Recent Advances in Extremal Combinatorics
December 3-7, 2018
University of Oxford

Abstracts

Peter Allen (London School of Economics)

Title: Optimal packings of sparse graphs

Abstract: I will describe (another) randomised approach to finding perfect packings of sparse graphs in quasirandom or complete host graphs. The packing algorithm is simple and I will describe it completely; the analysis is not so simple.

In particular, we prove that Ringel's Conjecture and the Tree Packing Conjecture are true for almost all trees (or sequences of trees, respectively).

This is joint work with Julia Boettcher, Dennis Clemens, Jan Hladky, Diana Piguet and Anusch Taraz.

Paul Balister (University of Memphis)

Title: Covering systems

David Ellis (Queen Mary University London)

Title: Families of permutations with a forbidden intersection

Abstract: A family of permutations is said to be 't-intersecting' if any two permutations in the family agree on at least t points. It is said to be (t-1)-intersection-free if no two permutations in the family agree on exactly t-1 points. Deza and Frankl conjectured in 1977 that for any positive integer t, a t-intersecting family of permutations in S_n can be no larger than a coset of the stabiliser of t points, provided n is large enough depending on t; this was proved by the speaker and independently by Friedgut and Pilpel in 2008. We give a new proof of a stronger statement: namely, that for any positive integer t, a (t-1)-intersection-free family of permutations in S_n can be no larger than a coset of the stabiliser of t points, provided n is large enough. This can be seen as an analogue for permutations of seminal results of Frankl and Furedi on families of k-element sets. Our proof is partly algebraic and partly combinatorial; it is more 'robust' than the original proofs of the Deza-Frankl conjecture, using a combinatorial 'quasirandomness' argument to avoid many of the algebraic difficulties of the original proofs.

Based on joint work with Noam Lifshitz (Bar Ilan University).

Asaf Ferber (MIT)

Title: On the singularity of random symmetric matrices --new approach, new bound

Abstract: Let M_n be an $n\times n$ symmetric matrix with entries in ± 1, chosen uniformly at random. It is widely conjectured that M_n is singular with probability at most $(2+o(1))^{-n}$. On the other hand, the best known upper bound on the singularity probability of M_n, due to Vershynin (2011), is 2^{-n^c}, for some unspecified small constant $c > 0$ (this improves on a polynomial bound due to Costello, Tao, and Vu (2005), and a bound of the form $n^{-\omega(1)}$due to Nguyen (2011)).

In this talk, using a novel combinatorial approach, we show that the probability of singularity of M_n is at most $2^{-n^{1/4}}$. We also discuss improvements for other models of discrete random matrices.

Joint work with Vishesh Jain (PhD student in MIT).

Jacob Fox (Stanford University)

Title: A solution to the Burr-Erdős problems on Ramsey complete sequences.

Abstract: A sequence A of positive integers is r-Ramsey complete if for every r-coloring of A, every sufficiently large integer can be written as a sum of the elements of a monochromatic subsequence. Burr and Erdős proposed several open problems in the early 1980s on how sparse can an r-Ramsey complete sequence be and which polynomial sequences are r-Ramsey complete. Erdős later offered cash prizes for two of these problems. We prove a result which solves the problems of Burr and Erdős on Ramsey complete sequences. The proof uses tools from probability, combinatorics, and number theory.

This is joint work with David Conlon.

Annika Heckel (University of Oxford)

Title: Colouring dense random graphs - when can we use the second moment method?

Abstract: In a colouring of a graph G, no two neighbouring vertices are coloured the same, and the chromatic number $\chi(G)$ is the minimum number of colours where this is possible. By definition any colour class is an independent set, so $\chi(G) \ge n/\alpha(G)$ (where $\alpha(G)$ is the independence number).

For the dense random graph $G(n, 1/2)$, Bollobás proved in 1987 that this lower bound is asymptotically sharp. In other words, there is a colouring with average colour class size $(1-o(1)) \alpha(G)$.

Using a combination of the first and second moment method and martingale concentration arguments, this can be sharpened to determine the average colour class size in an optimal colouring up to an additive $o(1)$, but no further. In this talk, we discuss the main obstacles when applying the second moment method to the number of colourings, or more generally to counts of other structures in $G(n,p)$.

