5_{\bar{C}'} \end{array} \\ \begin{pmatrix} 0 & \eta_{D,T}a & \lambda_2 c & 0 \\ -\eta_{D,T}a & M_{H'} & 0 & 0 \\ 0 & 0 & 0 & \kappa_{D,T}Y_1 \\ 0 & 0 & \kappa_{D,T}Y_2 & M'\eta \end{pmatrix} \end{array}$$

Zeros in first column protected by $\mathcal{U}(1) \times Z_2$ symmetry

Mass for doublet needs negative charge VEV

All GUT scale VEVs have positive $\mathcal{U}(1)$ charge

Superpotential must be holomorphic

$A^n(C\overline{C})^m$ type terms would destabilize $(B-L)$ VEV

Such terms forbidden by $\mathcal{U}(1)$ symmetry

$$W_\mu = HC'C' \Rightarrow \mu \sim M_{\mathsf{SUSY}} \sim 10^2 \text{ GeV}$$

This shows all order stability of doublet mass

Spectrum of doublets and triplets

$$\frac{M_{D_1} M_{D_2} M_{D_3}}{M_{T_1} M_{T_2} M_{T_3} M_{T_4}} = \frac{9}{4 M_{\text{eff}} \cos\gamma}$$

with $\quad \tan\gamma = \frac{\lambda_2 c}{3 Y_2}\,, \qquad \frac{1}{M_{\text{eff}}} = \left(M_T^{-1}\right)_{11} = \frac{M_{H'}}{\lambda_1^2 a^2}$

M_{eff} controls $d=5$ proton decay: $\mathcal{A}(d=5) \propto 1/M_{\text{eff}}$

$M_{\text{eff}} \sim 10^{19}$ realized naturally, as $M_{H'} \ll M_{\text{GUT}}$

Spectrum heavy states from 45

$$M_{e^c_A} = M_3 = \frac{1}{2} M_8 \equiv \frac{1}{2} M_\Sigma\,, \quad \text{with} \quad M_\Sigma = \frac{2\lambda'_A}{6\lambda_A + \lambda'_A} M_A\,.$$

$M_3 : M(1,3,0), \quad M_8 : M(8,1,0), \quad M_{e^c} : M(1,1,1)$

Remaining states are in $10 + \overline{10}$ of $SU(5)$

Spectrum from 10 + 10-bar

$$M(\Psi^{10}) = \begin{array}{c} \\ \Psi^{10}_A \\ \Psi^{10}_C \\ \Psi^{10}_{C'} \end{array} \!\!\begin{array}{c} \begin{array}{ccc} \overline{\Psi}^{\overline{10}}_A & \overline{\Psi}^{\overline{10}}_{\bar{C}} & \overline{\Psi}^{\overline{10}}_{\bar{C}'} \end{array} \\ \begin{pmatrix} M_\Psi & 0 & X_1 \\ 0 & 0 & \kappa_\Psi Y_1 \\ X_2 & \kappa_\Psi Y_2 & M'\eta \end{pmatrix} \end{array} ,$$

with $\Psi = (u^c, q, e^c)\,,\quad \kappa_\Psi = (2,1,0)\,,\quad M_\Psi = (0,0,M_\Sigma/2)$

$$X_{1,2} = 4a_{1,2}zc/M_*$$

$$\mathcal{U}^c_1\mathcal{U}^c_2 = Y_1Y_2(4+\tilde{p}^2)\,,\quad \mathcal{Q}_1\mathcal{Q}_2 = Y_1Y_2(1+\tilde{p}^2)\,,\quad \mathcal{E}^c_1\mathcal{E}^c_2 = Y_1Y_2\hat{p}^2$$

with $\tilde{p}^2 = \dfrac{|X_1|^2}{|Y_1|^2} = \dfrac{|X_2|^2}{|Y_2|^2}\,,\quad \hat{p}^2 = \tilde{p}^2\left|1 - \dfrac{M_\Sigma M'}{2X_1X_2}\eta\right|$

