Contents 1988

1988-1

January 16, 17: Goodwillie, Feigin, Tzygan result on the cyclic complex of a semi-direct product $R \oplus M$.

January 22: On the bi-functor $(K, L) \mapsto K \oplus_R L$.

January 23, 24: Calculations with $A = R/I$ where $R = T(V)$. Calculations with $R \oplus \Omega^1_R$, $R = T(V)$. Maps of complexes of $R$-bundles $\mathcal{B}(R) \otimes_R \mathcal{B}(R)$.

January 26: On a differential in the spectral sequences needed for the exact sequences

$$0 \to \mathcal{I}C_{2n+1}(A) \to I^{n+1}/[I, I^n] \xrightarrow{d'} H_1(R, I^n) \to \mathcal{I}C_{2n}(A) \to 0,$$

$$0 \to HC^2_n(A) \to HC_{0}(R/I^n+1) \xrightarrow{d'} H_1(R, R/I^n) \sigma \to HC_{2n-1}(A) \to 0.$$

January 27: On maps $(I \otimes_R)^{p+1} \to (I \otimes_R)^p$.

January 28: Resolution of paradox concerning $\mathcal{B}(R) \otimes_R R[\Delta(1)] \otimes_R \mathcal{B}(R) \to \mathcal{B}(R)$.

January 29: Horizontal differential $\sigma : \Sigma(I \otimes_R)^{p+1} \to (I \otimes_R)^p$.

January 30: To find the $S$-operator on the double complex with columns $\Sigma(I \otimes_R)^{p+1}$.

February 2: The reduced complex $\mathcal{C}(R)$ is quasi-isomorphic to $CC(A)/CC(C)$.

February 16: Notes on Walhausen’s big theorem, cyclic nerve of a category and Goodwillie’s geometric trace map. Also notes on Wodzicki’s spectral sequence.

February 19: Reduced cyclic complex of $R \leftarrow I$ where $R$ is unital. Morita invariance of Hochschild and cyclic homology.


February 26: Atiyah-Bott-Lefschetz formula for a Riemann surface.

February 28: Cartier’s residue construction.

February 29: Reconstructing how Tate found his 1-dimensional residue from Cartier’s definition.

March 1: On the isomorphism $\text{Ext}^1_A(M, N) = \text{Der}(A, \text{Hom}_k(M, N))/\text{ImHom}(M, N)$.

March 2: Calculations with $A$-module resolutions, cup products and non-commutative differential forms.

March 3: On Beilinson and Schechtman paper, ”Determinant bundles and Virasoro algebras”.

March 4: The Virasoro algebra $\hat{\mathfrak{g}}_E$ of $(X, E)$ where $E$ is a vector bundle over a curve $X$.

March 5: The Riemann-Roch theorem on a complete non-singular curve.

March 8: Generalization of the Cartier-Grothendieck residue symbol. Residues over an algebraic curve. Cyclic 1-cocycle associated to an Lefschetz operator on a circle. Tate’s method for constructing a residue as presented by Beilinson-Schechtman.

March 10, 11: More on the Beilinson-Schechtman residue construction.
1988-2


March 13: Beilinson-Schechtman account of Tate’s approach to residues.

March 14, 15: Deriving a formula for the cyclic 2-cycle associated to a square zero extension.

March 16: On Goodwillie’s theorem that a nilpotent extension induces an isomorphism on cyclic homology.

March 17: Alternative approaches to Connes homomorphism.

March 18: More on Connes homomorphism. Interpreting $HC_*(A)$ as the universal recipient group for traces of higher orders.

March 19: Stinespring circle of ideas.

March 20: Representing cyclic cocycles using iterated bimodule extensions.

March 25: Quillen’s method for producing Hochschild classes.

March 26: Calculations related to Tate’s theory concerning a ring $C$ with ideals $I_1, I_2$ such that $C = I_1 \oplus I_2$.

1988-3

March 27: Using a map $\rho : A \to B$ with $\rho(1) = 1$ to make $A \oplus A \otimes B \otimes A$ into an algebra.

March 28: Using Stinespring methods to construct odd cocycles attached to an extension.

March 29, 30, 31: More on Tate’s theory for $R = I_1 \oplus I_2$ where $I_1$ and $I_2$ are ideals. Multiplier algebra.

April 1: Calculations for an ideal $I$ in $R$ where $I^2 = I$.

April 2: Calculations for an algebra $R$, $e$ an idempotents in $R$, $F = 2e - 1$ and $\delta : R \to M$ a derivation where $M$ is an $R$-module.

