SYLLABUS AND READING LIST FOR ESKIN-KLEINBOCK COURSE

Syllabus (14 lectures)

1. General introduction, Birkhoff's Ergodic Theorem vs. Ratner's Theorems on unipotent flows; measure classification implies classification of orbit closures; uniform convergence and the theorem of Dani-Margulis; the statement of the Oppenheim Conjecture.
2. The case of $\mathrm{SL}(2,\mathbb{R})$ (the mixing argument). We will be loosely following Ratner's paper [18].
3. The classification of invariant measures for the action of a unipotent subgroup of $\mathrm{SL}(2,\mathbb{R}) \ltimes \mathbb{R}^2$ (modulo non-divergence).
4. Quantitative non-divergence of unipotent flows and quasi-polynomial maps: the case of $\mathrm{SL}(2,\mathbb{R})$.
5. Quantitative non-divergence in higher rank.
6. Oppenheim and Quantitative Oppenheim (lower bounds). The work of Dani-Margulis [2].
7. Quantitative Oppenheim (upper bounds) [4]. Systems of inequalities.
8. The work of Mozes-Shah [17] (limits of ergodic measures are ergodic).
9. Equidistribution of translates and applications to Diophantine equations. We will follow parts of [6] and [7].
10. Equidistribution of translates and applications to improvement of Dirichlet's Theorem [12, 14].

11-12. Applications of non-divergence to metric Diophantine approximation: Baker-Sprindžuk Conjectures and their generalizations, Diophantine exponents of affine subspaces [9, 10, 11].

13. Counting problems with billiards (slightly off-topic).
14. The work of Margulis-Goetze on quadratic forms.
15. If time allows: an overflow lecture. Possibly Oppenheim in signature $(2, 2)$ and applications to quantum chaos.

Prerequisites and suggested reading

Basic notions in ergodic theory and some basics on Lie groups (see the prerequisites for the Einsiedler-Lindenstrauss course). Recommended reading: [19, Ch. 1], [1, Ch. I–V], [21, Ch. 2]. Some of the topics covered in the course are surveyed in [3], [8], [15], [16]. More background is contained in the book [20] and the article [13] from the Handbook of Dynamical Systems. See also the bibliography below for references to some research papers which we may be following at some point in the course.

References

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[4] A. Eskin, G. A. Margulis and S. Mozes, *Upper bounds and asymptotics in a quantitative version of the Oppenheim conjecture*, Ann. Math. **147** (1998), 93–141.

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[7] A. Eskin, S. Mozes and N. Shah, *Unipotent flows and counting lattice points on homogeneous varieties*, Ann. Math. **143** (1996), 253–299.

[8] D. Kleinbock, *Some applications of homogeneous dynamics to number theory*, in: **Smooth ergodic theory and its applications**,Proc. Sympos. Pure Math., 69, Amer. Math. Soc., Providence, RI, 2001, pp. 639–660.

[9] ———, *An extension of quantitative nondivergence and applications to Diophantine exponents*, Preprint (2005), `arXiv:math/0508517`.

[10] D. Kleinbock, E. Lindenstrauss and B. Weiss, *On fractal measures and diophantine approximation*, Selecta Math. **10** (2004), 479–523.

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[14] D. Kleinbock and B. Weiss, *Dirichlet's theorem on diophantine approximation and homogeneous flows*, Preprint (2006), `arXiv:math/0612171`.

[15] G. A. Margulis, *Oppenheim conjecture*, in: **Fields Medalists' lectures**, WorldSci. Publishing, River Edge, NJ, 1997, pp. 272–327.

[16] ______, *Diophantine approximation, lattices and flows on homogeneous spaces*, in: **A panorama of number theory or the view from Baker's garden**, Cambridge Univ. Press, Cambridge, 2002, pp. 280–310.

[17] S. Mozes and N. Shah, *On the space of ergodic invariant measures of unipotent flows*, Ergodic Theory Dynam. Systems **15** (1995), no. 1, 149–159.

[18] M. Ratner, *Raghunathan's conjectures for* $\mathrm{SL}(2, \mathbf{R})$, Israel J. Math. **80** (1992), no. 1-2, 1–31.

[19] D. Morris, **Ratner's theorems on unipotent flows**, Chicago Lectures in Mathematics. University of Chicago Press, Chicago, IL, 2005.

[20] A. Starkov, **Dynamical systems on homogeneous spaces**, Translations of Mathematical Monographs, Vol. 190, Amer. Math. Soc., Providence, RI, 2000.

[21] R. J. Zimmer, **Ergodic theory and semisimple groups**, Birkhaüser, 1984.