Mini-course on RANDOM POLYMERS

• 2010 Clay Mathematics Institute Summer School
July 11 – August 7, 2010

• 14th Brazilian School of Probability
August 2–7, 2010

Buzios, Brazil

Lectures: Frank den Hollander
(6 sessions, 1.5 hours each; 3 on polymers without disorder, 3 on polymers with disorder).

Tutorials: Francesco Caravenna and Nicolas Pétrélis
(5 sessions, 1.5 hours each).

Literature:

- G. Giacomin, *Random Polymer Models*, Imperial College Press, London, 2007.
- F. den Hollander, *Random Polymers*, Lecture Notes in Mathematics 1974, Springer, Berlin, 2009.

1. Introduction

Lecture: What is a polymer? What types of polymers occur in nature? What types of questions are targeted? What is the model setting? The central role of free energy and phase transitions. Effect of disorder. Two basic models: simple random walk and self-avoiding walk.

Tutorial: Techniques to prove existence of free energy (subadditivity, concentration of measure). Examples where existence remains open.

2. Polymer collapse

Lecture: Competition between repulsion and attraction. An undirected model: three phases (extended, collapsed, localized), phase diagram, critical exponents. A directed model: explicit computation of free energy via

generating function, associated singularity analysis. Apply force to one end of collapsed polymer.

Tutorial: Details of generating function computation.

3. Polymer + homogeneous interface

Lecture: Homogeneous pinning model, excursions away from interface, free energy, phase transition, properties of critical curve, pinning vs. wetting, force and re-entrant phase transition.

Tutorial: Order of phase transition. Effect of different geometries of interface.

4. Polymer + random interface

Lecture: Random pinning model, free energy, phase transition, properties of critical curve, relevant vs. irrelevant disorder, derivation of variational formulas for critical curves based on annealed and quenched large deviation principles for excursions.

Tutorial: Explain background of annealed and quenched large deviation principle for random words cut out from random letter sequence. Relative entropies.

5. Copolymer + two immiscible fluids

Lecture: Random copolymer model, free energy, phase transition, properties of critical curve, upper and lower bounds, numerics, weak interaction limit, variational approach.

Tutorial: Proof of lower bound on critical curve. Proof that order of phase transition is at least two (smoothing by disorder).

6. Polymer + random potential

Lecture: Directed polymer in random environment, key martingale, phases of strong and weak disorder, successive estimates on critical temperature.

Tutorial: –