If we introduce certain conditions on the colour class sizes in order to circumvent these obstacles, the second moment method can be used to yield some very sharp results. In particular, both the equitable chromatic number (where colour class sizes may differ by at most 1) and the $(\alpha-2)$-bounded chromatic number (where we avoid colour classes of size α and $\alpha-1$) are concentrated on at most two consecutive values in $G(n, N/2)$.

Joint work with Konstantinos Panagiotou.

Mihyun Kang (TU Graz)

Title: Topological aspects of random graphs

Abstract: In this talk we shall discuss various topological aspects of random graphs. How quickly does the genus of a random graph grow as the number of edges increases? How dramatically can a small number of random edges increase the genus of a randomly perturbed graph? How do topological constraints (such as imposing an upper bound on the genus) influence the structure of a random graph?

Dan Král' (Masaryk University and University of Warwick)

Cycles of length three and four in tournaments

Linial and Morgenstern conjectured that, among all tournaments with a given density d of cycles of length three, the density of cycles of length four is minimized by a random blow-up of a transitive tournament with all but one parts of equal sizes. We prove the conjecture for d>=1/36 using methods from spectral graph theory. We also demonstrate that the structure of extremal examples is more complex than expected and give its full description for d>=1/16.

This talk is based on joint work with Timothy Chan, Andrzej Grzesik and Jonathan Noel.

Michael Krivelevich (Tel Aviv University)

Title: Complete minors in graphs without sparse cuts

Abstract: We show that if G is a graph on n vertices with all degrees comparable to d and without sparse cuts, in an appropriately defined quantitative sense, then G contains a complete minor of order $\sqrt{nd/\log d}$.

As a corollary we determine the order of a largest complete minor one can guarantee in d-regular graphs with large eigenvalue gap, in random d-regular graphs, and in jumbled graphs.

A joint work with Rajko Nenadov.

Daniela Kuhn (University of Birmingham)

Title: On a conjecture of Erdos on locally sparse Steiner triple systems

Abstract: Given a set X of size n, a collection S of 3-subsets of X is a Steiner triple system of order n if every 2-subset of X is contained in exactly one of the triples of S (so a Steiner triple system of order n can also be viewed as a decomposition of the n-vertex complete graph into edge-disjoint triangles). A famous theorem of Kirkman says that there exists a Steiner triple system of order n if and only if n equals 1,3 mod 6. In 1976, Erdos conjectured that one can find so-called "sparse" Steiner triple systems. Roughly speaking, the aim is to have at most j-3 triples in S on every set of j points, which would be best possible. (Triple systems with this sparseness property are also referred to as having high girth.)

We prove this conjecture asymptotically by analysing a natural generalization of the random triangle removal process. Our result also solves a problem posed by Lefmann, Phelps and Rodl as well as Ellis and Linial in a strong form. Moreover, we pose a conjecture which would generalize the Erdos conjecture to Steiner systems with arbitrary parameters and provide some evidence for this.

Joint work with Stefan Glock, Allan Lo and Deryk Osthus.

Shoham Letzler (University of Cambridge)

Dense induced bipartite subgraphs in triangle-free graphs

The problem of finding dense induced bipartite subgraphs in H-free graphs has a long history, and was posed 30 years ago by Erdős, Faudree, Pach and Spencer. We obtain two results in this direction. We show that every K_t-free graph with minimum degree d contains an induced bipartite graph with minimum degree at least c_t*log(d)/log log(d), thus confirming (asymptotically) a conjecture of Esperet, Kang and Thomassé. We also obtain nearly optimal bounds for this problem in the case of dense triangle-free graphs.

This is joint work with Matthew Kwan, Benny Sudakov and Tuan Tran.

Eoin Long (University of Oxford)

Title: Cycle-complete Ramsey numbers

Abstract: The Ramsey number $r(C_{\ell},K_n)$ is the smallest natural number N such that every red/blue edge-colouring of a clique of order N contains a red cycle of length ℓ or a blue clique of order n. In 1978, Erd\H{o}s, Faudree, Rousseau and Schelp conjectured that $r(C_{\ell},K_n) = (\ell-1)(n-1)+1$ for $\ell \geq n\geq 3$ provided $(\ell,n) \neq (3,3)$. In this talk I will discuss a recent proof of this conjecture for large ℓ, and a strong form of a conjecture due to Nikiforov, showing that $r(C_{\ell},K_n) = (\ell-1)(n-1)+1$ provided $\ell \geq \frac {C\log n}{\log \log n}$, for some absolute constant $C>0$. Up to the value of C this is tight, and answers two further questions of Erd\H{o}s et al. up to multiplicative constants.