Gauge boson spectrum

$$M^2(X,Y) = g^2a^2 \equiv M_X^2 \ , \quad M^2(X',Y') = M_X^2(1+p^2)$$

$$M^2(V_{u^c,\bar{u}^c}) = M_X^2(4+p^2) \ , \quad M^2(V_{e^c,\bar{e}^c}) = M_X^2 p^2 \ , \quad \text{with} \quad p^2 = \frac{4c^2}{a^2}$$

Note: $p = \tilde{p}$ in the model

$\Rightarrow$ Higher SUSY in $\{10 + \overline{10}\}$ spectrum

$\Rightarrow$ Threshold corrections from $\{10 + \overline{10}\}$ cancels

$\Rightarrow$ Model almost as predictive as minimal SUSY $SU(5)$

Threshold corrections

$$\alpha^{-1}(\Lambda) = \alpha_i^{-1}(M_Z) - \frac{b_i}{2\pi} \ln \frac{\Lambda}{M_Z} + \Delta_{i,w}^{(2)} + \Delta_i^{\text{GUT}} ,$$

Weak scale threshold

Case 1 : $\tan\beta = 3$, $m_0 \simeq 300$ GeV, $m_{1/2} \simeq 352.4$ GeV

$\Rightarrow \Delta_{i,w}^{(2)} \simeq (0.2602,\ 0.349,\ 1.207)$

Case 2 : $\tan\beta = 3$, $m_0 \simeq 930$ GeV, $m_{1/2} \simeq 146.8$ GeV

$\Rightarrow \Delta_{i,w}^{(2)} \simeq (0.4021,\ 0.2264,\ 0.9483)$;

Case 3 : $\tan\beta = 3$, $m_0 \simeq 1.97$ TeV, $m_{1/2} \simeq 146.8$ GeV ,

$\Rightarrow \Delta_{i,w}^{(2)} \simeq (0.6431,\ 0.4739,\ 1.209)$.

GUT scale threshold

$$\ln \frac{M_{\text{eff}} \cos\gamma}{M_Z} = \frac{5\pi}{6}\left(3(\alpha_2^{-1} + \Delta^{(2)}_{2,w} - \frac{1}{6\pi}) - 2(\alpha_3^{-1} + \Delta^{(2)}_{3,w} - \frac{1}{4\pi}) - (\alpha_1^{-1} + \Delta^{(2)}_{1,w})\right)$$

$$- \ln \frac{4\kappa^{5/2}}{9} + \ln \frac{(4+p^2)^{3/2}(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)^{3/2}(1+p^2)^2} + \ln \frac{p}{\hat{p}}$$

$$\ln \frac{\left(M_X^2 M_\Sigma\right)^{1/3}}{M_Z} = \frac{\pi}{18}\left(5(\alpha_1^{-1} + \Delta^{(2)}_{1,w}) - 3(\alpha_2^{-1} + \Delta^{(2)}_{2,w} - \frac{1}{6\pi}) - 2(\alpha_3^{-1} + \Delta^{(2)}_{3,w} - \frac{1}{4\pi})\right)$$

$$+\frac{1}{6}\ln\kappa - \frac{1}{6}\ln\frac{(4+p^2)(1+\tilde{p}^2)^2}{(4+\tilde{p}^2)(1+p^2)^2} - \frac{1}{3}\ln\frac{p}{\hat{p}}\ ,$$

Very similar to minimal SUSY $SU(5)$ threshold

A single new parameter beyond $SU(5)$

$\Rightarrow$ better $\alpha_3(M_Z)$

M_{H_C} of $SU(5)$ replaced by $M_{\text{eff}} \cos\gamma$

$\Rightarrow d = 5$ proton decay under control

Correlation between d=5 and d=6 proton decay

$$M_{\rm eff} \simeq 10^{18}{\rm GeV}\cdot\left(\frac{10^{16}{\rm GeV}}{M_X}\right)^3\left(\frac{3}{\tan\beta}\right)\left(\frac{1/25}{r}\right)\frac{\exp[2\pi(\Delta^{(2)}_{2,w}-\Delta^{(2)}_{3,w}-\delta\alpha_3^{-1})]}{1.42\cdot 10^{-2}}\,.$$

