A q-weighted Robinson-Schensted Algorithm

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Main references

This talk is mainly based on the following two papers:

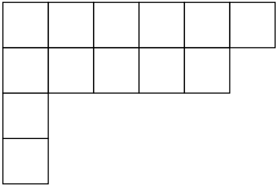
- Neil O'Connell and Yuchen Pei. A q-weighted version of the Robinson-Schensted algorithm.
  Electronic Journal of Probability, 18(0), October 2013
- Yuchen Pei. A symmetry property for q-weighted Robinson-Schensted and other branching insertion algorithms.
  Journal of Algebraic Combinatorics, 40(3):743–770, November 2014

Outline

- The (classical) Robinson-Schensted (RS) algorithm
- A q-weighted Robinson-Schensted (qRS) algorithm
- A symmetry property for the qRS algorithm

Young diagrams and tableaux

- Fix a positive integer ℓ.
- A partition / Young diagram: $\lambda = (\lambda_1, \ldots, \lambda_\ell) \in \mathbb{N}_{\geq 0}^{\ell}$ such that $\lambda_i \geq \lambda_{i+1} \forall i$.
- Size of a diagram: the number of boxes $|\lambda| := \lambda_1 + \cdots + \lambda_\ell$.
- A tableau: a Young diagram filled with numbers of $[\ell]$ non-decreasing along the rows and increasing along the columns.
- Shape of a tableau: the corresponding Young diagram.



$\ell = 5$, $\lambda = (6, 5, 1, 1, 0)$, $|\lambda| = 13$.

Young diagrams and tableaux

- Fix a positive integer ℓ.
- A partition / Young diagram: $\lambda = (\lambda_1, \ldots, \lambda_\ell) \in \mathbb{N}_{\geq 0}^{\ell}$ such that $\lambda_i \geq \lambda_{i+1} \forall i$.
- Size of a diagram: the number of boxes $|\lambda| := \lambda_1 + \cdots + \lambda_\ell$.
- A tableau: a Young diagram filled with numbers of $[\ell]$ non-decreasing along the rows and increasing along the columns.
- Shape of a tableau: the corresponding Young diagram.

1	2	2	2	3	5
2	3	4	4	5	
3					
5					

$\ell = 5$, a tableau with shape $\lambda = (6, 5, 1, 1, 0)$.

Young diagrams and tableaux

- Fix a positive integer ℓ.
- A partition / Young diagram: $\lambda = (\lambda_1, \ldots, \lambda_\ell) \in \mathbb{N}_{\geq 0}^{\ell}$ such that $\lambda_i \geq \lambda_{i+1} \forall i$.
- Size of a diagram: the number of boxes $|\lambda| := \lambda_1 + \cdots + \lambda_\ell$.
- A standard tableau: a Young diagram of size n filled with numbers exactly from $\{1, 2, \ldots, n\}$ increasing along the rows and increasing along the columns.
- Shape of a tableau: the corresponding Young diagram.

1	3	5	6	8	10
2	4	7	11	12	
9					
13					

$\ell = 5$, a standard tableau with shape $\lambda = (6, 5, 1, 1, 0)$.

Background of the RS algorithm

- Let d_λ be the number of standard tableaux of shape λ.
- Initial motivation:

$$\sum_{\lambda \vdash n} d_\lambda^2 = n!.$$

- This asks for a correspondence between permutations and pairs of tandard tableaux of the same shape.

The rule of insertion

To insert k into tableau T, which has λ_j^i number of letters no greater than i in the jth row:

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
2. If $k = \infty$ then we are done; otherwise go to step 1.

Robinson-Schensted algorithm: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
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Suppose we insert a 3 into

$$\begin{array}{llllll}
1 & 2 & 2 & 2 & 3 & 5 \\
2 & 3 & 4 & 4 & 5 & \\
3 & & & & & \\
5 & & & & &
\end{array}$$

Robinson-Schensted algorithm: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
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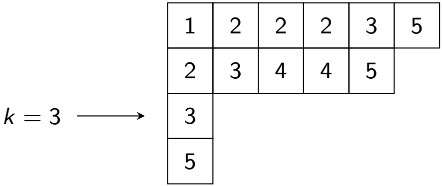

$k = 3 \longrightarrow$ (third row)

1	2	2	2	3	5
2	3	4	4	5	
3					
5					

Figure: $j = 3, \lambda_2^2 = \lambda_3^3 = 1$

Robinson-Schensted algorithm: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
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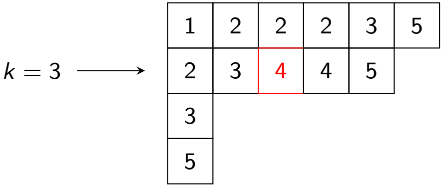

