Random Polymers and Algebraic Combinatorics
Abstracts of Talks

Guillaume Barraquand
Title: Random walks in Beta-distributed random environment.

Abstract: We will introduce a family of random walks in random space-time environment, with Beta-distributed transition probabilities. This model is exactly solvable, in the sense that the law of the (finite time) position of the walker can be completely characterized by Fredholm determinantal formulas. This allows to study in particular the large deviation principle satisfied by the random walk, and we can prove a limit theorem towards the Tracy-Widom distribution for the second-order corrections to the LDP. Based on a joint work with Ivan Corwin.

Dan Betea
Title: Asymptotics for some Schur processes.

Abstract: We study and compute limit shapes and fluctuations for various classes of Schur processes, including pyramid partitions recently introduced by Young, more general steep tilings of Bouttier, Chapuy and Corteel, and the Aztec diamond. If time permits, symmetric versions will be discussed as well.

Alexey Bufetov
Title: Hall-Littlewood positive homomorphisms of the algebra of symmetric functions

Abstract: Multiplicative linear functionals of the algebra of symmetric functions which are positive on all Hall-Littlewood functions (or, more generally, Macdonald functions) play an important role in asymptotic representation theory and its applications. However, the classification of all such functionals remains conjectural. In the talk I will describe a recent progress related to this conjecture. The talk is based on joint works with V. Gorin and L. Petrov.

Reda Chhaibi
Title: RS dynamics via random walks on groups over non-Archimedean fields

Abstract: The classical dynamics via the Robinson-Schensted (RS) correspondence naturally leads to non-intersecting particle systems. And the RS correspondence is now understood to be a tensor product dynamic in disguise, leading the way to many possible generalizations. In this talk, I would like to highlight a different approach where the tensor product operation can be viewed as exactly a matrix product in groups over non-Archimedean fields. This yields a probabilistic proof that non-Archimedean Whittaker functions are exactly characters, a formula known as the Shintani-Casselman-Shalika formula.

Sunil Chhita
Title: The two-periodic Aztec diamond

Abstract: Random tilings of large Aztec diamonds share many of the statistical properties observed in random matrix theory. The two-periodic Aztec diamond features three macroscopic phases which are sometimes called solid, liquid and gas. In this talk, we introduce the model and talk about recent results of the behavior of the dominoes at the liquid-gas boundary.

Francis Comets
Title: Polymer localization: what do we learn from the log-gamma model.

Abstract: As a general fact, directed polymers in random environment are localized in the so-called strong disorder phase. In this talk, based on a joint work with Vu-Lan Nguyen, we will consider the exctly solvable model with log-gamma environment, introduced recently by Seppalainen. For the stationary model and the point-to-line version, the localization can be expressed as the trapping of the end-point in a potential given by an independent random walk.

Sylvie Corteel
Title: Combinatorics of exclusion processes with open boundaries

Abstract: We study the combinatorics of the two-species asymmetric simple exclusion process with open boundaries studied among others by Masaru Uchiyama. We give a combinatorial interpretation of the partition function thanks to the Matrix Ansatz and the combinatorics of lattice paths. This brings us to a general positivity conjecture on "Koornwinder moments". We also present a generalization of the staircase tableaux and TAT tableaux of Mandelshtam and Viennot which gives a combinatorial interpretation of the stationary distribution. This is joint work with Lauren Williams (Berkeley) and also with Olya Mandelshtam (Berkeley) for the tableaux parts.

Manon Defosseux
Title: A Brownian Motion in a moving boundary domain and affine lie algebras.

Philippe Di Francesco
Title: Whittaker functions, cluster algebras and MacDonald difference operators

Abstract: Whittaker functions have recently reappeared in the context of random polymers and special McDonald processes. We revisit their earlier life in representation theory: they were originally built out of Whittaker vectors, for which we give a new statistical weighted path formulation, valid for simple and affine Lie algebras as well as the quantum algebra U_q(sl_n). We show how this formulation bypasses the classical derivation of Toda-type differential/difference equations satisfied by these functions. We then consider graded tensor products of current algebra g[t]-modules, and show that their characters

obey difference equations, generalizing the difference Toda equation, allowing for viewing graded characters as generalized Whittaker functions. This is done using a constant term expression for the characters using a solution of the quantum Q-system, a set of non-commuting integrable recursion relations that are particular mutations of a quantum cluster algebra attached to the Lie algebra g.
Finally, we obtain a new compact expression for graded sl_n characters by constructing a representation of the quantum Q-system via generalized McDonald-Ruijsenaars difference operators. (based on joint works with R. Kedem, and R. Kedem and B. Turmunkh).