Joint work with Peter Keevash and Jozef Skokan.

Richard Montgomery (University of Cambridge)

Title: Spanning spheres in 3-graphs

Frank Mousset (Tel Aviv University)

Title: The upper tail for triangle counts in random graphs

Abstract: Let X denote the number of triangles in the random graph G(n,p) and consider the problem of determining the asymptotic behaviour of the upper tail rate function r_a(n,p) = log P[X > (1+a)E[X]] for a given positive constant a. Improving various earlier estimates, Chatterjee (2012) and DeMarco and Kahn (2012) independently proved that if np >> log(n), then r_a(n,p) = Theta(n^2p^2 log(1/p)). Since then, there has been considerable interest in determining the precise asymptotics of the (bounded) function c_a(n,p) = r_a(n,p)/(n^2p^2 log(1/p)). Partial results on this problem have been obtained by Chatterjee and Dembo (2014), Eldan (2016), Cook and Dembo (2018), and Augeri (2018) in various ranges of the density p. The most recent of these results, due to Augeri, determined the asymptotics of c_a(n,p) in the range where n^{-1/2} log^4(n) << p << 1. I will present a result that gives the asymptotics of c_a(n,p) in the larger range where n^{-1} log(n) << p << 1. Our proof relies on a simple variation on a classical moment argument due to Janson, Oleszkiewicz, and Ruciński (2004). With more work, our method also allows us to determine the rough asymptotic structure of the random graph when conditioned on the upper tail event.

This is joint work with Matan Harel and Wojciech Samotij from Tel Aviv University.

Rajko Nenadov (ETH Zürich)

Title: Triangle-factors in pseudorandom graphs

Abstract: A result of Johansson, Kahn, and Vu states that a threshold for the property that every vertex of G(n,p) is contained in a triangle coincides, in the order of magnitude, with a threshold for much stronger property of containing a triangle-factor. We show that a similar phenomenon is, up to a log n factor, even more pronounced in pseudorandom graphs in the following sense: On the one hand, we prove that if L = o(d^2 / (n log n)) then every (n,d,L)-graph contains a triangle-factor, assuming the obvious divisibility condition. On the other hand, constructions by Alon and Krivelevich, Sudakov, and Szabo show that for L > Cd^2 / n, for some large C, there are infinitely many (n,d,L)-graphs which do not contain even a single triangle.

Deryk Osthus (University of Birmingham)

Title: Resolution of the Oberwolfach problem

Abstract: The Oberwolfach problem, posed by Ringel in 1967, asks for a decomposition of the complete graph of order 2n+1 into edge-disjoint copies of a given 2-factor. We show that this can be achieved for all large n.

We actually prove a significantly more general result, which allows for decompositions into more general types of factors. In particular, this also resolves the Hamilton-Waterloo problem for large n.

Joint work with Stefan Glock, Felix Joos, Jaehoon Kim and Daniela Kuhn.

Oleg Pikhurko (University of Warwick)

Title: On a conjecture of V.T. Sos on size convergence

Abstract: Vera Sos conjectured in 2012 that every graphon W is uniquely determined, up to a weak isomorphism, by the distribution of the number of edges in its all k-vertex random samples. The conjecture has been proved in the quasi-random case, that is, when W is a constant function. We investigate the validity of this conjecture for graphons of special form.

Joint work with Oliver Cooley and Mihyun Kang.

Alexey Pokrovskiy (Birkbeck University of London)

Title: Rainbow tree decompositions

Abstract: A spanning rainbow tree in a coloured graph is a spanning tree all of whose edges have different colours. A number of open problems in graph theory ask whether a certain class of coloured graphs has a spanning rainbow tree decomposition. For example Brualdi and Hollingsworth conjectured that 1-factorized complete graphs have spanning rainbow tree decompositions, whereas Rota conjectured that n-vertex multigraphs which are the union of n-1 monochromatic trees have spanning rainbow tree decompositions. This talk will be about recent techniques for finding spanning rainbow trees. Progress on both of the above conjectures will be discussed.

This is joint work with Bucic, Kwan, Sudakov, and Montgomery.

