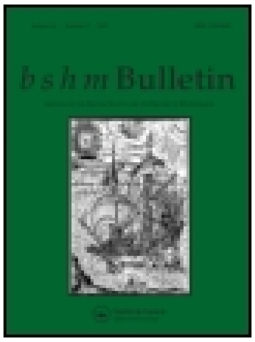

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Christopher Hollings, Ursula Martin & Adrian Rice

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The early mathematical education of Ada Lovelace

CHRISTOPHER HOLLINGS and URSULA MARTIN
University of Oxford, UK

ADRIAN RICE
Randolph-Macon College, USA

Ada, Countess of Lovelace, is remembered for a paper published in 1843, which translated and considerably extended an article about the unbuilt Analytical Engine, a general-purpose computer designed by the mathematician and inventor Charles Babbage. Her substantial appendices, nearly twice the length of the original work, contain an account of the principles of the machine, along with a table often described as 'the first computer program'. In this paper we look at Lovelace's education before 1840, which encompassed older traditions of practical geometry; newer textbooks influenced by continental approaches; wide reading; and a fascination with machinery. We also challenge judgements by Dorothy Stein and by Doron Swade of Lovelace's mathematical knowledge and skills before 1840, which have impacted later scholarly and popular discourse.

Introduction

In June 1833 the seventeen year-old Ada Byron and her mother, Lady Byron, attended a demonstration by Charles Babbage of a prototype of his Difference Engine. Designed to calculate and print successive values of a formula using the method of finite differences, and thus reduce the labour of producing mathematical tables, it captivated them both. Lady Byron described it as a 'thinking machine',[1] while Ada went to lectures on the engine by Dionysius Lardner, read his article in the *Edinburgh Review* (Stein 1985, 46), and studied Babbage's engineering drawings. Many years later, Sophia De Morgan recalled that:

> Miss Byron, young as she was, understood its working, and saw the great beauty of the invention.
>
> (De Morgan 1882, 89)

In 1840–41 Ada, by now the Countess of Lovelace,[2] studied calculus and higher mathematics with Sophia's husband, Augustus De Morgan, and in 1843 published the paper for which she is now famous: her translation, with extensive appendices, of Luigi Menabrea's *Notions sur la Machine Analytique de M. Charles Babbage* (Lovelace 1843).

[1]Dep. Lovelace Byron (Bodleian Library, Oxford), Box 77, Lady Byron to Dr William King, 21 June [1833], f. 217r. Subsequent references to this archive will appear in the text within square brackets; 'Dep. Lovelace Byron, Box *n*' will be abbreviated as 'LB *n*'. Other abbreviations employed will be 'ADM' for De Morgan and 'AAL' for Lovelace.
[2]In 1835 Ada Byron married Lord William King (unrelated to Lady Byron's friend Dr William King) who became the Earl of Lovelace in 1838. For readability we refer to her as 'Ada Byron' or 'Ada Lovelace'.

In the two hundred years since her birth, opinions of Lovelace's ability have ranged from 'genius' to 'charlatan'. In her lifetime she was famous as the daughter of the poet Lord Byron, brought up by her mother following her parents' acrimonious separation shortly after her birth: subsequently her life, as much as her scientific work, has inspired numerous biographies and creative endeavours.

In the next section we review assessments of Lovelace's mathematical ability, and their reception in technical and popular writing. In the third section we consider Lovelace's eclectic mathematical education before 1840, indicating the focus on practical mathematics and synthetic geometry, coupled with an interest in mechanical devices, and wide reading of the science writing of the day. In the final section (Analysis) we emphasize that Lovelace's mathematical education spanned the transition from an earlier practical tradition to a more analytic approach based on continental mathematics. This underpins our challenge to influential claims by Dorothy Stein, based on assessments of Lovelace's correspondence with Mary Somerville, that she was mathematically weak, and by Doron Swade, that in 1834 she lacked the background to have studied Lardner's article on Babbage's Difference Engine.

