

Calibrated Submanifolds

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LMS–CMI Research School: An Invitation to Geometry and Topology via G_2

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Outline

Lecture 1: Minimal submanifolds; introduction to calibrations

Minimal submanifolds: definition and examples

First variation of volume; mean curvature vector; minimal graph equation

Calibrations and calibrated submanifolds; calibrated submanifolds are volume-minimizing

Calibrations and holonomy

Lecture 2: Complex and special Lagrangian submanifolds

Calibrations and complex submanifolds in Kähler manifolds; Wirtinger's inequality

Special Lagrangian calibration

Examples of special Lagrangian submanifolds in Euclidean space and in Calabi–Yau manifolds

Lecture 3: Constructing calibrated submanifolds; nonlinear elliptic PDE

Methods for constructing calibrated submanifolds

Nonlinear elliptic PDE: regularity and methods of solution; gluing problems

Deformations of special Lagrangian submanifolds

Special Lagrangian isometric embeddings

Resolving special Lagrangian self-intersections

Lecture 4: The angle theorem; associative and coassociative submanifolds

The angle theorem; Lawlor necks and Nance calibrations

Associative and coassociative calibrations; relationship with complex and special Lagrangian geometry

Examples of associative and coassociative submanifolds

Dirac operator and deformations of associative submanifolds

Lecture 5: Coassociative and Cayley submanifolds; gauge theory

Deformations of coassociative submanifolds

Coassociative isometric embeddings

Cayley calibration: relationship with other calibrated geometries; examples

Deformations of Cayley submanifolds

Yang–Mills connections; G_2 instantons; gauge theory and calibrated geometry