

FRACTAL AND MULTI-FRACTAL PROPERTIES OF SLE

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This six hour mini-course will focus on path properties of the Schramm-Loewner evolution. I will assume that the students have some familiarity with SLE. (The course will be given in the third week. The course by Vincent Beffara which will be given in the first two weeks should give the necessary background on SLE.) The path properties to be discussed include:

- Existence and Hölder continuity of the curve
- Hausdorff dimension of the curve
- Green's function for SLE
- Natural parameterization of the path

As part of the tools for proving results I will also discuss

- The reverse Loewner flow
- Estimate of moments of derivatives for the reverse flow

For background in SLE, references are

- W. Werner (2004). Random planar curves and Schramm-Loewner evolutions, Ecole, d'Eté de Probabilités Saint-Flour XXXii - 2002, Lectures Notes in Math by 1840, Springer-Verlag.
- G. Lawler (2005). *Conformally Invariant Processes in the Plane*, AMS
- G. Lawler (2009). Schramm-Loewner Evolution in *Statistical Mechanics*, IAS/Park City Math Series **16**, S. Sheffield and T. Spencer, ed., AMS.

This course will talk about material from research papers. Links to the papers for which I am a co-author are available at

www.math.uchicago.edu/~lawler/papers.html

- S. Rohde and O. Schramm (2005). Basic properties of SLE, *Annals of Math.* **161**, 879–920.
- V. Beffara (2008). The dimension of the SLE curves, *Annals of Probab.* **26**, 1421–1452.
- G. Lawler (2009), Multifractal analysis of the reverse flow for the Schramm-Loewner evolution, *Progress in Probability* **61**, Birkhäuser, 73–107.
- F. Johansson and G. Lawler, Optimal Hölder exponent for the SLE path, preprint.

- F. Johansson and G. Lawler, Almost sure tip multifractal spectrum for the tip of an SLE curve, preprint.
- G. Lawler and S. Sheffield, The natural parameterization for the Schramm-Loewner evolution, preprint.