Generalized Geometry and Noncommutative Algebra

Abstracts of Talks

MICHAEL BAILEY

Title: Weakly holomorphic structures and generalized complex geometry

Abstract: A generalized complex structure may be understood as a "weakly holomorphic" Poisson structure: given an integrability condition, a generalized complex structure "integrates" to a symplectic groupoid equipped with a weak holomorphic structure, i.e., a compatible holomorphic structure on the associated stack. Similarly, generalized complex branes may be represented as weakly holomorphic coisotropic submanifolds. I will explain these weakly holomorphic structures, say something about their construction, and briefly discuss the implications for deformation quantization.

PIETER BELMANS

Title: Hilbert schemes of points and noncommutative planes and quadrics

Abstract: Recently an embedding of the derived category of a noncommutative plane (resp. quadric) into the derived category of a deformation of the Hilbert scheme of 2 points on the plane (resp. quadric) has been constructed. I will recall the idea behind these constructions. The limited functoriality for Hochschild cohomology then gives a morphism between the second Hochschild cohomologies of the Hilbert scheme and the (noncommutative) surface, and I will explain why one can expect a compatibility with the Hochschild-Kostant--Rosenberg decomposition in this case.

HENRIQUE BURSZTYN

Title: Morita equivalence in Poisson geometry and deformation quantization

Abstract: We will explain how deformation quantization provides a concrete link between the geometric notion of Morita equivalence of Poisson structures (due to P. Xu, based on symplectic dual pairs) and algebraic Morita equivalence of their star products. A central role in this connection is played by “B-fields”, native to generalized geometry.

DANIEL CHAN

Title: Noncommutative Mori contractions

Abstract: Noncommutative algebraic geometry has made significant strides in importing natural geometric ideas into the noncommutative context to great effect. However, with the exception of analogues of closed embeddings, there are still few examples of morphisms of noncommutative spaces. This talk is built around the theme that moduli spaces are useful in studying noncommutative geometry because the universal object can serve as a bimodule and hence potentially a functor defining a morphism. From this perspective, we examine some morphisms from noncommutative surfaces to commutative curves which are akin to $\mathbb{P}^1$-bundles. They are in a sense, noncommutative Mori contractions.

RYUSHI GOTO

Title: Scalar curvature as moment map in generalized Kähler geometry

Abstract: It is known that scalar curvature arises as the moment map in ordinary Kähler geometry. In pursuit of this analogy, we construct a moment map in generalized Kähler geometry, which is regarded as “Scalar curvature” in a generalized Kähler manifold. From the view point of the moment map, the notion of generalized Kähler-Einstein structure is introduced. Holomorphic Poisson structures on Kähler manifolds yield unobstructed deformations of generalized Kähler structures. We discuss the moment maps of these Poisson deformations and show that certain Poisson structures give intriguing deformations generalized Kähler -Einstein structures.

CHRIS HULL

Title: Strings, geometry and duality

Abstract: One of the ways in which string theory differs from conventional field theories is that it has duality symmetries, which allow the construction of so-called non-geometric backgrounds, such as T-folds which have T-duality transition functions. String theory on a torus requires the introduction of dual coordinates conjugate to string

winding number. This leads to novel geometry in a doubled space, with non-trivial dynamics in the full doubled space-time. The geometry and physics of doubled space-time will be developed and discussed.

DANIEL HUYBRECHTS

Title: Kuznetsov's Calabi-Yau categories: introduction and applications

Abstract: The talk will start with a gentle introduction into Kuznetsov's construction of (fractional) Calabi-Yau categories associated with hypersurfaces. The relation to matrix factorizations will be mentioned, but the talk will mainly focus on the Fourier-Mukai aspects. As an application we will discuss a new proof of the Global Torelli theorem for cubic fourfolds (joint work with Jørgen Rennemo).

COLIN INGALLS

Title: *b-minimal model*

Abstract: We extend results of Chan and Ingalls concerning the minimal model program for orders over surfaces to all dimensions. A b-divisor gives a number for all divisors of all models of a variety. We show that every b-log pair has a terminal resolution. We further show that if one carries out log contractions of terminal b-log pairs remain terminal. As an application, one obtains a decoration from a Brauer class and that this can be used to give a minimal model program for orders over varieties in all dimensions. This work is the joint work of: Daniel Chan, Kenneth Chan, Louis de Thanho er de Volcsey, Colin Ingalls, Kelly Jabbusch, Sandor Kovacs, Rajesh Kulkarni, Boris Lernerand Basil Nanayakkara.

