Calibrated Submanifolds

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LMS–CMI Research School: An Invitation to Geometry and Topology via G_2
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Outline

Lecture 1: Minimal submanifolds; introduction to calibrations
Minimal submanifolds: definition and examples
First variation of volume; mean curvature vector; minimal graph equation
Calibrations and calibrated submanifolds; calibrated submanifolds are volume-minimizing
Calibrations and holonomy

Lecture 2: Complex and special Lagrangian submanifolds
Calibrations and complex submanifolds in Kähler manifolds; Wirtinger’s inequality
Special Lagrangian calibration
Examples of special Lagrangian submanifolds in Euclidean space and in Calabi–Yau manifolds

Lecture 3: Constructing calibrated submanifolds; nonlinear elliptic PDE
Methods for constructing calibrated submanifolds
Nonlinear elliptic PDE: regularity and methods of solution; gluing problems
Deformations of special Lagrangian submanifolds
Special Lagrangian isometric embeddings
Resolving special Lagrangian self-intersections

Lecture 4: The angle theorem; associative and coassociative submanifolds
The angle theorem; Lawlor necks and Nance calibrations
Associative and coassociative calibrations; relationship with complex and special Lagrangian geometry
Examples of associative and coassociative submanifolds
Dirac operator and deformations of associative submanifolds

Lecture 5: Coassociative and Cayley submanifolds; gauge theory
Deformations of coassociative submanifolds
Coassociative isometric embeddings
Cayley calibration: relationship with other calibrated geometries; examples
Deformations of Cayley submanifolds
Yang–Mills connections; G_2 instantons; gauge theory and calibrated geometry