

The usual notion of differentiability in infinite-dimensional Banach spaces is Fréchet-differentiability. It can be viewed as a straight forward generalization of the finite-dimensional notion. The important feature of Fréchet differentiability is the validity of the chain-rule. However, there are different generalizations, and a new one is sc-differentiability. This notion also has a chain-rule. Sc-smoothness requires an additional structures on the Banach spaces, so-called sc-structures.

The striking difference between "Fréchet-smooth" and "sc-smooth" can be seen when studying maps  $r : U \rightarrow U$ , satisfying  $r \circ r = r$ , i.e. retractions, where  $U$  is an open subset of a Banach space. If  $r$  is Fréchet-smooth, then  $r(U)$  is necessarily a submanifold of  $U$ . However, there are sc-smooth examples where  $r(U)$  is finite-dimensional, but has locally varying dimension. There are also examples where a connected  $r(U)$  has finite-dimensional as well as infinite-dimensional parts. If we consider pairs  $(O, E)$ , where  $O$  is a subset of the sc-Banach space  $E$  and the image of an sc-smooth retraction, we obtain new local models for smooth spaces. We even can define the tangent of  $T(O, E)$  by  $(TO, TE)$ , where  $TO = Tr(TU)$ . Noting that by the chain-rule  $Tr \circ Tr = Tr$  we see that  $TO$  is again an sc-smooth retract. As it turns out the definition does not depend on  $r$  as long as  $O$  is the image of  $r$ . We also can define what sc-smooth maps between local sc-models are. Evidently many constructions known from differential geometry can be carried over to a new "sc-retraction based differential geometry". Manifolds become M-polyfolds and orbifolds become polyfolds.

A nonlinear elliptic differential operator can usually be interpreted as a Fredholm section of a Banach space bundle and, given enough compactness, can be studied topologically. Many interesting problems in geometry are related to elliptic problems which show a lack of compactness, like bubbling-off. However these problems usually have fancy compactifications. Two such problems of interest are Gromov-Witten theory and the more general symplectic field theory. Due to serious compactness and transversality issues it is difficult to study them in a classical Banach manifold set-up. However, it turns out that they are much more easily described in the sc-retraction based differential geometry.

Finally, there is a generalization of the classical nonlinear Fredholm theory to the sc-world, which also has a build in Sard-Smale-type perturbation and transversality theory. In its applications to symplectic field theory the solution spaces are the compactified moduli spaces.