$k = 3 \longrightarrow$

1	2	2	2	3	5
2	3	4	4	5	
3					
5					

Figure: $j = 2, \lambda_1^2 > \lambda_2^3$

Robinson-Schensted algorithm: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
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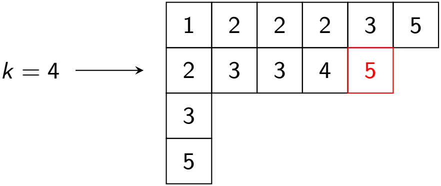

$k = 4 \longrightarrow$

1	2	2	2	3	5
2	3	3	4	5	
3					
5					

Figure: $j = 2,\lambda_1^3 > \lambda_2^4$

Robinson-Schensted algorithm: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
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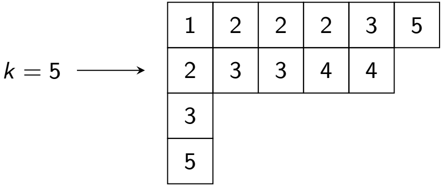

1	2	2	2	3	5
2	3	3	4	4	
3					
5					

Figure: $j = 2, \lambda_1^4 = \lambda_2^5$

Robinson-Schensted algorithm: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
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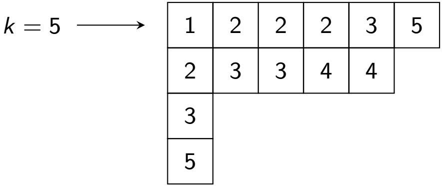

1	2	2	2	3	5
2	3	3	4	4	
3					
5					

Figure: $j = 1$

Robinson-Schensted algorithm: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
2. If $k = \infty$ then we are done; otherwise go to step 1.

$k = \infty$

1	2	2	2	3	5	5
2	3	3	4	4		
3						
5						

Figure: $j = 1$

The Markov chain in the rank-$(\ell - 1)$ case

- Take a random word w: given $a_i \geq 0$ and $a_1 + \cdots + a_\ell = 1$, let w_i be i.i.d. with $\mathbb{P}(w_i = j) = a_j$.
- Denote by $\mathcal{T}_\lambda$ the set of all the tableaux of shape λ. Define the Schur polynomial s_λ to be

$$s_\lambda(a) := \sum_{P \in \mathcal{T}_\lambda} a^P = \frac{\det(a_i^{\lambda_j + \ell - j})_{1 \leq i,j \leq \ell}}{\det(a_i^{\ell - j})_{1 \leq i,j \leq \ell}}.$$

  Here $a^P := a_1^{\#1\text{s in } P} a_2^{\#2\text{s in } P} \dots a_\ell^{\#\ell\text{s in } P}$.
- Starting with the empty tableau, the shape of the tableau is an $\ell-$dimensional Markov chain, with transition kernel (O'Connell'03):

$$p(\mu, \lambda) = \frac{s_\lambda(a)}{s_\mu(a)} \mathbb{I}_{\mu \nearrow \lambda}.$$

Remarks

- Greene's Theorem $+$ matrix input (RSK) $\rightarrow$ directed percolation, (Schur functions)
- $(\max, +) \rightarrow (+, \times)$: geometric RSK and directed polymer model (O'Connell'12, Corwin-O'Connell-Seppäläinen-Zygouras'14), (Whittaker functions)
- q-Whittaker functions interpolate between the Schur and the Whittaker functions $\rightarrow$ motivation for a q-RS algorithm.

A q-weighted RS algorithm

Higher-rank q case

- In general, given a tableau T associated with (λ^i_j), for $0 \leq q \leq 1$ define probabilities

$$f_0(P,k,j) = 1 - q^{\lambda^{k-1}_{j-1} - \lambda^k_j}; \quad f_1(P,k,j) = \frac{1 - q^{\lambda^{k-1}_{j-1} - \lambda^k_j}}{1 - q^{\lambda^{k-1}_{j-1} - \lambda^{k-1}_j}}; \quad k \geq 1$$

$$f_0(P,1,j) = f_1(P,1,j) = 1.$$

- The algorithm of q-inserting a $k \in [\ell]$ into a tableau P (O'Connell-Pei'13) is described as follows (initialise $j = k$, $b = 0$):
  1. With probability $1 - f_b(P,k,j)$ set $j \leftarrow j - 1$ and $b = 0$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$ and $b = 1$.
  2. If $k = \infty$ then we are done; otherwise go to step 1.

qRS: an example

Suppose we q-insert a 3 into

$$P = \begin{array}{cccccc} 1 & 2 & 2 & 2 & 3 & 5 \\ 2 & 3 & 4 & 4 & 5 & \\ 3 & & & & & \\ 5 & & & & & \end{array}$$