April 3: On studying cyclic homology using the cyclic complex applied to DG chain algebras. Higher traces. Program and problems.

April 4, 5: On length 2 chain algebra resolution of $\mathbb{C}[\mathbb{Z}/2]$.

April 6: Calculations in an algebra $C$ with a given subalgebra $A$ and an idempotent.

April 7, 8: Chern-Simons algebra.

April 8: On subshifts of finite type, Markov partitions, Cuntz-Krieger $C^*$-algebras,

April 9: Return to $C = A * \mathbb{C}[\mathbb{Z}/2]$.

April 10: Calculations and insights connected with a representation of an algebra with an idempotent.

April 11: Details for a letter to Kassels.

1988-4

April 11, 14: Letter to Kassel in which Qillen describes his latest ideas.


April 15, 16: Subshift of finite type.


April 23: Return to studying cyclic theory of a cochain algebra $C^*(A,B)$ with a 1-cochain $\rho : A \to B$ such that $d\rho + \rho^2 \in C^2(A,I)$.

April 24: Abstract construction of cyclic cocycles attached to unbounded $p$-summable Fredholm modules $(A, H, D)$. Traces on $B/I^\infty$.

April 28 - May 2: Cuntz-Krieger $C^*$-algebras.

May 2: Cyclic homology and extensions.

May 3: Deriving exact sequences for $HC_*(\mathcal{A}) = HC_*(\mathbb{C} \otimes \mathcal{A})$.

May 4, 5: Using Quillen’s Chern-Simons formula for an even Connes homomorphism to prove an index theorem.

May 6: Review earlier work on cyclic cocycles and left invariant forms on groups of gauge transformations. Understanding the $S$-operator.

May 7: Restricting cocycles to a direct summand. More on the $S$-operator. Cyclic cocycles associated to a connection.

May 8: On a super-algebra and an odd involution $F$ such that $[F, \cdot]^2 = 0$. Connes cocycles and two proofs.

May 9, 10: The relationship between Connes cocycles and the $S$-operator. Cyclic cocycles on $C^\infty(M)$.

May 11: Finite dimensional algebras and the $S$-operator.

May 12, 13: Transgression calculation. Formalism explaining odd Connes cocycles.

May 14: Dilation process. Equivalence of cocycles obtained by dilation. Transgression and super-connection methods.

May 15: Geometric approach to super-connection forms.

May 16: Review of formalism used in constructing Connes cocycles. $S$-operator. Proof that different Connes cocycles are related by the $S$-operator.

May 17: The difference between character forms associated to two flat connections on a bundle. Calculations applying super-connection methods to deriving a Chern-Simons form based on a path $d + tA$ of connections.

May 10: BRS-geometry. Witten’s Lagrangian for topological QFT using equivariant differential forms for $g$-manifold.

May 20: Witten’s formula for the Chern-Weil homomorphism on a $G$-manifold with a free $G$-action.

May 21: Example of gauge fixing and associated formulas.

May 22: An integral formula for Witten’s map. Atiyah’s idea.

May 24: Universal relations in the double cochain algebra $C^*(A, \Omega_A^*)$ with canonical element $\theta \in C^1(A, \Omega^*_A)$. Review of the Dubois-Violette, Talon, Viallet results. Relation between the BRS algebra of DVTV and anomalies.

May 25: Chern-Simons forms - components of $\int_0^1 dt \text{tr}(\theta e^{t\theta + (t^2 - t)\theta^2})$.

May 26: Splitting of the Chern-Simons transgression form and cyclic formalism. Review of Connes cocycles and also Quillen’s method.
May 27, 28, 29: The $S$-operation for odd cocycles belonging to an extension.


May 31: Methods for producing left invariant forms on a group of gauge transformations and cyclic cocycles.

1988-6

June 1: Discussion of anomalies. Using material in a paper of Dubois-Violette, Talon and Viallet to produce cyclic cocycles associates to a connection.

June 2: On $\Omega(\mathcal{G} \times M)^{\mathcal{G}} = \Omega(\mathcal{G})^{\mathcal{G}} \otimes \Omega(M)$, where $P$ is a $G$ bundle over $M$, $\mathcal{G} = \text{Aut}(P)$.

June 3, 4: On Bismut forms for a manifold with a $S^1$ action. Cyclic theory using the bicomplex $(A \times A^{\otimes n}, b, B)$ for a unital algebra $A$. Constructing homotopy equivalences between the space of pairs of projectors $e, e'$ on a Hilbert space with $e - e'$ compact and $\text{Im} e$ is of infinite dimension and codimension, and the restricted Grassmannian.