This paper complements our work on Lovelace's studies with Augustus De Morgan in 1840 and 1841 (Hollings *et al.* 2017), in which contextual analysis allows us to present a corrected ordering of the archive material, and thus to challenge further claims by Stein of Lovelace's mathematical weakness (Stein 1985, 84, 89–91), and to assess De Morgan's favourable judgment of her potential for serious mathematical work [LB 339, ADM to Lady Byron, 21 January 1844, f 2]. That paper presents De Morgan's skill as a teacher in developing Lovelace's habits of study, repairing gaps in her earlier education, and introducing her to research questions of the day; and shows Lovelace's emerging mathematical strengths of attention to detail, interest in big questions, and desire to tackle problems from first principles. These two papers appear to be the first studies by historians of mathematics of the Lovelace archive, and taken together provide an assessment of her mathematical background prior to her most famous work.

Changing perceptions of Lovelace

Lovelace was respected by her scientific peers: for example Babbage wrote to Faraday in 1843 of

> that Enchantress who has thrown her magical spell around the most abstract of Sciences and has grasped it with a force which few masculine intellects (in our own country at least) could have exerted over it.
>
> (James 1996, 164)

Augustus De Morgan, while supporting the widespread view that women were not strong enough for the demands of mathematics (see Winter 1998 for an analysis), wrote to her mother of her 'power of thinking … utterly out of the common way for any beginner, man or woman', continuing:

> Had any young beginner, about to go to Cambridge, shown the same power, I should have prophesied first that his aptitude at grasping the strong points and the real difficulties of first *principles* would have very much lowered his chance of

> being senior wrangler; secondly, that they would have certainly made him an original mathematical investigator, perhaps of first rate eminence.
>
> [LB 339, ADM to Lady Byron, 21 January 1844, f 2]

On her death, an anonymous obituary recorded (Examiner 1852):

> The Countess of Lovelace was thoroughly original, and the poet's temperament was all that was hers in common with her father. Her genius, for genius she possessed, was not poetic, but metaphysical and mathematical, her mind having been in the constant practice of investigation, and with rigour and exactness.

Similar references occur throughout the nineteenth century, for example to the 'noble and all-accomplished' lady, author of the 'profound, luminous and elegant notes' forming 'by far the most instructive' part of the Menabrea translation (Timbs 1860, x), who wrote 'with all the thorough knowledge of detail which a woman can so well master' and displayed 'a surprising knowledge of analysis' (Farr 1871, 45). Ada Lovelace's daughter-in-law, Mary, Countess of Lovelace, quoted this obituary in her Epilogue to the first biography of Lady Byron (Colburn Mayne 1929), and noted Ada's position as a member of the scientific elite:

> Her few real friendships were with people of intellectual power, Babbage for instance, and Mrs Somerville, who had a great appreciation of her. Her mathematical teacher, Professor Augustus De Morgan, said of her that if she had been a man she could have been Senior Wrangler.

The volume includes the complete letter from De Morgan to Lady Byron, so the observant reader can see that this is precisely the opposite of what De Morgan said.

The work of Babbage and Lovelace was never entirely forgotten: Hartree (1949, 70) asserted that 'she must have been a mathematician of some ability', and noted her remark in the 1843 paper that the engine can only 'do whatever we know how to order it to perform': in his 1950 paper on whether a machine can think, Alan Turing challenged this view, which he called 'Lady Lovelace's objection'.

The industrialist B V Bowden edited *Faster than thought*, an influential collection of essays on the theory and practice of computing, which included both Lovelace's paper, and a short biography, based on Bowden's conversations with her descendants. He described her as 'a mathematician of great competence' (Bowden 1953, xi) with 'a profound understanding of the principles of the machine' (Bowden 1953, 23). A striking indication of the way class, as well as gender, shaped opinions of Lovelace, is the review of Bowden's (1953) book which begins:

> We are accustomed to seeing pictures of sleekly streamlined modern electronic computers with similarly endowed, female operators poised at the controls. It is surprising then to find a book on these machines starting off with a handsome, old-fashioned portrait of a nineteenth-century lady mathematician.

Bowden, Hartree, and Turing had sufficient computing background to read Lovelace's paper, and it was thanks to Bowden that B H Neumann became the first mathematician to look at her correspondence with Augustus De Morgan, though he felt that her mathematics ‘owed even more’ to Babbage (Neumann 1973, 95).