What's the probability of getting the same tableau

$$\tilde{P} = \begin{array}{ccccccc} 1 & 2 & 2 & 2 & 3 & 5 & 5 \\ 2 & 3 & 3 & 4 & 4 & & \\ 3 & & & & & & \\ 5 & & & & & & \end{array}$$

produced by the classical Robinson-Schensted algorithm?

qRS: an example

1. With probability $1 - f_b(P, k, j)$ set $j \leftarrow j - 1$ and $b = 0$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$ and $b = 1$.
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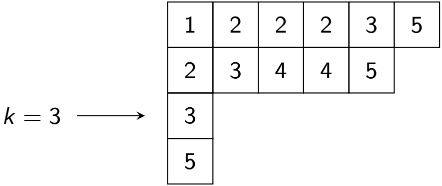

1	2	2	2	3	5
2	3	4	4	5	
3					
5					

Figure: $j = 3, \lambda_2^2 - \lambda_3^3 = 0$, $1 - f_0(P, 3, 3) = 1$.

qRS: an example

1. With probability $1 - f_b(P, k, j)$ set $j \leftarrow j - 1$ and $b = 0$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$ and $b = 1$.
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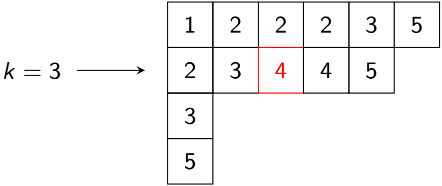

$k = 3 \longrightarrow$

1	2	2	2	3	5
2	3	4	4	5	
3					
5					

Figure: $j = 2, f_0(P, 3, 2) = 1 - q^{\lambda_1^2 - \lambda_2^3} = 1 - q^2$.

qRS: an example

1. With probability $1 - f_b(P, k, j)$ set $j \leftarrow j - 1$ and $b = 0$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$ and $b = 1$.
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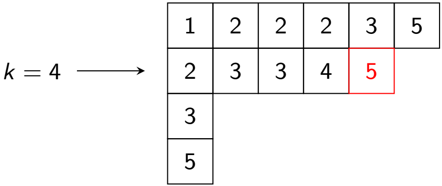

$k = 4 \longrightarrow$

1	2	2	2	3	5
2	3	3	4	5	
3					
5					

Figure: $j = 2, f_1(P, 4, 2) = \frac{1-q^{\lambda_1^3-\lambda_2^4}}{1-q^{\lambda_1^3-\lambda_2^3}} = \frac{1-q}{1-q^3}$.

qRS: an example

1. With probability $1 - f_b(P, k, j)$ set $j \leftarrow j - 1$ and $b = 0$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$ and $b = 1$.
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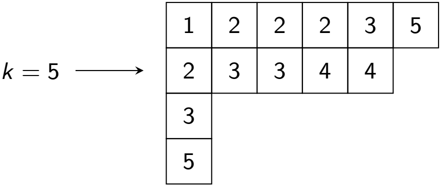


$k = 5 \longrightarrow$

1	2	2	2	3	5
2	3	3	4	4	
3					
5					

Figure: $j = 2{,}1 - f_1(P, 5, 2) = 1 - \frac{1-q^{\lambda_1^4-\lambda_2^5}}{1-q^{\lambda_1^4-\lambda_2^4}} = 1.$

qRS: an example

1. With probability $1 - f_b(P, k, j)$ set $j \leftarrow j - 1$ and $b = 0$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$ and $b = 1$.
2. If $k = \infty$ then we are done; otherwise go to step 1.

$k = 5 \longrightarrow$

1	2	2	2	3	5
2	3	3	4	4	
3					
5					

Figure: $j = 1, f_0(P, 5, 1) = 1$

qRS: an example

1. If $\lambda_{j-1}^{k-1} = \lambda_j^k$ and $j > 1$ then set $j \leftarrow j - 1$; otherwise k displaces the first number s in jth row of the tableau that is greater than k ($s = \infty$ and k is appended at the end of the row if no such number exists) and set $k \leftarrow s$.
2. If $k = \infty$ then we are done; otherwise go to step 1.