ULF LINDSTRÖM

Title: Supersymmetry, sigma models and geometry

Abstract: There is an interesting interplay between the number of supersymmetries of a non-linear sigma model and the geometry of its target space. In this presentation I will describe that relation with emphasis on sigma models that are maps from a two (even) dimensional domain with (2,2) supersymmetry. This allows for a complete description of Generalized Kähler geometry away from irregular points and elucidates the triple role of the generalized Kähler potential.

WENDY LOWEN

Title: The Grothendieck construction and Hochschild cohomology

Abstract: We discuss a linear version of the Grothendieck construction and two applications to Hochschild cohomology. Firstly we introduce a natural approach to obtaining Mayer-Vietoris sequences, and secondly we develop an approach for endowing the Gerstenhaber-Schack complex of an arbitrary prestack with an L infinity structure governing deformations. In both cases, we discuss applications to schemes.

RUXANDRA MORARU

Title: Hermitian-Einstein equations over generalized Kähler manifolds

Abstract: In this talk, we discuss an analogue of the Hermitian-Einstein equations for generalized Kähler manifolds. We explain in particular how these equations are equivalent to a notion of stability for generalized holomorphic bundles, and that there is a Kobayahsi-Hitchin-type correspondence between solutions of these equations and stable bundles. Moreover, we discuss the deformation theory of stable generalized holomorphic bundles on generalized Kähler manifolds and, time permitting, describe their moduli spaces in some specific examples.

SHINNOSUKE OKAWA

Title: Compact moduli of marked noncommutative del Pezzo surfaces

Abstract: I will talk about a certain GIT construction of compactified moduli spaces of marked noncom-mutative del Pezzo surfaces by Tarig Abdelgadir, myself, and Kazushi Ueda. The construction is based on a choice of a full strong exceptional collection of the derived category of a noncommutative del Pezzo surface.

MARTIN ROCEK

Title: Gauging isometries of Generalized Kähler geometries

Abstract: I will give a review of how nonlinear sigma models relate to Generalized Kähler geometry, discuss how isometries can act on these geometries, give a number of examples including a few recent results, and some open problems.

JUSTIN SAWON

Title: Deformations of categories and generalized complex manifolds

Abstract: The first order deformations of a complex manifold X are parametrized by $H^1(T)$but if we regard it as a generalized complex manifold its deformations are parametrized by the larger degree two cohomology of polyvector fields $HT^2(X)$. Moreover, $HT^2(X)$ can be identified with the degree two Hochschild cohomology of X, which parametrizes first order deformations of the category of coherent sheaves on X. We describe some apparent compatibilities between these two points of view, for abelian varieties, K3 surfaces, and certain higher-dimensional holomorphic symplectic manifolds.

GEOFFREY E. SCHNEIDER

Title: Recursively generating formality quasi-isomorphisms

Abstract: Solutions to Deligne's conjecture are given as constructions of a sequence of quasi-isomorphisms usually defined via transcendental methods (e.g. configuration space integrals). In this talk we will see that the use of operadic methods allow us to "demystify" these quasi-isomorphisms such that they can be computed recursively over $\mathbb{Q}$. The conditions under which such a recursive construction can be carried out also hold for operads used in a generalization of Deligne's Conjecture replacing Ger with BV, the operad governing Batalin–Vilkovisky algebras, and even more generally with an operad Calc governing the algebraic structure of the Hochschild cohomology-homology pair.

PAVOL ŠEVERA

Title: Deformation quantization in simple cases

Abstract: The simple observation that the tensor product of two commutative algebras can be non-commutative if the algebras are in a braided monoidal category provides deformation quantization of a nice class of Poisson manifolds. The examples I will describe are a quantization of Lie bialgebras (or Poisson-Lie groups) and, more generally, a quantization of moduli spaces of flat connections on a surface with a decorated boundary.

