SYLLABUS AND READING LISTS FOR EINSIEDLER-LINDENSTRAUSS COURSE

Syllabus (14 lectures)

1. General introduction, preview of some applications, and Cassels-Swinnerton-Dyer isolation theorem.
2. Leafwise measures and their properties.
3-4. Entropy, Bowen balls, elements from Ledrappier-Young theory, and relation to recurrence.
5. Product lemma for leafwise measures and the high entropy method.
6. The low entropy method.
7. Classification of measures invariant under a maximally split torus: combining the high and low entropy methods.
8-9. Application: arithmetic quantum unique ergodicity.
10. Variational principal, closed invariant sets for some multidimensional diagonalizable actions and their topological entropy.
11. Application: values of product of linear forms and the set of exceptions to Littlewood's conjecture.
12. Periodic torus orbits: discriminant, volume, and distribution properties of periodic orbits.
13. Periodic torus orbits: Periodic geodesics on the modular surface: Dukes theorem, the Linnik method.
14. Periodic torus orbits: Equidistribution of periodic torus orbits on the space of 3D Lattices.

Prerequisites

We will assume knowledge of some basic notions in ergodic theory (ergodicity, unique ergodicity, ergodic decomposition, pointwise ergodic theorem, Poincare recurrence,...) as well as the measure theoretic notions of conditional expectations and conditional measures, for example as covered in [EW, Chp. 2,4-5].

Formally our treatment of ergodic theoretic entropy will be self-contained; however it is recommended that students come with some prior knowledge on the subject as can be found in essentially any standard reference on ergodic theory (e.g. [Pet, Chp. 5-6], or the relevant chapter(s) in [Rud]). Another possibility is [KH, Chp. 4].

Suggested reading

(Note that there may be some overlap with the Eskin-Kleinbock course.)

Textbooks/tutorials (recommended reading): [Lin4], [Mor, Chp. 1], [BM, I-V]. A survey of some of the topics covered in the course can be found at [Lin2].

Some papers related to the course[1] [Lin3, EKL, Lin1, ELMV]

References

[BM] M. Bachir Bekka and Matthias Mayer. *Ergodic theory and topological dynamics of group actions on homogeneous spaces*, volume 269 of *London Mathematical Society Lecture Note Series*. Cambridge University Press, Cambridge, 2000.

[EKL] Manfred Einsiedler, Anatole Katok, and Elon Lindenstrauss. Invariant measures and the set of exceptions to Littlewood's conjecture. *Ann. of Math. (2)* **164**(2006), 513–560.

[ELMV] Manfred Einsiedler, Elon Lindenstrauss, Philippe Michel, and Akshay Venkatesh. The distribution of periodic torus orbits on homogeneous spaces, 2006. submitted (42 pages).

[EW] Manfred Einsiedler and Thomas Ward. Ergodic Theory: with a view towards Number Theory. in preparation, some chapters available online., 2007.

[KH] Anatole Katok and Boris Hasselblatt. *Introduction to the modern theory of dynamical systems*, volume 54 of *Encyclopedia of Mathematics and its Applications*. Cambridge University Press, Cambridge, 1995. With a supplementary chapter by Katok and Leonardo Mendoza.

[Lin1] Elon Lindenstrauss. Arithmetic quantum unique ergodicity and Adelic dynamics. to appear in the proceedings of Current Developments in Mathematics conference, Harvard 2004 (30 pages), 2005.

[Lin2] Elon Lindenstrauss. Rigidity of multiparameter actions. *Israel J. Math.* **149**(2005), 199–226.

[Lin3] Elon Lindenstrauss. Invariant measures and arithmetic quantum unique ergodicity. *Ann. of Math. (2)* **163**(2006), 165–219.

[Lin4] Elon Lindenstrauss. Some examples how to use measure classification in number theory. In *Equidistribution in number theory, an introduction*, volume 237 of *NATO Sci. Ser. II Math. Phys. Chem.*, pages 261–303. Springer, Dordrecht, 2007.

[Mor] Dave Witte Morris. *Ratner's theorems on unipotent flows*. Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 2005.

[Pet] Karl Petersen. *Ergodic theory*, volume 2 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1983.

[Rud] Daniel J. Rudolph. *Fundamentals of measurable dynamics*. Oxford Science Publications. The Clarendon Press Oxford University Press, New York, 1990. Ergodic theory on Lebesgue spaces.

[1] Available at `http://www.math.princeton.edu/~elonl/Publications/index.html`