$k = \infty$

1	2	2	2	3	5	5
2	3	3	4	4		
3						
5						

Figure: $j = 1$

$$\mathbb{P}(\text{obtaining } \tilde{P} \text{ after inserting 3 into } P) = 1 \cdot (1 - q^2) \cdot \frac{1 - q}{1 - q^3} \cdot 1 \cdot 1.$$

qRS: an example

All possible outputs of q-inserting a 3 into P:

$$\begin{array}{lllllll} 1 & 2 & 2 & 2 & 3 & 5 & 5 \\ 2 & 3 & 3 & 4 & 4 & & \\ 3 & & & & & & \\ 5 & & & & & & \end{array} \qquad (1-q^2)\frac{1-q}{1-q^3};$$

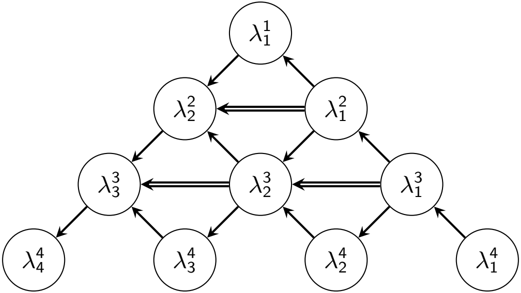
$$\begin{array}{lllllll} 1 & 2 & 2 & 2 & 3 & 4 & 5 \\ 2 & 3 & 3 & 4 & 5 & & \\ 3 & & & & & & \\ 5 & & & & & & \end{array} \qquad (1-q^2)\left(1-\frac{1-q}{1-q^3}\right);$$

$$\begin{array}{lllllll} 1 & 2 & 2 & 2 & 3 & 3 & 5 \\ 2 & 3 & 4 & 4 & 5 & & \\ 3 & & & & & & \\ 5 & & & & & & \end{array} \qquad q^2.$$

The q-Whittaker functions

Define $(n)_q = (1-q)(1-q^2)\dots(1-q^n)$. Define a function κ on $\mathcal{T}_\lambda$ by

$$\kappa(P) = \frac{\prod_{\textcircled{a} \Rightarrow \textcircled{b}} (a-b)_q}{\prod_{\textcircled{c} \to \textcircled{d}} (c-d)_q}.$$



The q-Whittaker functions

- Define the q-binomial coefficients

$$\begin{bmatrix} m+n \\ n \end{bmatrix}_q := \frac{(m+n)_q}{(n)_q(m)_q}.$$

- The explicit formula for κ:

$$\kappa(P) = \frac{1}{\prod_{1\le i\le \ell-1}(\lambda_i-\lambda_{i+1})_q} \prod_{1\le j<k\le \ell} \begin{bmatrix} \lambda_j^k - \lambda_{j+1}^k \\ \lambda_j^{k-1} - \lambda_{j+1}^k \end{bmatrix}_q .$$

- Then the q-Whittaker function parameterised by partition λ is defined as (Gerasimov-Lebedev-Oblezin'10)

$$\Psi_\lambda(a) = \sum_{T\in\mathcal{T}_\lambda} \kappa(P)a^P.$$

The q-Whittaker functions as eigenfunctions

The q-Whittaker fuctions Ψ_a are also eigenfunctions of the Hamiltonian L of q-deformed $\mathrm{gl}(\ell;\mathbb{C})$-Toda chain (Ruijsenaars'90, Gerasimov-Lebedev-Oblezin'11):

$$L\Psi_a(\lambda) = \left(\sum_{i=1}^{\ell} a_i\right)\Psi_a(\lambda),$$

where the integral operator L has kernel

$$K(\lambda,\mu) = \begin{cases} 1 - q^{\lambda_i - \lambda_{i+1}+1}, & \text{if } \mu = \lambda + \mathrm{e}_i \text{ for some } 1 \le i \le \ell - 1; \\ 1, & \text{if } \mu = \lambda + \mathrm{e}_\ell; \\ 0, & \text{otherwise.} \end{cases}$$

The q-Whittaker functions as the Macdonald polynomials

Define $(a,q)_n := (1-a)(1-aq)\dots(1-aq^{n-1})$.
The (type A) Macdonald polynomial (Macdonald'95) is

$$P_\lambda(a;q,t) = \sum_{P\in\mathcal{T}_\lambda} a^P \prod_{\substack{2\le i\le \ell\\ 1\le j\le i-1\\ 1\le k\le i-j}} \frac{\dfrac{\left(q^{\lambda_k^{i-1}-\lambda_{k+j-1}^{i-1}} t^j;q\right)_{\lambda_{k+j-1}^{i-1}-\lambda_{k+j}^{i}}}{\left(q^{\lambda_k^{i-1}-\lambda_{k+j-1}^{i-1}+1} t^{j-1};q\right)_{\lambda_{k+j-1}^{i-1}-\lambda_{k+j}^{i}}}}{\dfrac{\left(q^{\lambda_k^{i}-\lambda_{k+j-1}^{i-1}} t^j;q\right)_{\lambda_{k+j-1}^{i-1}-\lambda_{k+j}^{i}}}{\left(q^{\lambda_k^{i}-\lambda_{k+j-1}^{i-1}+1} t^{j-1};q\right)_{\lambda_{k+j-1}^{i-1}-\lambda_{k+j}^{i}}}},$$

and

$$\Psi_a(\lambda) = \prod_{i=1}^{\ell-1} (\lambda_i - \lambda_{i+1})_q^{-1} P_\lambda(a;t=0).$$

Coupling two randomnesses

- We apply the qRS algorithm to the random word w.
- (O'Connell-Pei'13) The shape evolves as a Markov chain with transition kernel

$$P(\mu, \lambda) = \frac{\Psi_a(\lambda)}{\Psi_a(\mu)} K(\mu, \lambda) \mathbb{I}_{\mu \nearrow \lambda}.$$

- This interpolates between the classical case ($q = 0$) and the directed polymer case ($q \to 1$).
- Part of the stochastic tableau also evolves as q-TASEP with the wedge initial condition.