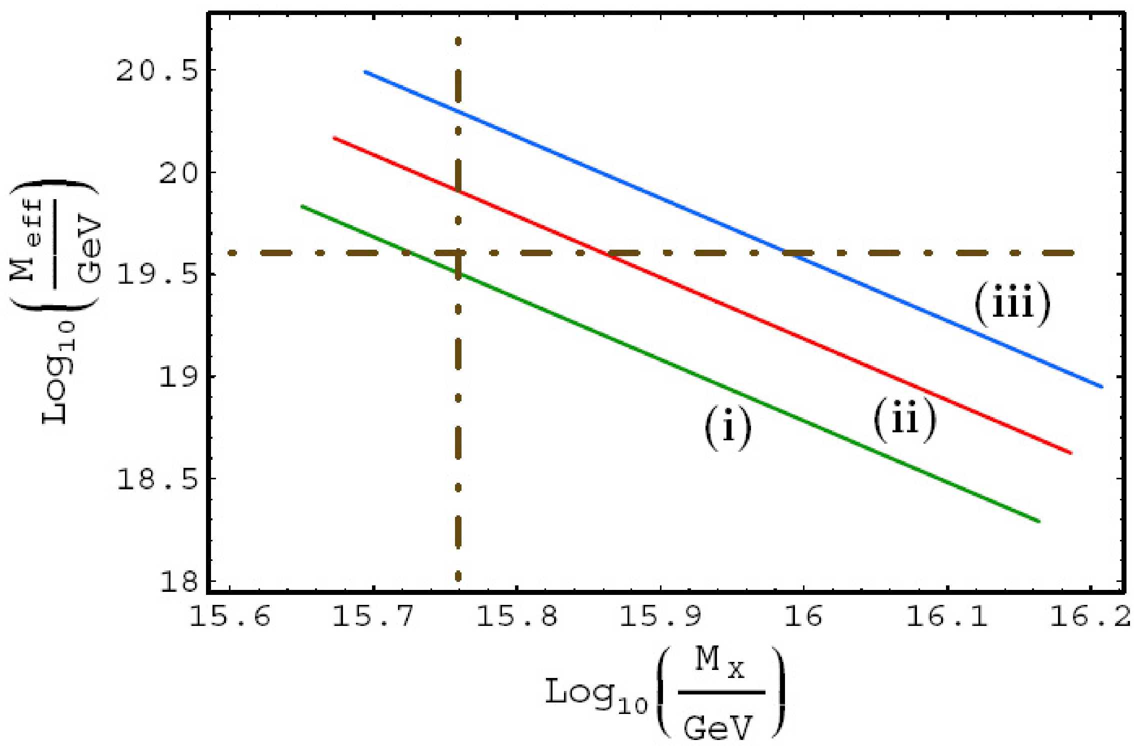
$$r \equiv M_\Sigma/M_X \ll 1, \quad r = (1/25 - 1/200)$$

$$\alpha_3^{-1}(M_X) = \alpha_3^{-1}(M_Z) + \Delta^{(2)}_{3,w} - \frac{1}{4\pi} + \frac{3}{2\pi}\ln\frac{M_X}{M_Z}$$

$$-\frac{1}{2\pi}\ln\frac{M_X^4}{M_{T_1}M_{T_2}M_{T_3}M_{T_4}} + \frac{3}{2\pi}\ln r - \frac{1}{2\pi}\ln\frac{M_X^2}{\mathcal{U}_1^c\mathcal{U}_2^c} - \frac{1}{\pi}\ln\frac{M_X^2}{\mathcal{Q}_1\mathcal{Q}_2}\,.$$