Elnur Emrah
Title: The shape functions of certain exactly solvable inhomogeneous planar corner growth models

Abstract: We consider two kinds of inhomogeneous corner growth models with independent waiting times W(i, j) in the first quadrant: (1) W(i, j) is distributed exponentially with parameter a_i+b_j, (2) W(i, j) is distributed geometrically with fail parameter a_ib_j. The a and b parameters are randomly chosen at the outset from a joint distribution that is stationary with respect to the nonnegative shifts and ergodic (separately) with respect to the positive shifts of the indices. Let P_a and P_b denote the marginal distributions of the a's and b's, respectively. We obtain variational formulas in terms of P_a and P_b for the shape functions of models (1) and (2) for typical realizations of a's and b's. We identify certain choices of P_a and P_b for which these formulas yield closed-form expressions for the shape function. To compute the shape function, we rely on explicit knowledge of certain stationary corner growth models.

Vadim Gorin
Title: Multilevel Dyson Brownian Motion and its edge limits

Abstract: The GUE Tracy-Widom distribution is known to govern the large-time asymptotics for a variety of interacting particle systems on one side, and the asymptotic behavior for largest eigenvalues of random Hermitian matrices on the other side. In my talk I will explain some reasons for this connection between two seemingly unrelated classes of stochastic systems, and how this relation can be extended to general beta random matrices. A multilevel extension of the Dyson Brownian Motion (which we construct with the use of Jack symmetric polynomials) will be the central object in the discussion. (Based on joint papers with Misha Shkolnikov).

Chris Janjigian
Title: Large deviations of the free energy in the O'Connell-Yor polymer

Abstract: This talk considers the model of a 1+1 dimensional directed polymer in a random environment due to O'Connell and Yor, which is a Brownian analogue of the classical lattice directed polymer models. The O'Connell-Yor model satisfies a strong analogue of Burke's theorem from queuing theory, which makes some objects of interest exactly computable. We will discuss how, using the Burke property, one can compute the positive moment Lyapunov exponents of the parabolic Anderson model associated to

the polymer and how this leads to a computation of the large deviation rate function with normalization n for the free energy of the polymer.

Kurt Johansson
Title: The two-time distribution in random growth

Abstract: I will discuss a recent result on the two-time distribution in the zero temperature Brownian semi-discrete directed polymer. This should be the universal two-time distribution in last-passage percolation and related random growth models. The starting point is certain determinantal transition functions originating in work by J. Warren. I will give some hints of the demonstration although the whole proof is long and very technical.

Rinat Kedem
Title: Fusion products and q-Whittaker functions

Abstract: The generalized Heisenberg spin chain is a prototypical example of a statistical model with a Bethe ansatz solution. The combinatorics of the spectrum, i.e. the linearized energy function and resulting partition function, yields a graded character formula for tensor product affine Lie algebra modules. In the simplest example, these are affine Demazure modules, and in the infinite-size limit, they give the space of states of the level-1 WZW conformal field theory. In the finite-dimensional case, the characters are Whittaker functions for the quantum (finite) algebra at special values of the parameters, or specialized Macdonald polynomials. I will explain the physics and representation theory of graded tensor products, and show how the characters are related to an integrable discrete evolution called the Q-system, a cluster algebra mutation.

A. N. Kirillov
Title: Rigged configurations and RSK

Christian Korff
Title: Non-commutative q-Whittaker polynomials and 2D TQFTs

Abstract: The RSK correspondence maps words into a pair of tableaux P and Q. The plactic algebra introduced by Lascoux and Schuetzenberger is obtained by identifying words which have the same P-tableau. In my talk I will review the construction of an (affine) quantum plactic algebra and use it to define analogues of q-Whittaker polynomials in its generators. These polynomials are shown to give rise to a family of 2D topological quantum field theories whose fusion coefficients define cylindric generalisations of skew Hall-Littlewood and q-Whittaker functions. The partition functions of the TQFTs can be neatly expressed in terms of a Cauchy type identity of the non-commutative q-Whittaker polynomials. If q=0 one recovers from this construction the sl(n) Verlinde ring.

Jeffrey Kuan
Title: Stochastic duality of ASEP with two particle types via symmetry of quantum groups of rank two

Abstract: We study two generalizations of the asymmetric simple exclusion process with two types of particles. Particles of type 1 can jump over particles of type 2, while particles of type 2 can only influence the jump rates of particles of type 1. We prove that the processes are self-dual and explicitly write the duality function. As an application, an expression for the r-th moment of the exponentiated current is written in terms of r-particle evolution. The construction and proofs of duality are accomplished using symmetry of the quantum groups Uq(sl3) and Uq(sp4), with each node in the Dynkin diagram corresponding to a particle type, and the number of edges corresponding to the jump rates.