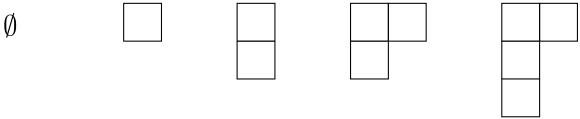
The symmetry property

The growth partitions

- A tableau T is identified with the shapes λ^i of its subtableaux with entries no greater than i.
- For a standard tableau, λ^{i+1} is obtained from λ^i by adding a box.
- An example:

$$T = \begin{array}{ll} 1 & 3 \\ 2 & \\ 4 & \end{array}$$

can be identified by the growing partitions



$\emptyset$ 1 11 21 211

$T = (1 \nearrow 11 \nearrow 21 \nearrow 211)$.

Permutation words

- Let $P(m)$ be the P-tableau after inserting $(w_1, w_2, \ldots, w_m)$.
- The RS algorithm also produces a recording tableau Q, identified with the growth of P over time:

$$Q = (\mathrm{sh}P(0) \nearrow \mathrm{sh}P(1) \nearrow \cdots \nearrow \mathrm{sh}P(n)).$$

- Given a permutation σ of length n, we identify it with its permutation word $(\sigma(1), \sigma(2), \ldots, \sigma(n))$.
- Applying the RS algorithm to a permutation, the output (P, Q) is a pair of standard tableaux.

RS correspondence: An example

Inserting $(1, 4, 2, 3)$:

$$P(0) = \emptyset \qquad\qquad Q(0) = \emptyset$$

$$P(1) = \begin{array}{l} 1 \end{array} \qquad\qquad Q(1) = \begin{array}{l} 1 \end{array}$$

$$P(2) = \begin{array}{l} 1 \\ 4 \end{array} \qquad\qquad Q(2) = \begin{array}{l} 1 \\ 2 \end{array}$$

$$P(3) = \begin{array}{ll} 1 & 4 \\ 2 & \end{array} \qquad\qquad Q(3) = \begin{array}{ll} 1 & 3 \\ 2 & \end{array}$$

$$P(4) = \begin{array}{ll} 1 & 4 \\ 2 & \\ 3 & \end{array} \qquad\qquad Q(4) = \begin{array}{ll} 1 & 3 \\ 2 & \\ 4 & \end{array}$$

The symmetry property for the classical RS algorithm

If we denote by $P(\sigma)$ and $Q(\sigma)$ the pair of tableaux obtained by applying the RS algorithm to σ, then

$$(P(\sigma^{-1}), Q(\sigma^{-1})) = (Q(\sigma), P(\sigma)).$$

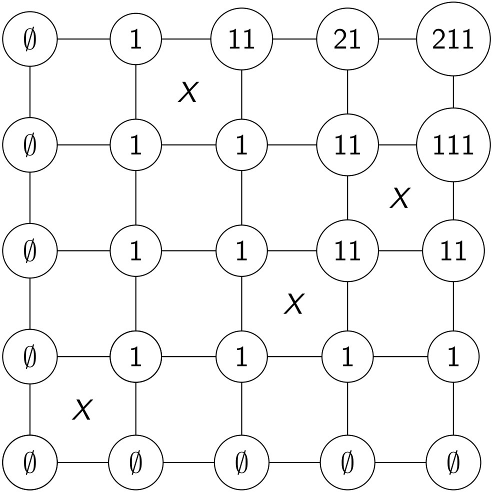
The symmetry property for the classical RS algorithm

If we denote by $P(\sigma)$ and $Q(\sigma)$ the pair of tableaux obtained by applying the RS algorithm to σ, then

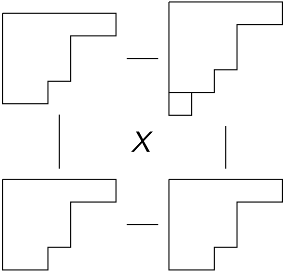
$$(P(\sigma^{-1}), Q(\sigma^{-1})) = (Q(\sigma), P(\sigma)).$$

This can be proved using the growth diagram technique introduced by Sergey Fomin (Fomin'94).