The first biographies to pay substantial attention to Lovelace focused on the Byron family. Malcolm Elwin (1975) described the demanding educational regime imposed by her mother, including basic mathematics, while the first full length biography, by Doris Langley Moore (1977), devoted just a few pages to the 1843 notes: quoting De Morgan’s judgement of Lovelace’s ability, Langley Moore summarized her letters to him as:

> … enquiring, speculating, arguing, filling pages with equations, problems, solutions, algebraic formulae, like a mathematician’s cabalistic symbols.
>
> (Langley Moore 1977, 99)

Langley Moore was also the first biographer to draw attention to what she called Lovelace’s considerable ‘arrogance’ and ‘self regard’: later authors have gone further and used Lovelace’s sometimes flamboyant remarks—for example ‘I do not believe that my father was (or ever could have been) such a Poet as I shall be an Analyst’ (Toole 1992, 215)—coupled with her numerous physical ailments, as evidence of mental instability. Most striking is Dorothy Stein (1985), who devotes a seventeen-page appendix to a posthumous diagnosis of manic-depression.

Stein was, however, the first biographer to analyse in detail Lovelace’s scientific writings. She argued that her command of mathematics was actually rather limited, pointing to several elementary errors, and her professed inability to handle algebraic substitutions as ‘evidence of the tenuousness with which she grasped the subject of mathematics’, claiming that such evidence ‘would be difficult to credit about one who succeeded in gaining a contemporary and posthumous reputation as a mathematical talent, if there were not so much of it’ (Stein 1985, 90).

Indeed Hyman (1982), in his biography of Babbage, while respecting Lovelace’s achievements, had already questioned the 1843 paper as evidence of her mathematical reputation, observing guardedly that while ‘mathematicians all over Europe thought her a splendid addition to their number’ (Hyman 1982, 196), Lovelace

> …worked under Babbage’s careful guidance and [the programs] are student exercises rather than original work. Indeed there is not a scrap of evidence that Ada ever attempted original mathematical work.
>
> (Hyman 1982, 198)

Doron Swade (2000, 166) went far further, dismissing Lovelace as a ‘talented beginner, a precocious novice’, challenging what he called ‘well-intentioned posthumous elegies’, such as Sophia De Morgan’s assertion that she quickly understood Babbage’s engine, on the grounds that Lovelace did not learn the ‘necessary elementary mathematics of the Engine (the method of finite differences)’ until 1841, in her studies with De Morgan. Even more forthright was Bruce Collier (1990) who described Lovelace as

> mad as a hatter … with the most amazing delusions about her own talents … the most overrated figure in the history of computing.

Betty Toole (1992) published transcriptions of a selection of Lovelace's letters, omitting most of the mathematical content, and argued that to judge Lovelace's mathematical ability solely on the basis of the questions she asked her teachers is misleading, as she only wrote to them when she had problems. Toole portrays Lovelace as 'a synthesizer and a visionary' who

> saw the need for a mathematical and scientific language which was more expressive and which incorporated imagination
>
> (Toole 1992, 2)

and uses Lovelace's phrase 'poetical science' [LB 44, AAL to Lady Byron, f 210] to characterize the approach.

Recent popular biographers have developed this line: for Woolley (2000, 276),

> Ada was not a great mathematician … as De Morgan implied…, unlike Mary Somerville, she really was not sufficiently engaged with the details to be one.

This is a misinterpretation of De Morgan who actually argued the opposite—that Somerville's achievement was pedantic compared to Lovelace's deeper understanding. Woolley asserts that

> Ada's notes are not to be assessed as a work of mathematics, but as a work of a more speculative, experimental nature … She showed what imagination could reveal that mathematics alone could not …. This is what poetical science had enabled her to do: see something that would remain invisible to the rest of the world for a century more…

Frustrated at ever more extravagant claims that Lovelace foresaw quantum mechanics, invented the CD, or brought about Silicon Valley, Haigh and Priestley (2015) argued for a more measured approach, observing that the search for heroines presents a distorted view of the overall development of computing: they use Google *n*-grams to show that references to Lovelace far outweigh those to others with a much greater claim to influence.

Controversies about Lovelace's mathematical talents, and in particular a reliance on Stein's 1985 analysis, have informed broader debate about her contribution. For example, Babbage scholar Allan Bromley is fairly neutral in Bromley (1982), which appeared before Stein's book, whereas by 1990 his essay in Aspray's (1990) overview of computing history, concludes 'Not only is there no evidence that Ada ever prepared a program for the Analytical Engine, but her correspondence with Babbage shows that she did not have the knowledge to do so'. Misa (2016) provides a welcome and thorough survey of the debate, concluding that the 1843 notes are 'a product of an intense intellectual collaboration'. The latest edition of Campbell-Kelly *et al.* (2013) remains unconvinced however.