SUSAN SIERRA

Title: A family of quantised projective spaces

Abstract: Let k be an algebraically closed field of characteristic zero. For any positive integer n, we construct a Calabi-Yau algebra $R(n)$, which induces a Poisson deformation of $k[x_0, \dots, x_n]$ and generalises a construction given by Pym when $n = 3$. Modulo scalars, the graded automorphism group of $R(n)$ is isomorphic to k, and we consider not only $R(n)$ but its Zhang twist $R(a, n)$ by the automorphism corresponding to a. Each $R(a, n)$ induces a Poisson structure on $k[x_0, \dots, x_n]$ in the semiclassical limit, and we study this structure. We show that the Poisson spectrum of the limit is homeomorphic to Spec $R(a, n)$, and explicitly describe Spec $R(a, n)$ as a union of commutative strata. This is joint work with Cesar Lecoutre.

PAUL SMITH

Title: 4-dimensional exotic elliptic algebras

Abstract: (Joint work with Alex Chirvasitu.) The algebras of the title are a family of Artin-Schelter regular algebras of global and GK dimension 4. They are noetherian domains and have most of the homological properties of the polynomial ring with its standard grading. They are constructed from Sklyanin algebras of dimension 4 by a cocycle twist or descent-like procedure that is well-enough behaved that one can transfer (some) properties from the Sklyanin algebras to the new algebras. Like the Sklyanin algebras their graded representation theory is controlled by an elliptic curve and a translation automorphism. We show the new algebras exhibit several new features (20 point modules, for example) that "test" our understanding of AS regular algebras. Their line modules are particularly interesting, and are parametrized by 7 curves in an appropriate Grassmanian, three of which are elliptic curves, and 4 of which are plane conics. They also provide new non-commutative analogues of quadric surfaces. (arXiv: 1502.01744)

TOBY STAFFORD

Title: Non-commutative minimal surfaces

Abstract: Twenty years ago Mike Artin proposed that one should classify non-commutative projectivesurfaces (or connected graded domains of Gelfand-Kirillov dimension 3) in a manner similar to the commutative theory: one first classifies the minimal models within a given birational class and then show that any other surface can be blown down a finite number of times to reach such a minimal model.

The generic non-commutative $\mathbb{P}^2$ is given by the Sklyanin algebra S and the minimal models birational to S are thought to be the Sklyanin algebra itself and Michel Van den Bergh's quadrics. In this lecture we will show, using our non-commutative version of blowing down, that these algebras are minimal and indeed are minimal in a very strong sense: let R be a Sklyanin algebra or Van den Bergh quadric. Then any connected graded, noetherian over-ring of R with the same graded ring of fractions equals R.

This is joint work with Rogalski and Sierra.

JEFFREY STREETS

Title: Generalized Kähler-Ricci flow

Abstract: Generalized Kähler-Ricci flow is a natural extension of the Kähler-Ricci flow, introduced by myself and Tian, which preserves generalized Kähler structure. In this talk I will introduce this equation, discuss the role of the Poisson structure in determining the structure of the flow, and describe global existence results in the case of real dimension 4.

JOEY VAN DER LEER DURÁN

Title: Blow-ups in generalized Kähler geometry

Abstract: It is a well-known fact that the blow-up of a Kähler manifold along a compact complex sub-manifold can again be endowed with a Kähler metric. In the context of generalized Kähler geometry this is only known to be true for some very specific cases, dealing only with points on complex surfaces. In this talk we will show how to extend this to submanifolds of higher dimension, and as an application we will construct generalized Kähler structures on the blow-up of the maximal torus in a Lie group which is a product of circles and three-spheres.

DANIEL WALDRAM

Title: Algebras from branes and their dual generalised geometries

Abstract: We discuss the generalised geometries dual to four-dimensional $N = 1$ superconformal theories. These are natural extensions of Sasaki-Einstein manifolds. In particular we consider their moduli spaces and how to identify the deformations that correspond to chiral operators in the field theory. Duality relates these generalised geometry deformations to elements of cohomology of a (non-commutative) field-theory algebra determined by a superpotential. We focus on the $N = 1$ deformations of $N = 4$ as a simple example.

CHELSEA WALTON

Title: Noncommutative McKay correspondence

Abstract: In this talk, I will discuss joint work with Kenneth Chan, Ellen Kirkman, and James Zhang on a version of the McKay correspondence for semisimple Hopf actions on Artin–Schelter regular algebras.

MAXIM ZABZINE

Title: Some thoughts on generalised Kähler potential

Abstract: I will explain the idea behind the generalised Kähler potential. I will present some observations which we accumulated during last 10 years. Eventually we have rather simple picture which requires proper mathematical framework.