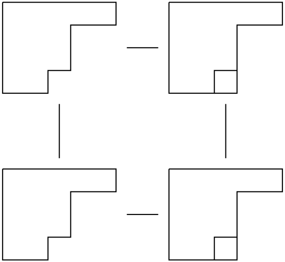
Proof of the symmetry property for $\sigma = (1, 4, 2, 3)$



The growth diagram local rules

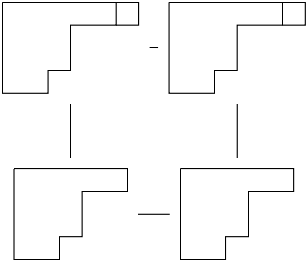


Case 1

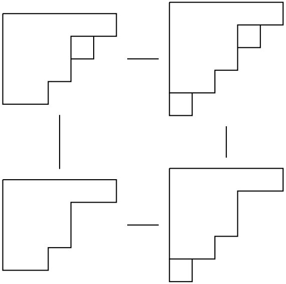


Case 2

The growth diagram local rules (cont'd)

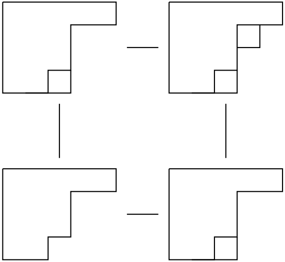


Case 3

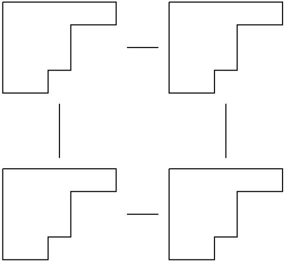


Case 4

The growth diagram local rules (cont'd)

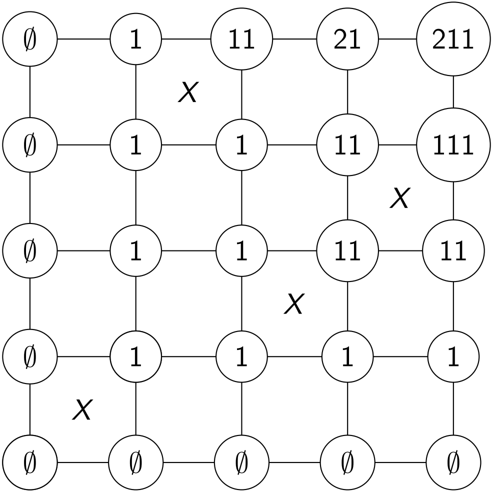


Case 5

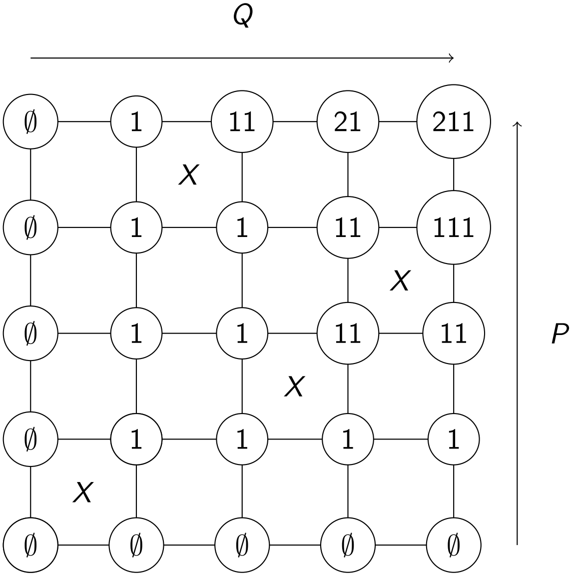


"Boring" intersection
of Case 2 and Case 3

Proof of the symmetry property for $\sigma = (1, 4, 2, 3)$



Proof of the symmetry property for $\sigma = (1, 4, 2, 3)$

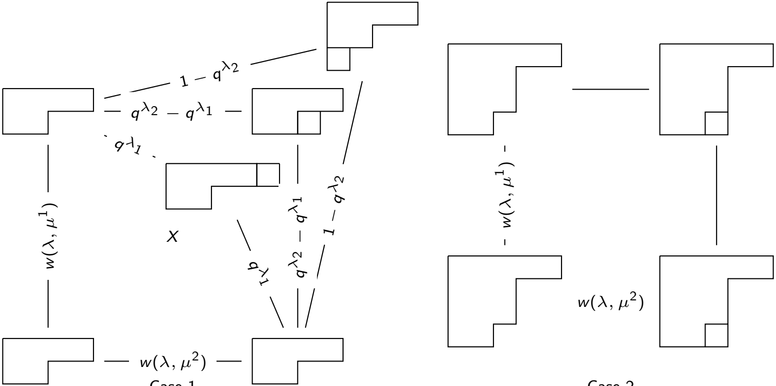


Symmetry property for the qRS

- Similarly we can pair the output tableaux of the qRS with recording Q tableaux.
- (Pei'14) The symmetry property states: if we denote by $\phi_\sigma(P, Q)$ probability of obtaining pair (P, Q), then