Thomas Lam
Title: Box-ball systems and cylindric loop Schur functions

Abstract: Box-ball systems are deterministic discrete dynamical systems that arise as the "crystal limit" of the six vertex model and similar lattice models. These dynamical systems have a natural birational/geometric lifting arising from the geometric R-matrix. I will talk about formulae for the integrals of motion of these systems in terms of cylindric loop Schur functions. This leads to explicit piecewise-linear formulae for soliton lengths in box-ball systems, orequivalently, for certain shapes in the corresponding rigged configuration. This is joint work with Pavlo Pylyavsky and Reiho Sakamoto.

James Martin
Title: Transfer matrices, tandem queues, and multi-type particle systems.

Abstract: I'll survey two related approaches to the description of certain multi-type particle systems, including ASEP, TASEP with multiple jump-rates, Hammersley's process and certain zero-range processes. The equilibria of such models have been constructed using products of "transfer matrices" and also using systems of queues in series. These two approaches share a common recursive structure (in which the equilibrium of an n-type system is derived from that of an (n-1)-type system). The queueing approach relies strongly on ideas of time-reversal (which can be seen as elaborations of Burke's theorem for a single-server queue). It is equally well-suited to describing models where mass is continuous rather than discrete, and can be used to describe the joint distribution of Busemann functions associated to trees of infinite geodesics in the last-passage percolation model, as the direction of the geodesics is varied.

Kostya Matveev
Title: q-randomized RSK's and random polymers.

Abstract: Robinson-Schensted-Knuth (row or column) insertion algorithm applied to a matrix of (Bernoulli or geometric) independent entries produces a random dynamics on interlacing particle arrays. In each of these four cases we introduce a q-deformation of a dynamics, which:

1) can be viewed as a q-randomized RSK insertion applied to a matrix of (Bernoulli or q-geometric) entries
2) at time t produces a distribution on the array from a family of q-Whittaker processes instead of Schur processes

3) has a 1D integrable marginal dynamics -- (Bernoulli or geometric) (q-TASEP or q-PushTASEP)
4) in the geometric case and q to 1 limit leads to partition functions of two models of log-Gamma polymers and geometric/tropical RSK. This is a joint work with Leo Petrov.

Mihai Nica
Title: Three ways to think about a certain model of vicious walkers

Abstract: I will present a particular model of non-crossing (so called "vicious") walkers which, by using the Robinson-Schensted-Knuth correspondence, can be thought of in two other ways described by evolving Young diagrams. By using simple properties of the RSK correspondence, this allows us to characterize the finite dimensional distributions of the walkers and allows us to see certain nice properties that are not otherwise apparent. The model scales to the Airy 2 process under the correct scaling.

Vu-Lan Ngyuen
Title: Variants of geometric RSK, geometric PNG and the multipoint distribution of the log-gamma polymer.

Abstract : Geometric (or tropical) RSK was introduced by Kirillov and extended in a matrix setting by Noumi-Yamada. O'Connell,Seppallainen and Zygouras reformulated it in terms of local moves. We show that the local move reformulation can be extended to cases where the input matrix is replaced by more general polygonal arrays. We also show that a rearrangement of the sequence of the local moves gives rise to a geometric version of the polynuclear growth model (PNG). These reformulations are used to obtain integral formulas for the Laplace transform of the joint distribution of the point-to-point partition functions of the log-gamma polymer at different space-time points. This is joint work with Nikos Zygouras.

Sergey Oblezin
Title: Whittaker functions and harmonic analysis

Abstract: Whittaker functions were introduced by Jacquet and Langlands in the end of 1960s, and since then play an important role in harmonic analysis for non-compact reductive groups. The local Langlands correspondence provides a relation between the Whittaker function and complete flag manifold of the complex dual group. Recently, in my paper with Gerasimov and Lebedev a notion of parabolic Whittaker function for the real group GL(n,R) have been introduced. These functions are related to the general (incomplete) flag varieties, and they are the missing part of the local harmonic analysis. In my talk I'll describe integral representations of the parabolic Whittaker functions and outline their basic properties and applications.