$$\alpha_G = (1/19 - 1/22)$$

Correlation of d=5 and d=6 proton decay



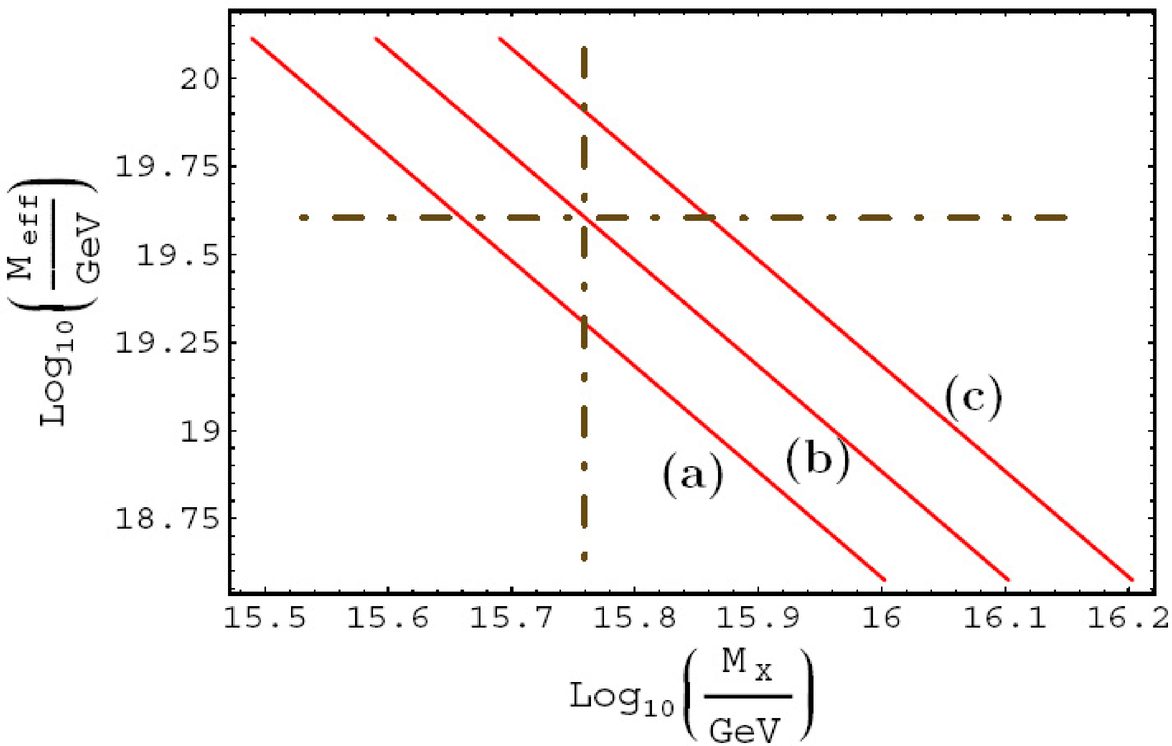
(i): $\alpha_3 = 0.1156$

(ii): $\alpha_3 = 0.1176$

(iii): $\alpha_3 = 0.1196$

$$r = 1/200, \quad \tan\beta = 3$$

Correlation of d=5 and d=6 proton decay



(a): $r = 1/50$

(b): $r = 1/100$

(c): $r = 1/200$

$$\alpha_3 = 0.1176, \quad \tan\beta = 3$$

d=5 Proton Decay in the model

$$\tau_p^{d=5}(p \to \bar{\nu} K^+) \simeq 2.3 \cdot 10^{33}\,\text{yrs} \times \left(\frac{0.01\,\text{GeV}^3}{\tilde{\beta}}\right)^2 \left(\frac{1.74}{A_S^t}\right)^2 \left(\frac{1.34}{A_L}\right)^2 \times$$

$$\times \left(\frac{M_{\text{eff}}}{1.51 \cdot 10^{19}\,\text{GeV}}\right)^2 \left(\frac{m_{\tilde{q}}}{1.8\,\text{TeV}}\right)^4 \left(\frac{125\,\text{GeV}}{M_{\tilde{W}}}\right)^2 .$$

