CLAY MATHEMATICS INSTITUTE WORKSHOP OUTLINE: RATIONAL POINTS AND RATIONAL CURVES

1. Scientific context

There is a well-known and very strong analogy between the existence and properties of rational points on varieties defined over number fields (or over finite fields, p-adic fields, etc.) and the existence and properties of rational curves on complex varieties. One example of this is the Lang conjecture together with its conjectural converse. Historically "rational curve" results have been far simpler than the analogous "rational point" results. This workshop will bring together experts in both types of results to discuss what has been proved on the rational curves side of the picture, what this might suggest on the rational points side of the picture, and possible directions for future research on rational curves inspired by questions about rational points.

A second purpose is to bring together experts to investigate a very focused question. Motivated by the success in the (negative) solution of Hilbert's Tenth Problem for $\mathbb{Z}$, Hilbert's Tenth Problem has been intensively investigated for other rings and fields. In particular, Hilbert's Tenth Problem has been settled in the negative for all function fields $K/\mathbb{C}$ with $\text{tr.deg.}(K/\mathbb{C}) \geq 2$. However the case of $K = \mathbb{C}(t)$ (and more generally, $\text{tr.deg.}(K/\mathbb{C}) = 1$) is unsettled. János Kollár pointed out that a "converse theorem" of Graber, Harris, Mazur and Starr suggests the general method used to settle Hilbert's Tenth Problem for $\text{tr.deg.}(K/\mathbb{C}) \geq 2$ cannot be extended to $\mathbb{C}(t)$. Unfortunately, this requires a stronger form of the "converse theorem" than is currently known. One goal of the workshop is to bring together experts (Kollár, Poonen, Graber, Harris, Mazur, Starr) to investigate the narrow question of whether the "converse theorem" can be extended and applied to the investigation of Hilbert's Tenth Problem for $\mathbb{C}(t)$.