

## Outline of content of Nordström's lecture series

(tentative—subject to revision based on audience needs)

One of the central questions in  $G_2$  geometry is: Which closed 7-manifolds admit metrics with holonomy  $G_2$ ? In our current understanding there is a gulf between the known necessary topological conditions, and the known existence results which provide a finite number of examples by gluing methods. The aim of the lecture series is to give a picture of both sides of this story. The first half discusses the necessary topological conditions, which requires an outline of characteristic class theory. The second half discusses one of the known constructions of closed  $G_2$ -manifolds.

### Lecture 1: Chern-Weil theory

- Characteristic classes of bundles on manifolds via curvature
- Definition of Chern, Pontrjagin and Euler classes

### Lecture 2: Characteristic classes and classifying spaces

- Chern, Pontrjagin and Euler classes with integer coefficients
- Stiefel-Whitney classes
- Spin structures

### Lecture 3: Necessary conditions for holonomy $G_2$

- Consequences of  $G_2$ -structures and holonomy  $G_2$
- Contrast with Calabi-Yau theorem (Holonomy  $SU(n)$ )

### Lecture 4-5: Twisted connected sum construction

- Asymptotically cylindrical holonomy  $SU(3)$  manifolds from algebraic geometry data
- Gluing and solution of the PDE
- Topological calculations
- Holomorphic curves and associatives