Discrete Structures: Harmonic Analysis and Probability
September 27-28, 2018
University of Oxford

Abstracts

Franck Barthe (Université de Toulouse)

Title: Spectral gap for some log-concave measures

Abstract: This work with Bo'az Klartag was initiated by a question on a specific measure related to the LASSO estimator. In an attempt to have a broader view on the question, we investigate Poincaré inequalities for some families of log-concave perturbations of product measures. We use techniques that were developped in order to tackle the famous Kannan-Lovasz-conjecture, which predicts the approximate value of the spectral gap of log-concave measures. Other key ingredients are the notion of Gaussian mixtures and a recent extension of the Gaussian correlation inequality. As an application we can confirm (up to logarithmic terms) the KLS conjecture for sections of proportional dimensions of unit balls of ℓ_p^n, for $1\le p <2$.

Sergey Bobkov (University of Minnesota)

Title: An extended Khinchine's theorem in the Metric Theory of Diophantine Approximations

Abstract: We will be discussing an extension of a well-known Khinchine's theorem on the simultaneous approximation of irrational numbers by rationals to the scheme of probability distributions. The question is motivated by the central limit problem for weighted sums of independent random variables.

Dario Cordero-Erausquin (Université Paris 6)

Title: Complex interpolation curves between real Banach spaces and Legendre duality

Abstract: Inspired by the theory of the homogeneous complex Monge-Ampère equation and of the polynomial hulls à la Alexander-Wermer-Slodkowsky, we introduce curves between (« real ») norms on $\R^{2n}=\C^n$ that enjoy some of the properties of complex interpolation, with which it coincides in the case of « complex » norms. In particular, we will show that the construction commutes with the Legendre transform, leading to an « exact » duality theory. This is a joint work with B. Berndtsson, B. Klartag and Y. Rubinstein.

Paata Ivanisvili (Princeton University and University of California, Irvine)

Title: Markov--Bernstein type estimates on the Hamming cube

Abstract: Hamming cube of dimension n can be considered as the set of all vectors of length n with coordinates plus or minus 1. Functions on the Hamming cube can be expanded into the corresponding Fourier--Walsh series of degree n. Approximating such functions by simpler functions, for example, Fourier--Walsh polynomials of smaller degree (living on low frequencies)

are of interests. I will speak about Bernstein--Markov type estimates, and its converse forms, for functions on the hamming cube living on low frequencies, and on high frequencies correspondingly. This is joint work with Alexandros Eskenazis.

Dong Li (Hong Kong University of Science and Technology)

Title: Fractional Laplacians on the Hamming cube and related issues

Abstract: I will discuss recent some dimension free Poincaré and Bernstein-type inequalities on the Hamming cube for functions with different spectral properties and for fractional Laplacians. Some related issues and sharp counterexamples will also be explained. Joint work with A. Volberg and P. Ivanisvili.

Ryan O'Donnell (Carnegie Mellon University)

Title: Fooling Polytopes

Abstract: We give a nearly efficient *deterministic* algorithm for approximately counting the number of 0/1-coordinate-points in high-dimensional polytopes. The two main technical tools are: a new multidimensional Berry--Eseen theorem under limited independence; and, a new multidimensional Littlewood--Offord theorem for polytopes. Joint work with Rocco Servedio (Columbia) and Li-Yang Tan (Stanford)

Stefanie Petermichl (Université de Toulouse)

Title: Sparse domination and dimensionless estimates for the Riesz vector

Abstract: It is known since the 1970s, formulated in the work by Gundy and Varopoulos, that certain classical operators such as the Hilbert or Riesz transforms have stochastic representation using the background noise process and harmonic extensions.

On the other hand, their point-wise domination by so-called sparse operators is known since 2015 by Nazarov-Lerner, Lacey, Conde-Rey, independently. These principles are originally based on stopping cubes and carry dimensional constants in several parts of the proof. Through a probabilistic argument, a trajectory-wise sparse domination with continuous parameter can be obtained, thus avoiding all occurrences of dimensional constants.

We use our technique to give a new, short proof of a dimensionless L^p estimate for the Bakry Riesz vector on Riemannian manifolds with bounded geometry - this includes the classical Riesz vector on Euclidean space as well as the Riesz vector on the Gauss space. Our proof has the advantage that it gives new dimensionless, sharp L^p estimates in the weighted setting, even in the presence of negative curvature.

Mark Rudelson (University of Michigan)

Title: Invertibility of adjacency matrices of random graphs

Abstract: Consider an adjacency matrix of a bipartite, directed, or undirected Erdos-Renyi random graph. If the average degree of a vertex is large enough, then such matrix is invertible with high probability. As the average degree decreases, the probability of the matrix being singular increases, and for a sufficiently small average degree, it becomes singular with probability close to 1. We will discuss when this transition occurs, and what the main reason for the singularity of the adjacency matrix is. This is a joint work with Anirban Basak.

Ramon van Handel (Princeton University)

Title: Mixed volumes and the Bochner method

Abstract: A classical technique in Riemannian geometry, known as the Bochner method, shows (among other things) that the spectral gap of the Laplacian is bounded from below when one has a lower bound on the curvature. This idea was developed in much greater generality by Bakry-Emery to study, for example, geometric inequalities for uniformly log-concave measures on R^n.

However, curvature assumptions effectively rule out passing from continuous to discrete situations. For example, as log-concave measures are unimodal, it is clearly impossible to approximate any nontrivial discrete distribution by log-concave measures. While one could introduce some limited discrete analogues of "curvature", curvature assumptions effectively render the continuous and discrete worlds disjoint.

In an ongoing investigation of certain problems in geometric analysis, we encountered a remarkable situation where this intuition is mysteriously broken. We consider certain weighted Laplacian operators on the sphere, due to Minkowski-Hilbert-Alexandrov, that are of great importance in convex geometry: the quadratic forms associated to these operators are precisely the mixed volumes of smooth convex bodies. By passing from smooth bodies to polytopes, these objects interpolate between continuous and discrete distributions. Nonetheless, it turns out that the Bochner method applies in this setting despite the manifest absence of any well-behaved curvature. The most basic incarnation of this idea, which has a one-line proof, yields perhaps (at least in my opinion) the simplest known proof of the Alexandrov-Fenchel inequalities. My aim is explain some of these ideas, what we would like to do with them, and some of the many things we do not understand. Joint work with Yair Shenfeld.

Alexander Volberg (Michigan State University)

Title: What is wrong with square function?

Abstract: Square function operator is one of the most widely used and thoroughly studied operator in harmonic analysis. However, there are still open questions about this object. This object is a discrete analog of stopping time of Brownian motion. The talk will be concerned with 3 problems about square function operator: 1) what is the Bellman function for Burgess Davis type estimates of this operator (L^p estimate), 2) what is Bellman function for Bollobas type estimate (weak L^1 estimate), 3) what is sharp end-point estimate of square function operator in weak weighted $L^2(w)$? All three question have equivalent PDE reformulations, but still questions 1 and 3 do not have complete answers till now.