A metabiography in the sense of Sapp (1990) might assess these varying accounts of Lovelace's life and contribution against changing contexts of class, gender, or mental stability; changing perceptions of mathematics amongst both professional mathematicians and the general public; or changing perceptions of how to present women scientists. We restrict our attention in this paper to Lovelace's mathematics, and adopt the approach of other recent work on women and science in treating her as a member of a scientific community, alongside Babbage, De Morgan and Somerville, rather than constraining her by a narrative of exceptionalism or marginality (Jones and Hawkins 2015).

Ada Byron's childhood education

It was the custom in early nineteenth-century England for upper-class girls to be educated by governesses, often rather strictly: the subjects offered might include some arithmetic, astronomy and geometry alongside languages and music (Hughes 1993, 71), though the typical level of attainment might be slight and involve much rote-learning (Hughes 1993, 42). Lady Byron, who had herself had a good mathematical education for the time, avoided this approach, considering learning to think more important than memorization [LB 119, ff 216–217], and did much of Ada's teaching herself. Recognized as an educational reformer (Taylor 1998), she followed the principles of the Swiss reformer Pestalozzi in establishing several village schools whose 'learning by action' combined text-books with physical work and practical skills. Although it is sometimes claimed that Lady Byron forced her daughter to learn mathematics to keep her from poetry, there is little evidence of this: however many letters seek advice from her physician William King, and her own childhood tutor, the Unitarian minister William Frend.

Her 'happy and intelligent' [LB 71, Lady Byron to William Frend, 23 August 1818, f 34] daughter started to learn her letters from her mother when she was nearly three. In 1821 a Miss Lamont was appointed as a temporary governess: she documented a demanding schedule,

> … lessons in the morning in arithmetic, grammar, spelling, reading, music, each no more than a quarter of an hour long – after dinner, geography, drawing, French, music, reading, all performed with alacrity and docility.
>
> [LB 118, item 5, Governess's diary, 14 May 1821, f 4]

In May 1821 the diary records,

> Commenced giving instructions to Miss Byron … The first trial was in arithmetic. She adds up sums of five or six rows of figures, with accuracy; she is deliberate and correct in the process, and takes an interest in the performance.
>
> [LB 118, item 5, Governess's diary, 14 May 1821, f 2r]

Miss Lamont was also required to keep a diary in Ada's name:

> I want to please Mama very much, that she and I may be happy together… The French has not interested so much as some others – and one night I was rather foolish in saying that I did not like arithmetic & to learn figures, when I did – I was not thinking quite what I was about. The sums can be done better, if I tried, than they are.
>
> [LB 118, Item 6, 'Ada's' journal]

Ada's later letters to her mother bubble over with enthusiasm: Italian, music, drawing, geography, her cat and unexpected kittens, summaries of the Sunday sermon, and her mathematics. Aged ten, she wrote about the so-called *rule of three*:

> I have been puzzling hard at a sum in the rule of three which I could not do, the question is 'If 750 men are allowed 22500 rations of bread per month, how many rations will a garrison of 1200 men require?'
>
> [LB 41, Ada Byron to Lady Byron, 1 June 1826, f 27r]

In the same letter, Ada said she was 'very desirous to master those overs in the sums of division' and added

> I think by the time you come back I may have learnt something about decimals, I attempted the double rule of three but I could not understand, however I will not give it up yet.
>
> [LB 41, Ada Byron to Lady Byron, 1 June 1826, ff 27r–27v]

Her questions indicate that rather than learning by rote, she wanted to understand. As she explained to her mother: '…the book does not teach as well as you do'.

Ada was introduced to geometry at the age of twelve, thanking her mother for the 'entertaining pamphlet you left me on that subject…, which I found very amusing indeed'. Although confessing that 'I am a little afraid of the Theorems', she nonetheless resolved to 'attack them boldly & do my best' [LB 41, Ada Byron to Lady Byron, 22 November 1828, f 78r]. Even in these childhood letters, we see an intelligent, inquisitive and tenacious mathematical learner who would go on to study higher mathematics, and enjoy doing so.