Janosch Ortmann
Title: Tracy-Widom asymptotics for a random polymer model with gamma-distributed weights

Abstract: We establish Tracy-Widom asymptotics for the partition function of a random polymer model with gamma-distributed weights recently introduced by Seppäläinen. We show that the partition function of this random polymer can be represented within the framework of the geometric RSK correspondence and consequently its law can be expressed in terms of Whittaker functions. This leads to a representation of the law of the partition function which is amenable to asymptotic analysis. In this model, the partition function plays a role analogous to the smallest eigenvalue in the Laguerre unitary ensemble of random matrix theory. Joint work with Neil O'Connell.

Yuchen Pei
Title: A q-weighted Robinson-Schensted algorithm

Abstract: I'll describe a q-weighted Robinson-Schensted algorithm transforming a random walk into a Markov chain related to the q-Whittaker functions, and a symmetry property this algorithm satisfies. This talk is partly based on joint work with Neil O'Connell.

Leonid Petrov
Title: Bethe ansatz for stochastic quantum integrable systems in infinite volume

Abstract: I will discuss the higher spin vertex model, which is a (discrete-time) stochastic quantum integrable system on the (half-)line. In various regimes, this model degenerates to both ASEP and q-TASEP, two well-known integrable discretizations of the Kardar-Parisi-Zhang equation. I will focus on Bethe ansatz integrability of the higher spin vertex model. This approach provides eigenfunctions of the model, which are nice symmetric rational functions generalizing the Hall-Littlewood symmetric polynomials. A certain Markov (self-)duality plus Fourier-like transforms associated with the eigenfunctions allow to write down exact formulas for observables of the system started from an arbitrary initial data

A.M. Povolotsky
Title: The generalized TASEP on the edge of jamming

Abstract: The generalized TASEP is the TASEP-like model equipped with an additional interaction enhancing formation of large jams. In particular, as the interaction strength varies, it interpolates between the TASEPs with parallel and backward sequential updates, and finally comes to the irreversible adsorption: all particles tend to stick together into a single cluster moving as one particle. While in the the most of the parameter range the system is expected to be characterized by the KPZ fluctuations, they become purely Gaussian in the jamming limit. In the talk we will describe the model, study the limits of the KPZ-universality and explain how the KPZ universality breaks down crossing over to the Gaussian behaviour.

Dan Romik
Title: A Pfaffian point process for totally symmetric self complementary plane partitions

Abstract: Totally Symmetric Self Complementary Plane Partitions (TSSCPPs) can be encoded as a family of nonintersecting lattice paths having fixed initial points and variable endpoints. The endpoints of the paths associated with a uniformly random TSSCPP of given order therefore induce a random point process, which turns out to be a Pfaffian point process. I will discuss conjectural formulas for the entries of the correlation kernel of this process, and a more general "rationality phenomenon", which if true implies the existence of an interesting limiting process describing "infinite TSSCPPs" as well as conjectural probabilities for the occurrence of certain connectivity patterns in loop percolation (a.k.a. the dense O(1) loop model).

Tomohiro Sasamoto
Title: A determinantal structure for the O'Connell-Yor polymer model

Abstract: For the Gaussian unitary ensemble (GUE), it is well known that the eigenvalues are determinantal because the probability density of the eigenvalues is written in the form of two determinants. For the O'Connell-Yor(OY) polymer model, a generating function of the partition function can be written as a Fredholm determinant but the underlying determinant structure is not well understood. We discuss a determinantal structure associated with the OY polymer model, which is related to the above-mentioned Fredholm determinant. We show this relation by introducing a generalization of Warren's Brownian motion in the Gelfand-Tsetlin cone. This is based on a collaboration with T. Imamura.

Hao Shen
Title: The strict-weak lattice polymer

Abstract: We discuss KPZ universality of the free energy fluctuation of the strict-weak polymer model. The proof of KPZ universality relies on the fact that the discrete time geometric q-TASEP model, studied earlier by A. Borodin and I. Corwin, scales to this polymer model in the limit q->1. This allows us to exploit the exact results for geometric q-TASEP to derive a Fredholm determinant formula for the strict-weak polymer, and in turn perform rigorous asymptotic analysis to show KPZ universality. This is a joint work with Ivan Corwin and Timo Seppäläinen.

Yi Sun
Title: A representation-theoretic proof of the branching rule for Macdonald polynomials

Abstract: We give a new representation-theoretic proof of the branching rule for Macdonald polynomials using the Etingof-Kirillov Jr. expression for Macdonald polynomials as traces of intertwiners of U_q(gl_n). In the Gelfand-Tsetlin basis, we show that diagonal matrix elements of such intertwiners are given by application of Macdonald's operators to a simple kernel. An essential ingredient in the proof is a map between spherical parts of double affine Hecke algebras of different ranks based upon the Dunkl-Kasatani conjecture.