



**Advances in Probability: Integrability, Universality, and Beyond**  
**September 28 - October 2, 2014**

**Abstracts of Talks**

**Nathanael Berestycki** (University of Cambridge)

Title: Liouville Brownian motion

Abstract: I will introduce and discuss a canonical notion of Brownian motion in the random geometry of Liouville quantum gravity, called Liouville Brownian motion. I will explain the construction and discuss some of its basic properties, for instance related to its heat kernel and to the time spent in the thick points of the Gaussian Free Field. Time permitting I will also discuss a derivation of the KPZ formula based on the Liouville heat kernel. Joint work with C. Garban, R. Rhodes and V. Vargas.

**Alexei Borodin** (Massachusetts University of Technology)

Title: Gaussian Free Field in beta ensembles and random surfaces

Abstract: The goal of the talk is to argue that the two-dimensional Gaussian Free Field is a universal and unifying object for global fluctuations of spectra of random matrices and random surfaces. This viewpoint leads to natural Gaussian processes on larger spaces which, despite their explicit covariance structure, so far lack conceptual understanding.

**Amir Dembo** (Stanford University)

Title: Universal asymptotic for certain particle systems

I will describe recent results about the asymptotic fluctuations in weakly asymmetric exclusion process and for competing Brownian particles. The former follows the KPZ equation and the latter leads to a Stefan problem and the additive stochastic heat equation. This talk is based on joint works with Li-Cheng Tsai, Manuel Cabezas, Andrey Sarantsev and Vladas Sidoravicius.

**Christina Goldschmidt** (University of Oxford)

Title: A line-breaking construction of the stable trees

Abstract: Consider a critical Galton-Watson tree whose offspring distribution lies in the domain of attraction of a stable law of parameter  $\alpha \in (1,2]$ , conditioned to have total progeny  $n$ . The stable tree with parameter  $\alpha \in (1,2]$  is the scaling limit of such a tree, where the  $\alpha = 2$  case is Aldous' Brownian continuum random tree. In this talk, I will discuss a new, simple construction of the  $\alpha$ -stable tree for  $\alpha \in (1,2]$ .

We obtain it as the closure of an increasing sequence of  $\mathbb{R}$ -trees built by gluing together line-segments one-by-one. The lengths of these line-segments are related to the increments of an increasing  $\mathbb{R}_+$ -valued Markov chain. For  $\alpha = 2$ , we recover Aldous' line-breaking construction of the Brownian continuum random tree based on an inhomogeneous Poisson process. This is joint work with Bénédicte Haas (Paris-Dauphine and ENS).

**Geoffrey Grimmett** (University of Cambridge)

Title: Self-avoiding walks on transitive graphs

Abstract: What can be said about the connective constant of a transitive graph? We describe weak and strict inequalities, and make a conjecture. In addition, a locality theorem is presented: if two transitive graphs agree on a large neighbourhood, then their connective constants are nearly equal. Joint with Zhongyang Li.

**Alice Guionnet** (Massachusetts University of Technology)

Title: Free analysis and random matrix theory

Abstract: In this talk we will consider loop equations appearing in free probability, random matrix theory and mean field interacting particles systems in a Coulomb gas type interaction. We will discuss the large dimension expansion of their solutions, as well as the universality of their fluctuations.

**Grégory Miermont** (ENS Lyon)

Title: Brownian surfaces

Abstract: We show that a random bipartite quadrangulation on a given compact orientable surface rescales, as the number of faces tends to infinity, to an object that can be understood as a Brownian map with the topology specified by the chosen surface. This is achieved by a surgical approach, which consists in cutting the maps along selected geodesics to obtain pieces that can be related to the classical spherical Brownian map. This is a joint work with Jérémie Bettinelli.

**Jason Miller** (Massachusetts Institute of Technology)

Title: Liouville quantum gravity as a mating of trees

Abstract: There is a simple way to “glue together” a coupled pair of continuum random trees to produce a topological sphere. The sphere comes equipped with a measure and a space-filling curve (which describes the “interface” between the trees). We present an explicit and canonical way to embed the sphere into the Riemann sphere. In this embedding, the measure is Liouville quantum gravity with parameter  $\gamma$  in  $(0,2)$ , and the curve is space-filling version of SLE with  $\kappa=16/\gamma^2$ . Based on joint work with Bertrand Duplantier and Scott Sheffield.