In line with Pestalozzi's ideas, practical skills were encouraged, whether playing with toy bricks [LB 118, Item 5] or dissecting a dragonfly [LB 118, Item 6]. Shortly before her father's death, Lady Byron wrote to him of Ada's 'mechanical ingenuity—her self-invented occupations being the manufacture of ships and boats' (Prothero 1904, 330). In 1828 she became fascinated by flying, asked Lady Byron for a book on the anatomy of birds and 'had great pleasure in looking at the wing of a dead crow'. She made some wings from paper, silk and feathers:

> My wings are going on prosperously but do not expect to see a pair of *well* proportioned wings though they are quite sufficiently so for me to explain to you all my ideas on the subject of *flying*.
>
> [LB 41, Ada Byron to Lady Byron, 7 April 1828, f 57v]

She hoped to write a book on 'Flyology', and she had plans for a flying machine that would be powered by steam, the cutting-edge technology of the age:

> a thing in the form of a horse with a steamengine in the inside so contrived as to move an immense pair of wings, fixed on the outside of the horse, in such a manner as to carry it up into the air while a person sits on its back.
>
> [LB 41, Ada Byron to Lady Byron, 7 April 1828, ff 57v–58r]

Shortly after Ada's thirteenth birthday, Lady Byron again sought the advice of Dr William King: while she was most interested in her daughter's education in 'Morals, History, and Political Economy', she also wanted Ada to study 'The principles of Natural History', and she encouraged her interests in astronomy. She described Ada's knowledge of mathematics:

> Arithmetic, the first part of Algebra & Paisley's [sic] "Practical Geometry". Present information on these subjects confined to some general ideas of the value of numbers, without facility in working them, & to some acquaintance with the problem in the 1st part of Paisley. Great interest in such pursuit.
>
> [LB 77, Lady Byron to Dr King, 7 January 1829, f 35r]

C W Pasley's (1814) *Practical geometry* was intended for self-study, or for use by military instructors. An approach very much in the spirit of Pestalozzi, it covered the basics of geometry, of the kind needed by soldiers, surveyors or engineers, entirely through practical instructions for making technical drawings, and stopping short of any explicit algebra or Euclidean geometry.

Despite interruptions from illness, and a lengthy foreign tour with her mother, Ada retained her interest in mathematics throughout her teenage years. In 1834, she wrote to William Frend about rainbows:

> I am very much interested on the subject just now, but I cannot make out one thing at all, viz: why a rainbow always appears to the spectator to be an arc of a circle. Why is it a curve at all, and why a circle rather than any other curve? I believe I clearly understand *how* it is that the colours are separated, and the different angles which the different colours must make with the original incident ray. I am not sure that I entirely understand the *secondary* rainbow.
>
> [LB 171, Ada Byron to William Frend, 15 March 1834, ff 127r–127v]

She speculated correctly:

> Is the spectator's eye supposed to be in the centre of the circle of which the arc of the rainbow forms a portion?

Motivated to repair the gaps in her mathematical education, in March 1834 she asked Dr King for further help. 'My wish', she wrote,

> is to make myself well acquainted with Astronomy, Optics &c; but I find that I cannot study these satisfactorily, for want of a thorough acquaintance with the elementary parts of Mathematics.
>
> [LB 172, Ada Byron to Dr King, 9 March 1834, f. 126v]

What she needed, she said, was a course in pure mathematics, by which she meant basic arithmetic, algebra, and geometry: the extent of mathematical education for most men who went to university, and a level reached by very few women. In his reply, King recommended the books he had used in his own university studies twenty-five years before, describing 'a Complete Cambridge Course':

> Begin with Euclid, then Plane & Spherical Trigonometry, which is found at the end of Simson's Euclid, then Vince's Plane & Spherical Trigonometry, then Bridge's Algebra.
>
> [LB 172, Dr King to Ada Byron, 15 March 1834, ff 128r–128v]

His advice on how to study was similarly outdated:

> The only mode of study of any use is $-1°$. to read and understand the proposition. 2°. Master the proposition by the help of the figure without the text. 3°. Do the same by drawing the figure yourself. 4°. Write down the proposition on paper : not verbatim, but still fully : and taking care to arrange the steps lucidly on paper not writing continuously as in the printed book. … 5°. Repeat all this by heart, without book or figure. 6°. Do the same in your walks, carrying the figure in your pocket on a Card if necessary. 7°. This must be repeated daily with the same proposition till it is as familiar to the mind as your own name. 8°. The words of each proposition … must be fixed in your mind, with their number, so that you could repeat the Book through.
>
> [LB 172, Dr King to Ada Byron, 15 March 1834, ff 128v–129r]