$\Rightarrow M_{\text{eff}} > 1.2 \times 10^{19}$ GeV **From experimental limit**

$M_{\text{eff}} < 5 \times 10^{19}$ GeV **From theory -- tuning**

Light gaugino and heavy scalars prefered

$p \to e^+\pi^0$ lifetime in SUSY SO(10) (yr)

$r \diagdown \alpha_3$	0.1156		0.1176		0.1196	
	A	B	A	B	A	B
1/50	6.7×10^{32}	3.9×10^{32}	2.5×10^{33}	1.4×10^{33}	8.1×10^{33}	4.7×10^{33}
1/100	1.7×10^{33}	1.1×10^{33}	6×10^{33}	3.5×10^{33}	2×10^{34}	1.2×10^{34}
1/200	4×10^{33}	3×10^{33}	1.5×10^{34}	9×10^{33}	5×10^{34}	3×10^{34}

A: $M_{\text{eff}} = 3 \cdot 10^{19}$ GeV B: $M_{\text{eff}} = 4.5 \cdot 10^{19}$ GeV

$r = M_\Sigma / M_X$

Lepton flavor violation in SUSY SO(10)

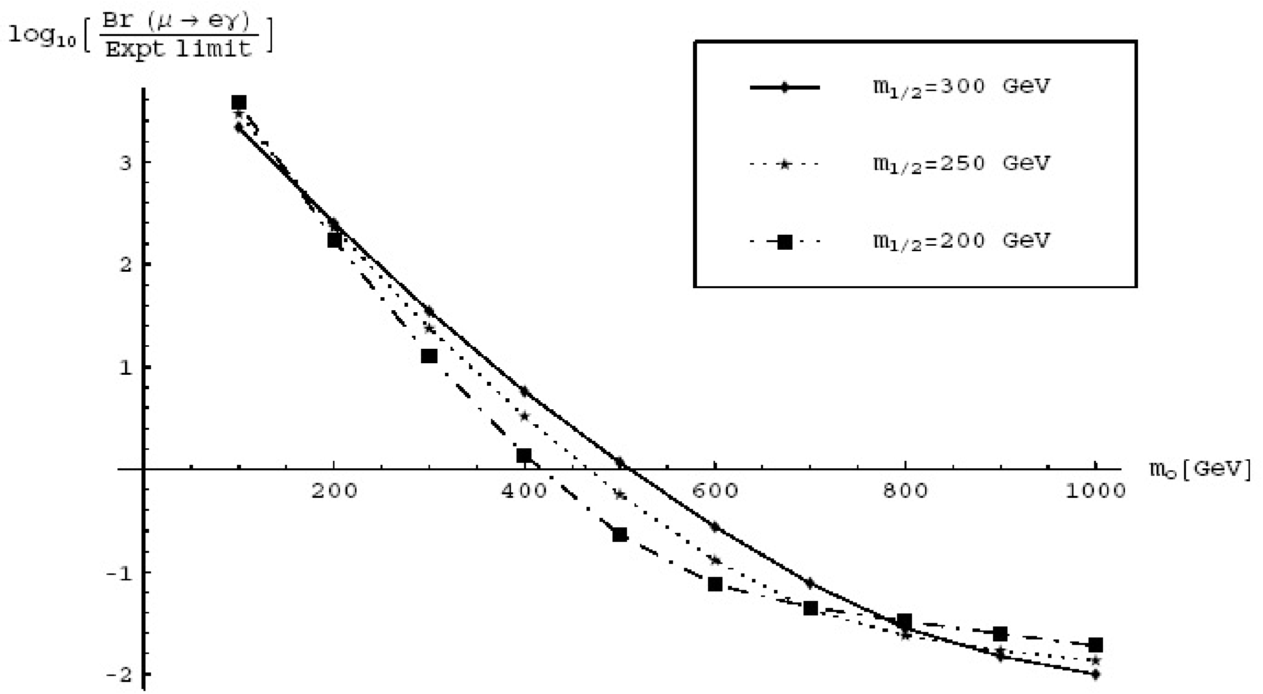


Fig. 1. Log of Br($\mu \to e\gamma$) divided by the experimental bound (1.2×10^{-11}) obtained for the SO(10) framework with $\ln(M^*/M_{GUT}) = 1$, $\tan\beta = 10$ and $\mu > 0$ vs m_o (in GeV) with $m_{1/2} = 200,\ 250$ and 300 GeV.

$$\Delta\hat{m}^2_{\tilde{b}_L} = \Delta\hat{m}^2_{\tilde{b}_R} = \Delta\hat{m}^2_{\tilde{\tau}_L} = \Delta\hat{m}^2_{\tilde{\tau}_R} \equiv \Delta \approx -\left(\frac{30m_o^2}{16\pi^2}\right)h_t^2\ ln(M^*/M_{GUT})$$