$$\phi_{\sigma^{-1}}(P, Q) = \phi_\sigma(Q, P).$$

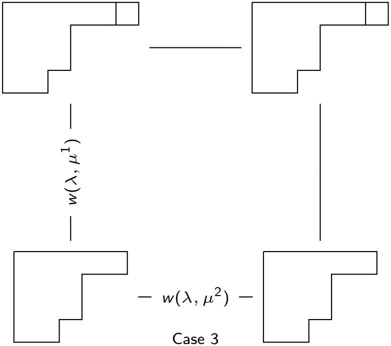
Branching growth diagram local rules



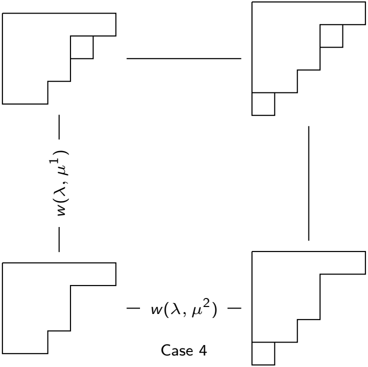
Case 1

Case 2

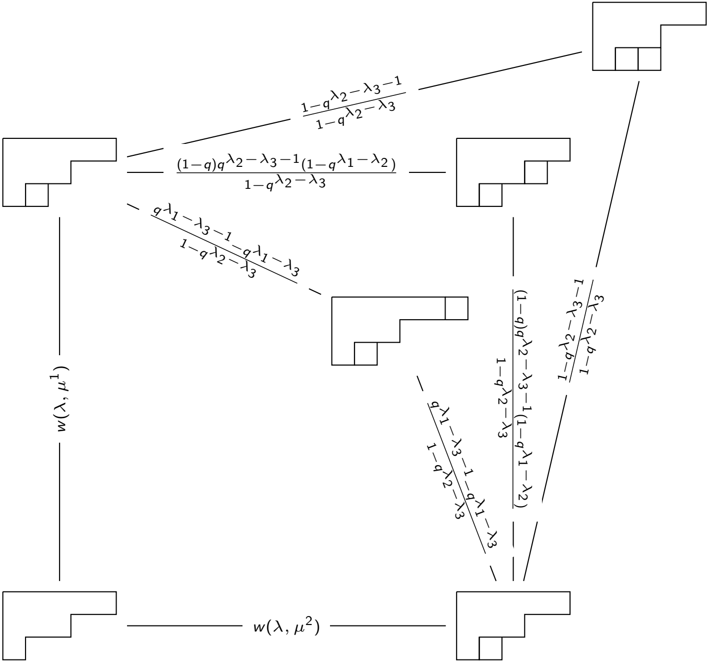
Branching growth diagram local rules (cont'd)