Nonetheless, Ada started work with enjoyment. Within a fortnight, she could report that she was

> getting on very well so far, with Euclid. I usually do four new propositions a day, and go over some of the old ones. I expect now to finish the 1st book in less than a week.
>
> [LB 172, Ada Byron to Dr King, 24 March 1834, f 131r]

By mid-April, Ada was conjecturing her own variation on Pythagoras' theorem:

> Can it be proved … that equilateral triangles being constructed on the sides of a right angled triangle, and also one on the hypotenuse, the sum of the triangles on the sides is equal to the triangle on the hypotenuse? … It strikes me that it ought to be as demonstrable as when the figures are four-sided & equilateral.
>
> [LB 172, Ada Byron to Dr King, 13 April 1834, f 132r]

'You will soon puzzle me in your studies' replied King, apologizing that

> When I was at College we had few problems deduced from Euclid. We got up a set of books and seldom went out of them, except the high men, i.e., the first 4 or 6 Wranglers, i.e., the men of the first class.
>
> [LB 172, Dr King to Ada Byron, 24 April 1834, f 133r]

In just seven weeks, she had reached the bounds of his expertise.

At about the same time, Ada met Mary Somerville, who, although of the same generation as Dr King, was at the forefront of the introduction of newer continental mathematical ideas to British readers, through her exposition of Laplace's

Mécanique céleste, published in 1831 as *The mechanism of the heavens*. The Somerville family chaperoned Ada Byron in London's scientific and literary society, introducing her both to her future husband, and to Charles Babbage. After her marriage, Ada maintained her mathematical interests, writing to Somerville in 1835:

> I now read Mathematics every day & am occupied in Trigonometry & in preliminaries to Cubic & Biquadratic Equations. So you see that matrimony has by no means lessened my taste for those pursuits, nor my determination to carry them on….
>
> [Somerville Papers (Bodleian Library, Oxford), Dep. c.367, Folder MSBY-3, Ada King to Mary Somerville, 1 November 1835, f 55v]

Her letter indicates that she was working through David Brewster's English edition of Legendre's *Elements of geometry and trigonometry*. She asked Somerville for help with an exercise (Legendre 1822, 288) deriving formulae for $\sin(a-b)$ and $\cos(a-b)$ from the addition formulae:

$$\sin\ (a+b) = \frac{\sin\ a\ \cos\ b + \sin\ b\ \cos\ a}{R} \tag{1}$$

$$\cos\ (a+b) = \frac{\cos\ a\ \cos\ b - \sin\ a\ \sin\ b}{R} \tag{2}$$

Somerville pointed out that the solution was simply a matter of substituting $-b$ for b, and indeed the formulae $\sin(-x) = -\sin x$ and $\cos(-x) = \cos x$ were introduced a few pages earlier, alongside the definitions of the trigonometric functions (Legendre 1822, 283). A couple of weeks later, Ada was asking a similar, and only slightly harder, question.

Stein describes this exchange as 'rather startling, for it reveals that Ada had not progressed very far; despite hard work, skill and ingenuity in the manipulation of symbols did not come easily to her. … It was a principle that would continue to elude her' (Stein 1985, 56).

This, then, was the extent of Lovelace's mathematical education prior to the beginning of her course of study with De Morgan in the summer of 1840.

Analysis

Lovelace's early mathematical education can be viewed in a broader context of changes in British mathematics (see for example Flood *et al.* 2011). 'Practical mathematics', exemplified by Pasley, the arithmetic and geometry needed for accounting, surveying, navigation and warfare, had long been a staple of education for anyone above the labouring classes. Those with the opportunity to move beyond this, might, like Lady Byron, have learned enough algebra to solve simple equations, and some Euclidean geometry and trigonometry from textbooks such as Vince (1800).

Frend and King were products of the old-fashioned university system described by King to Ada: her later mentors, Babbage, De Morgan, and Somerville, were at the heart of attempts to change it, and to align mathematical education and research with newer more rigorous approaches emerging from Europe (Craik 2016).

The widespread availability and use of elementary mathematics also led to a surprising familiarity with mathematics in popular culture—young ladies did examples from Euclid for pleasure, periodicals like *The Ladies' Diary* published mathematical questions and readers' answers, and astronomy was a popular pursuit. A number of works, such as Somerville's 1834 *On the connexion of the physical sciences*, set out to explain science to the general reader in mathematical terms, but without explicit mathematics.