Pati, Rastogi, Babu (2005)

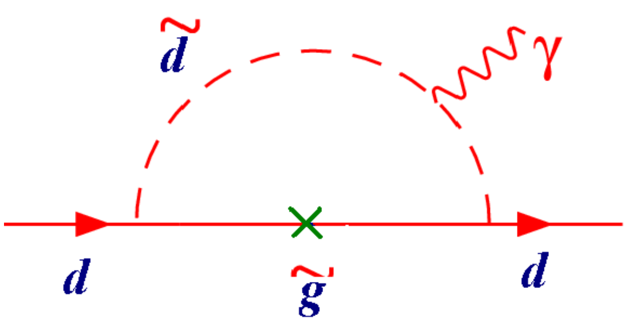
Electric dipole moments

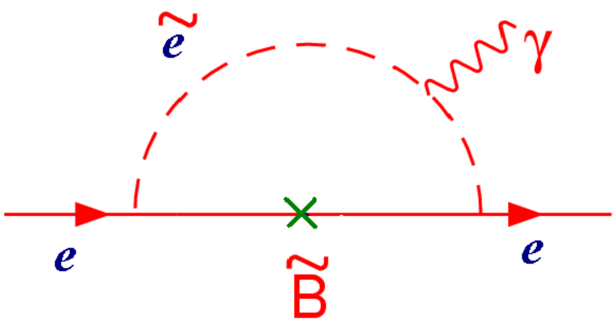
$$\mathcal{L}_{eff} = -\frac{i}{2} d_f \bar{\psi} \sigma_{\mu\nu} \gamma_5 \psi F^{\mu\nu}$$

Violates CP

Electron: $d_e(Exp) \leq 2.1 \times 10^{-27}$ e-cm

Neutron: $d_n(Exp) \leq 6.3 \times 10^{-26}$ e-cm





Phases in SUSY breaking sector must be small, < 0.001

Light gaugino and heavy squarks suppress EDM

Summary and Conclusions

- SUSY GUTs in 4d well motivated
- Doublet–triplet mass hierarchy can be stable to all orders
- GUT scale threshold effects surprisingly small
- Good values of $\alpha_3(M_Z)$ predicted
- $d = 5$ proton decay adequately but not fully suppressed
- $d = 5$ and $d = 6$ proton decay rates correlated
- Both $p \to \bar{\nu} K^+$ and $p \to e^+ \pi^0$ should be within reach of HyperKamiokande and DUSEL experiments
- Light gaugino and heavy scalars prefered
- SUSY FCNC under control by non-Abelian discrete symmetry