Asaf Shapira (Tel Aviv University)

Title: Efficient Removal without Efficient Regularity

Abstract: Obtaining an efficient bound for the triangle removal lemma is one of the most outstanding open problems of extremal combinatorics. Perhaps the main bottleneck for achieving this goal is that triangle-free graphs can be highly unstructured. For example, triangle-free graphs might have only regular partitions (in the sense of Szemerédi) of tower-type size. And indeed, essentially all the graph properties P for which removal lemmas with reasonable bounds were obtained, are such that every graph satisfying P has a small regular partition. So in some sense, a barrier for obtaining an efficient removal lemma for property P was having an efficient regularity lemma for graphs satisfying P.

In this paper we consider the property of being induced C4-free, which also suffers from the fact that a graph might satisfy this property but still have only regular partitions of tower-type size. By developing a new approach for this problem we manage to overcome this barrier and thus obtain a merely exponential bound for the induced C4 removal lemma. We thus obtain the first efficient removal lemma that does not rely on an efficient version of the regularity lemma. This is the first substantial progress on a problem raised by Alon in 2001, and more recently by Alon, Conlon and Fox.

Joint work with L. Gishboliner.

Jozef Skokan (London School of Economics)

Title: Decomposing tournaments into paths

Abstract: We consider a generalisation of Kelly's conjecture which is due Alspach, Mason, and Pullman from 1976. Kelly's conjecture states that every regular tournament has an edge decomposition into Hamilton cycles, and this was proved by K{\"u}hn and Osthus for all sufficiently large tournaments. The conjecture of Alspach, Mason, and Pullman concerns general tournaments and asks for the minimum number of paths needed in an edge decomposition of each tournament into paths. There is a natural lower bound for this number in terms of the degree sequence of the tournament and they conjecture this bound is correct for tournaments of even order.
Almost all cases of the conjecture are open and we prove many of them.

This is joint work with Allan Lo, Viresh Patel, and John Talbot.

Benny Sudakov (ETH Zürich)

Title: Nearly-linear monotone paths in edge-ordered graphs

Abstract: How long a monotone path can one always find in any edge-ordering of the complete graph K_n? This appealing question was first asked by Chvatal and Komlos in 1971, and has since attracted the attention of many researchers, inspiring a variety of related problems. The prevailing conjecture is that one can always find a monotone path of linear length, but until now the best known lower bound was $n^{2/3-o(1)}$. We almost close this gap, proving that any edge-ordering of the complete graph contains a monotone path of length $n^{1-o(1)}$.

Joint with: M. Bucic, M. Kwan, A. Pokrovskiy, T. Tran and A. Wagner

Lutz Warnke (Georgia Tech)

Title: Large girth approximate Steiner triple systems

Abstract: In 1973 Erdos asked whether there are n-vertex partial Steiner triple systems with arbitrary high girth and quadratically many triples. (Here girth is defined as the smallest integer $g \ge 4$ for which some g-element vertex-set contains at least g-2 triples.)

We answer this question, by showing existence of approximate Steiner triple systems with arbitrary high girth. More concretely, for any fixed $\ell \ge 4$ we show that a natural constrained random process typically produces a partial Steiner triple system with $(1/6-o(1))n^2$ triples and girth larger than ℓ. The process iteratively adds random triples subject to the constraint that the girth remains larger than ℓ. Our result is best possible up to the o(1)-term, which is a negative power of n.

Based on joint work with Tom Bohman.

Liana Yepremyan (University of Oxford)

Title: On the number of symbols that forces a transversal

Abstract: Akbari and Alipour conjectured that any Latin array of order n with at least $n^2/2$ symbols contains a transversal, or equivalently, every proper-edge coloring of the complete bipartite graph $K_{n,n}$ with $n^2/2$ colours contains a rainbow perfect matching. In this talk we will present a proof of this conjecture in a stronger sense: we show that $n^{399/200}$ colours suffice. This is joint work with Peter Keevash.

Yufei Zhao (MIT)

Title: A reverse Sidorenko inequality

Abstract: We prove a number of tight graph homomorphism inequalities, where, for a fixed H, we wish to maximize the number of homomorphism from G to H (after exponentially normalizing by the size of G) under certain degree constraints on G (e.g., d-regular). One highlight of our results is that, among d-regular graphs of the same size, a disjoint complete bipartite graphs has the most number of proper q-colorings. Our results also extend to irregular graphs and list colorings. These results settle a number of conjectures by Kahn, Galvin-Tetali, Galvin, and Cohen-Csikvári-Perkins-Tetali.

Joint work with Ashwin Sah, Mehtaab Sawhney, and David Stoner.