Case 3

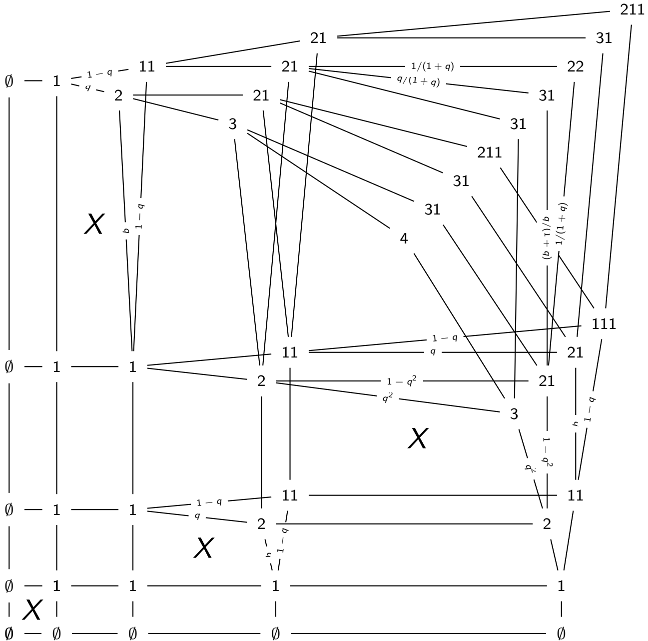


Case 4

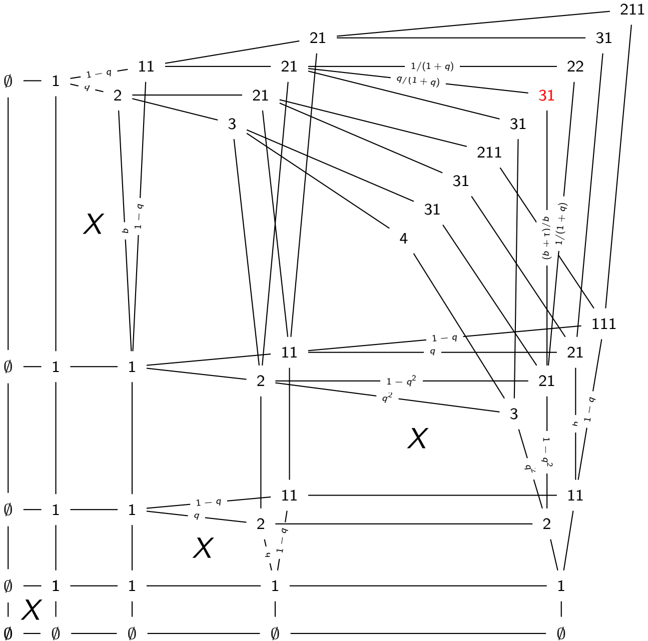

$\frac{1-q^{\lambda_2-\lambda_3-1}}{1-q^{\lambda_2-\lambda_3}}$
$\frac{(1-q)q^{\lambda_2-\lambda_3-1}(1-q^{\lambda_1-\lambda_2})}{1-q^{\lambda_2-\lambda_3}}$
$\frac{q^{\lambda_1-\lambda_3-1}-q^{\lambda_1-\lambda_3}}{1-q^{\lambda_2-\lambda_3}}$
$w(\lambda, \mu^1)$
$\frac{(1-q)q^{\lambda_2-\lambda_3-1}(1-q^{\lambda_1-\lambda_2})}{1-q^{\lambda_2-\lambda_3}}$
$\frac{1-q^{\lambda_2-\lambda_3-1}}{1-q^{\lambda_2-\lambda_3}}$
$\frac{q^{\lambda_1-\lambda_3-1}-q^{\lambda_1-\lambda_3}}{1-q^{\lambda_2-\lambda_3}}$
$w(\lambda, \mu^2)$

Case 5

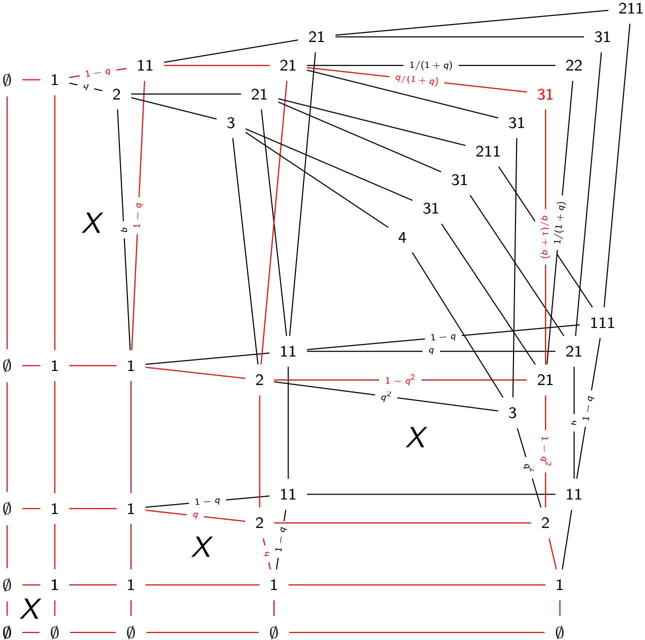
Proof of the symmetry property for permutation $(1, 4, 2, 3)$



Proof of the symmetry property for permutation $(1, 4, 2, 3)$



Proof of the symmetry property for permutation $(1, 4, 2, 3)$



Proof of the symmetry property for permutation $(1, 4, 2, 3)$

This contributes the probability $q \cdot (1 - q^2) \cdot (1 - q) \cdot q/(1 + q) = q^2(1 - q)^2$ to the pair

$$(P, Q) = \left(\begin{array}{ccc} 1 & 2 & 4 \\ 3 & & \end{array} , \begin{array}{ccc} 1 & 3 & 4 \\ 2 & & \end{array} \right)$$

Recent development

- Borodin-Petrov'13: The row insertion version.
- Matveev-Petrov'15: The next talk - no spoiler :)

Thank you for your attention!

Neil O'Connell and Yuchen Pei.
A *q*-weighted version of the Robinson-Schensted algorithm.
Electronic Journal of Probability, 18(0), October 2013.

Yuchen Pei.
A symmetry property for *q*-weighted Robinson-Schensted and other branching insertion algorithms.
Journal of Algebraic Combinatorics, 40(3):743–770, November 2014.