June 5, 6: Linking different cocycles belonging to an extension by the $S$-operator.

June 7: Connes approach to the $S$-operator.

June 10, 11: The isomorphism of $A \ast A$ and $\Omega_A^\ast$. Classical limit filtration construction $\oplus_{p \geq 0} h^p F_q A$ in the context of Clifford and exterior algebras, Weyl and polynomial algebras.

June 12: Construction of Connes cocycles. Connes entire cocycles paper.

June 14: Account of conversation with Stora about gauge fixing and ghosts.

June 15: The BRS algebra and $H^*(\mathcal{G})$.

June 20, 22: Calculations using the left $T(A)/(1 - \rho(1_A))$ module structure on $\Omega_A^\ast$.

June 23: Understanding BRS (examples from Itzykson).

June 24: Finding an additive description of $C = \begin{pmatrix} eCe & eCe \\ \tau Ce & \tau Ce \end{pmatrix}$

June 26: Definitive picture of the relationship between $A \ast A$ and $\Omega_A$.

1988-7

June 27: A good construction of Connes cocycles.

June 28: On the exact sequence

$$0 \rightarrow \overline{H} C_{2n-1}(A) \rightarrow I^n/[I,I^{n-1}] \rightarrow I^{n-1} \otimes_B \Omega^1_B \otimes_B .$$

June 30, July 1: On finding a simple analytical expression for the index given by pairing a Dirac operator with a gauge transformation.

July 4: For $(E, \nabla, \rho)$ over a manifold $M$, on finding a Dirac operator analogue of the associated odd character form.

July 5: Pairing $\alpha \in KK(C(M), \mathbb{C})$ with $KK(\mathbb{C}, C(M)) \rightarrow KK(\mathbb{C}, \mathbb{C})$.

July 7: Wave packet transform on $\mathbb{R}$.

July 8: Return to index theory on the circle.

July 10, 11, 12: More index theory on the circle and analysis of an approach working over the cotangent bundle.

July 13: Return to associating an index to an odd Dirac operator and a gauge transformation.
1988-8

July 15: On reconciling approaches in cyclic theory given by index theory and Lie algebra cohomology of matrices.

July 16: Connecting the index function and the $\eta$ function.

July 18: Proof that Connes cocycles are connected by the $S$-operator.

July 19: Arguments involving the deformation

$$\tilde{F} = \frac{1}{\sqrt{1 + t^2}} \begin{pmatrix} F & 0 \\ 0 & -F \end{pmatrix} + t \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

July 20, 21: Review of formulas.

July 24: More on $\tilde{F} = \frac{1}{\sqrt{1 + t^2}} \begin{pmatrix} F & t \\ t & -F \end{pmatrix}$. Review of some steps of the proof of the $S$-relation for Connes cocycles.

July 26: Finding the $n$th order path algebra. On the $S$-operator relation between cyclic cocycles $\text{tr}(\delta \rho + \rho^2)^n$.

July 27, 28: Calculations for $A = R/I$ where $R$ is a free algebra. Resulting problems.

July 29: Proof that free algebras have trivial cyclic homology.

August 1 - 4: On constructing a natural map $CC(R \leftarrow I) \leftarrow CC(A)$ for $A = R/I$, starting from a lifting $\rho : A \rightarrow R$. General discussion of DG algebras and coalgebras.

1988-9

Quillen’s own index for August 21 - November 11, 1988

August 21: DG algebras, DG coalgebras and twisting cochains.

August 23: Cyclic homology of $k[G]$ where $G$ is a free group.

September 1, 2: Computing the $U(V)$ equivariant forms representing characteristic forms on Grass$(V)$.

September 5: Leray spectral sequence for a principal bundle. The spectral sequence with $E_1^{pq} = H^q(h, \Lambda^p(G/h)^*) \Rightarrow H^{p+q}(G)$ where $h$ is a subalgebra of the Lie algebra $G$. The triangle of complexes $CC(C) \rightarrow CC(A) \rightarrow CC(A)$.

September 6: Showing Connes homomorphisms $HC_{n-1}(A) \rightarrow I^n/[I, I^{n-1}]$ are compatible with the $S$-operation.

September 7: Formalism explaining why character forms are linked by the $S$-operator.

September 8: On the double complex $CC(R \leftarrow I)$ of the $I$-adic filtration of $\Omega^*_R \otimes_R$. Understanding the maps $\Sigma(I \otimes_R)^{n+1}_R \rightarrow (I \otimes_R)^n_R$ from the viewpoint of derived categories.

