Harmonic Analysis and Related Areas
September 24 – 28, 2017

Abstracts of Talks

Nalini Anantharaman (Université de Strasbourg)

Title: Quantum ergodicity on graphs: spectral and spatial delocalization

Abstract: We discuss the notion of “quantum ergodicity" on large graphs, which is a form of delocalization of eigenfunctions. We have established it on large regular graphs with Etienne Le Masson, and more recently with Mostafa Sabri for some families of non-regular graphs, making a connection with spectral delocalization on infinite trees.

Jonathan Bennett (University of Birmingham)

Title: Perturbed Brascamp—Lieb inequalities

Abstract: The classical Brascamp—Lieb inequality is a far-reaching common generalisation of many well-known functional inequalities, such as the multilinear H\”older, Loomis—Whitney, and sharp Young convolution inequalities. Here we describe several ``perturbed “ forms of the Brascamp—Lieb inequality that have come to the fore in harmonic analysis in recent years. Emphasis will be placed on the surprisingly effective method of induction-on-scales in these contexts, and how it can, in some situations, be tightened enough to access endpoint estimates.

Ciprian Demeter (Indiana University)

Title: Decoupling beyond uniform sets

Abstract: I will explore additive energies, exponential sums and decouplings for Cantor-like sets and beyond.

Polona Durcik (California Institute of Technology)

Title: Entangled multilinear forms and applications

Abstract: We discuss L^p estimates for multilinear singular integral forms with the so-called entangled structure. As an application we show that if 1<p<\infty, p not equal to 2, and d is large enough, an arbitrary measurable set in R^d x R^d of positive upper Banach density contains corners (x,y),(x+s,y),(x,y+s) such that the l^p norm of the side s attains all sufficiently large real values. Furthermore, we discuss quantitative cancellation estimates for the simplex Hilbert transform. The talk is based on joint works with V. Kovač, L. Rimanić and C. Thiele.

Ben Green (University of Oxford)

Title: The arithmetic Kakeya conjecture of Katz and Tao

Abstract: In around 2003, Katz and Tao formulated a purely arithmetic statement which, if true, would imply the Kakeya conjecture. I will report on some joint work with Imre Ruzsa in which we played around with the conjecture, coming up with various equivalent formulations as well as establishing some lower bounds for the problem and a finite field version of it.

Izabella Laba (University of British Columbia)

Title: Lower bounds for the maximal directional Hilbert transform

Abstract: We prove that the maximal directional Hilbert transform associated with an infinite set of directions cannot be bounded on L^p(R^n) for any n>1 and any finite p. This follows from an optimal lower bound for finite sets of directions that matches the upper bounds for lacunary sets due to Demeter, Di Plinio and Parissis. Our proof extends an earlier argument of Karagulyan for p=2 and n=2. (Joint work with Alessandro Marinelli and Malabika Pramanik.)

Elon Lindenstrauss (Hebrew University of Jerusalem)

Title: Bourgain's discretized projection theorem, revisited

Abstract: One of the cornerstone of arithmetic combinatorics in its applications to questions of equidistribution and spectra is a theorem of Bourgain regarding projections of discrete sets. These have also consequences regarding Hausdorff dimension of projections of planar sets. I will present a new, more quantitative variant of this result joint with Péter Varjú.

Stefanie Petermichl (Université de Toulouse III)

Title: Higher order Journé commutators

Abstract: This research completes the line begun by Coifman, Rochberg, Weiss, then by Cotlar, Ferguson, Sadosky, Lacey of characterization of (multi parameter) BMO spaces via L^p boundedness of commutators.

The testing condition for boundedness is formulated via tensor products of Riesz transforms. The boundedness then self improvemes to hold for all commutators with arbitrary Journé operators.

These types of questions stem from operator theory via Hankel and Toeplitz forms and target (weak) factorisation of Hardy spaces. The tools used go deep into modern advances in dyadic harmonic analysis, while preserving the Ansatz from classical operator theory.

Tom Sanders (University of Oxford)

Title: The cost of commuting

Abstract: A number of problems in additive combinatorics and allied areas are often initially asked in a commutative setting, and then recast more generally. This incurs a cost, and in this talk we shall discuss this cost in the context of the idempotent theorem from harmonic analysis. For a group G the idempotent theorem describes the structure of sets whose (indicator function has) Fourier transform has bounded L_1-norm in a suitable sense. (Such functions occur when considering representations of $L_1(G)$, for example.) There are different quantitative aspects of this question and in this talk we shall discuss new bounds in certain aspects of the non-commutative idempotent theorem and how these difficulties relate to a lack of commutativity.

Christopher Sogge (Johns Hopkins University)

Title: On the concentration of eigenfunctions

Abstract: I shall present some results in global harmonic analysis that concern properties of

eigenfunctions on compact Riemannian manifolds. Using local arguments we can show that L^p norms of eigenfunctions over the entire manifold are saturated if and only if there are small balls (if p is large) or small tubular neighborhoods of geodesics (if p is small) on which the eigenfunctions have very large L^p mass. Neither can occur on manifolds of nonpositive curvature, or, more generally, on manifolds without conjugate points.

Christoph Thiele (Universität Bonn)

Title: Some results on directional operators

Abstract: We present some joint work with Francesco DiPlinio, Shaoming Guo, and Pavel Zorin-Kranich. First we prove a Littlewood-Paley diagonalization result for bi-Lipschitz perturbations of the identity map on the real line. This result entails a number of corollaries for the Hilbert transform along lines and monomial curves in the plane. Second, we prove a square function estimate for a single scale directional operator.

Trevor Wooley (University of Bristol)

Title: Nested Efficient Congruencing and relatives of Vinogradov's mean value theorem

Abstract: Vinogradov's mean value theorem and its relatives provide estimates for mean values of exponential sums of utility in a wide spectrum of applications, such as estimating the zero-free region of the Riemann zeta function, in Diophantine problems such as that of Waring, and in establishing equidistribution of polynomial sequences. The efficient congruencing method employs variants of Hensel's lemma (as applied to polynomial congruences) in a p-adic concentration argument, improving the classical versions of Vinogradov's mean value theorem almost to the point of proving the bounds conjectured to hold. We now apply a nested version of the method, which involves a novel variant of Hensel's lemma in averaged form, so as to establish the main conjecture in Vinogradov's mean value theorem in full. This was recently established by Bourgain, Demeter and Guth using their l^2-decoupling method. The new method is flexible enough to apply in non-classical settings. Aside from sketching out the key ideas of the method, we will outline some extensions and mention some results beyond efficient congruencing and decoupling.

Josh Zahl (University of British Columbia)

Title: Breaking the 3/2 barrier for unit distances in three dimensions