Although Lovelace was enthusiastic about mathematics and science, her background when she started studying with De Morgan in 1840 was very patchy. There is evidence of childhood arithmetic, Pasley's practical geometry, a little Euclid, and observational astronomy; she had started to tackle the somewhat old-fashioned books recommended by Dr King, and been introduced to more recent treatments by Mary Somerville. While King, Somerville, and Frend are sometimes described as 'tutors', they did not provide regular face-to-face instruction, but were unpaid family friends, who made suggestions for what to read, which Ada followed up with questions by letter when she encountered difficulties. It is hard to interpret remarks like 'I read Mathematics every day': did she work hard at every point, or skim material she found less congenial? Certainly, her later studies with De Morgan seem to be where she learned more reflective habits of study, to set realistic expectations, and to go more slowly (Hollings *et al.* 2017).

The letters show evidence of an enquiring mind, stretching her correspondents through formulating her own questions about rainbows or variants of Euclid. She retained her childhood interest in mechanical devices, keen to get hold of mathematical models to accompany her studies of spherical trigonometry, and pleased to accompany her mother on a tour of factories in 1834 (Stein 1985, 46). She, and her mother and husband, also read the latest scientific works, for example the Bridgewater Treatises of Babbage (1837) and Whewell (1833), and works by Somerville and von Humboldt.

We can now return to the claims of Stein and Swade regarding the extent of Lovelace's mathematical knowledge prior to her studies with De Morgan.

Stein inferred from Ada's seemingly trivial questions to Mary Somerville about trigonometry that her mathematical (and, in particular, algebraic) ability in the mid-1830s was low. However, Ada's questions become more understandable when one realizes that, until she read Legendre, her study of trigonometry had been entirely in the older synthetic tradition, in which algebra was not used. Dr King had told her to study Vince's *Trigonometry*, which proved (1) and (2) by an elaborate geometric construction (Vince 1800, 55), without reference to 'negative' angles, or formulae such as $\sin(-x) = -\sin x$: only lengthy verbal explanations in terms of 'complements' and 'supplements' of angles, in place of the crisper algebraic treatment of later work. Thus Ada's query shows merely that she had overlooked the importance of the newer algebraic formulae, which she had first encountered only a few pages before.

We turn now to Swade's claim that Lovelace, who did not study the theory of finite differences until 1841, lacked the knowledge to understand Dionysius Lardner's 1834 paper on Babbage's Difference Engine (Lardner 1834). Lardner was a science popularizer: his 64-page paper combined advocacy for the importance of the engines, a plea for their construction, and an idea of the mechanisms 'conveying, even to readers unskilled in mathematics, some satisfactory notions of a general nature on this subject' (Lardner 1834, 267). The method of differences occupies only eight pages, explained entirely through a lengthy worked arithmetical example, with no reference to the mathematical theory. A further fourteen pages explain the representation of decimal numbers by dials, and the mechanisms—the 'wheel work'—for addition and carrying. The rest of the paper comprises a lengthy account of the production of tables by hand; arguments for the importance of the Difference Engine in reducing error and speeding up the process; and a request that the government set up an enquiry into how it might be constructed without delay. It was exactly Lardner's intention that the paper should be accessible to a general reader, and someone like Lovelace, who combined an enthusiasm for and ability in elementary mathematics, an interest in machinery, and a familiarity with popular science writing, would surely have been able to grasp his account of the method of differences and the working of the engine.

For the next few years Lovelace focused more on her family than her mathematics, and it was not until 1840 that she resumed her studies with Augustus De Morgan, rapidly developing more advanced mathematical skills (Hollings *et al.* 2017). This paper has described her education up to that point, and is the first investigation of her mathematical knowledge and skills situated in the context of the books she studied, and the mathematical culture of the time. We have shown how assertions by Stein of Lovelace's lack of mathematical ability, and Swade's claim of her early lack of understanding of the workings of the Difference Engine, have influenced the trajectory of later scholarly and popular work, with a focus shifting from Lovelace's technical achievements to her more 'imaginative' writing. Thus, in rebutting these assertions, we have brought into question widely circulated assessments of Lovelace's mathematical competence, as well as showing the important role of specialist historians of mathematics in studying mathematical archives.

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