**Ashkan Nikeghbali** (Universität Zürich)

Title: The circular unitary ensemble: microscopic scale, ratios and the Riemann zeta function

Abstract: We provide some convergence results for the characteristic polynomial of random unitary matrices on the microscopic scale and as a consequence establish several facts about ratios of characteristic polynomials and correlations for the log of the characteristic polynomial, thus answering some open problems on that scale. These answers exhibit a natural random analytic function. We shall also give evidence about the connection between this object and the Riemann zeta function and its logarithmic derivative.

**Neil O'Connell** (University of Warwick)

Title: Geometric RSK, Whittaker functions and random polymers

Abstract: The Robinson-Schensted-Knuth (RSK) correspondence is a combinatorial bijection which plays an important role in the theory of Young tableaux and provides a natural framework for the study of last passage percolation and longest increasing subsequence problems. In this talk I will explain how a 'geometric' (or 'de-tropicalized') version of the RSK mapping provides a similar framework for the study of Whittaker functions and random polymers, mainly based on recent joint works with Ivan Corwin, Timo Seppalainen and Nikos Zygouras.

**Yuval Peres** (Microsoft)

Title: Random walks on groups and the Kaimanovich-Vershik 1983 conjecture

Abstract: Let  $G$  be an infinite group with a finite symmetric generating set  $S$ . The corresponding Cayley graph on  $G$  has an edge between  $x, y$  in  $G$  if  $y$  is in  $xS$ . Kaimanovich-Vershik (1983), building on fundamental results of Furstenberg, Derrienc and Avez, showed that  $G$  admits non-constant bounded harmonic functions iff the entropy of simple random walk on  $G$  grows linearly in time; Varopoulos (1985) showed that this is equivalent to the random walk escaping with a positive asymptotic speed. Kaimanovich and Vershik (1983) also described the lamplighter groups (groups of exponential growth consisting of finite lattice configurations) where (in dimension at least 3) the simple random walk has positive speed, yet the probability of returning to the starting point does not decay exponentially. They conjectured a complete description of the bounded harmonic functions on these groups; In dimension 5 and above, their conjecture was proved by Anna Erschler (2011). I will discuss the background and present a proof of the Kaimanovich-Vershik conjecture for all dimensions, obtained in joint work with Russ Lyons; the case of dimension 3 is the most delicate.

**Jeremy Quastel** (University of Toronto)

Title: The Kardar-Parisi-Zhang equation and its universality class

Abstract: The KPZ equation is a canonical continuum equation for random interface growth, as well as a member of a broad class of models characterized by unusual scale and distribution of fluctuations. We will survey progress which has been achieved on the equation in the last few years, as well as models in the universality class.

**Fabio Toninelli** (University Lyon 1)

Title: Height fluctuations in interacting dimers

Abstract: Perfect matchings of  $Z^2$  (also known as non-interacting dimers on the square lattice) are an exactly solvable 2D statistical mechanics model. It is known that the associated height function behaves like a massless gaussian field, with the variance of height gradients growing logarithmically with the distance (see e.g. Kenyon, Okounkov, Sheffield '06). As soon as dimers mutually interact, the model is not solvable any more. However, tools from constructive field theory allow to prove that, as long as the interaction is small, the height field still behaves like a gaussian log-correlated field. Work in collaboration with A. Giuliani and V. Mastropietro.

**Craig Tracy** (University of California, Davis)

Title: Susceptibility of the two-dimensional Ising model

**Vincent Vargas** (ENS Paris)

Title: Some new estimates on the Liouville heat kernel

Abstract: Liouville Brownian motion (LBM) is the natural diffusion process associated to the so-called Liouville measure (formally) defined by the exponential of the Gaussian Free Field, i.e. Gaussian multiplicative chaos with Green kernel. In this talk, I will consider the Liouville heat kernel, defined as the density of the LBM with respect to the Liouville measure. More precisely, I will present regularity estimates and non trivial off-diagonal bounds for the Liouville heat kernel. Based on jointwork with P. Maillard, R. Rhodes and O. Zeitouni.

**Ofer Zeitouni** (Weizmann Institute of Science, New York University)

Title: Maxima of the two-dimensional Gaussian Free Field and relatives.

Abstract: I will describe recent advance in the study of the maximum (and near maxima) of the planar Gaussian free field.