September 12: Formula for the natural map from $CC(A)$ to the Connes $(b, B)$ category.

September 13: To find a formula for the $S$-operator $\overline{HC}_{2n}(A) \xrightarrow{S} \overline{HC}_0(A) = \overline{A}/[A, A]$ when $A$ is unital.

September 15: Understanding the cyclic theory of $C^*(A, R) = \text{Hom}_{K}(\text{Bar}(A), R)$.

September 16: On the character forms $\text{tr} \left( \frac{\gamma}{m} \right) \in R^n/[R, R]$ where $(R^*, \delta)$ is a DGA, $\rho \in R^1$ and $\gamma = \delta \rho + \rho^2$.

September 17: Calculations for $R = A^* \otimes B$ where $A^*$ is a DGA and $B$ an algebra with ideal $I$. 
September 18: On $\Omega_A = \Omega_A^1 \otimes_A$.

September 20: Proving the $S$-relation between Connes cocycles attached to a Fredholm module.

September 22: On the $S$-relation between the canonical maps from the cyclic complex to De Rham cohomology.

September 23: Clarifying signs encountered in cyclic homology calculations.

September 28: On the bicomplex made from the reduced Hochschild complex with the $(b,B)$ operators.

September 30: On producing bivariant classes in $R\text{Hom}_S(CC(A),CC(B))$.

October 1: Finding a coalgebra structure on $B^N(A)$, the standard normalized bar resolution of $A$ as an $A$-bimodule.

October 2: $F$-modules.

October 3: On interpreting the Hochschild complex. Left invariant forms on gauge transformation groups.

October 4: More invariant theory for gauge transformation groups. Linking up Chern-Simons homomorphisms giving even Connes homomorphisms $HC_{2n}(A) \rightarrow HC_0(R/I^{n+1})$ to Connes graded $F$-cycles.

October 5: $K$-pairings. Calculations for idempotents $e, e'$ over $R$ which are congruent modulo an ideal $I$. Linking up Connes approach to even cyclic cocycles with Quillen’s approach based on Chern-Simons forms.

October 6: On the DGA of (bar)cochains with values in the Cuntz algebras $A\ast A$ and $(A\ast A)\hat{\otimes}\mathbb{C}[F]$.

October 7: On obtaining Jaffe-Lesmiewski cocycles, Cyclic forms.

October 12: Review of the cyclic formalism and Hochschild cohomology.

October 13: The map $\Sigma\mathcal{G} \rightarrow U$ where $\mathcal{G}$ is a group of gauge transformations acting on a space of connections. The map induced by associating to a connection the Cayley transform of the Dirac operator. Constructing odd equivariant forms for the conjugation action of $U(n)$ on itself.

October 14: More on equivariant functions for $U(H)$ acting by conjugation on $U^\infty(U, -1)$.

October 19: Comments on a non-unital $A$-bimodule algebra, on a $*$-algebra acting on a Hilbert space, on Hochschild cohomology, on Lie algebra cochains and on invariant theory.

October 20: Notes on an equivariant vector bundle over a Lie group, on applications to the Fredholm module $(A, H, F)$ and on Lie algebra cochains.

October 21: Lie algebra interpretation of Hochschild cochains which come from a super-trace on $A \ast A$.

October 22: Review of interests.

October 26: Review of the link between the Weil algebra and the Narasimhan-Ramanan theorem viewpoints. Equivariant forms on $U(n)$ with the conjugation action.

October 27: On understanding the JLO big cocycle by considering the cochain algebra $C(A, L(H))[\sigma]$ and using the super-connection $\delta + \theta + X\sigma$.

October 28: Establishing the $S$-relation between cyclic cocycles. Comment on $\Omega^1_R \otimes_R$.

November 3: The exactness of $R^\otimes^4 \rightarrow R^\otimes^3 \rightarrow R \otimes V \otimes R \rightarrow 0$. 

1988-10
November 4: A simple construction of the exact sequence $0 \to c \otimes A \overset{i}{\to} C^\otimes 2 \to C^\otimes 3$ by duality and exactness of $0 \to A[1] \otimes C \overset{i}{\to} C \otimes C \to C \otimes C^\otimes 2$.

November 9: Calculations in the unbounded Fredholm situation.

November 11: Big cochains associated to a bundle with a connection and a cycle.

Lecture Notes

Notes for some lectures on cyclic cohomology.