NEWSTEAD'S SCIENTIFIC WORK

Newstead's research has dealt with the study of vector bundles in the context of algebraic geometry. He has used a combination of classical geometric methods, topological methods and geometric invariant theory, and in latter years has worked with many collaborators.

A recurrent topic throughout his career has been the study of the topology of the moduli space of vector bundles on curves (see 2, 3, 6, 23, 28, 29). In rank 2, he computed Betti numbers, complete sets of generators and relations and compared the intermediate jacobian of the moduli space with the jacobian of the curve. Analogous complete results for higher rank have only been obtained very recently by Kirwan and Jeffrey.

A second topic of interest has been the study (and proof of non-existence in some cases) of universal (or Poincaré) bundles (see 8, 31, 40, 42). A systematic study of Picard bundles defined as direct images of twists of the universal bundles, has been started, in particular from the point of view of stability (see 34, 35).

Newstead must be credited with the first construction of a compactification of the moduli space of vector bundles on a singular curve (in the last chapter of his first book I). Although this theory has been extended to more general curves, his description is still the more widely used.

It has long been conjectured that the moduli space of vector bundles of fixed rank and determinant over a curve is rational. Newstead settled this conjecture in some cases (see 9). It was proved for the case in which the rank and degree are coprime by King and Schofield in 1999. The general case is still open and is one of the main problems remaining in this area.

In more recent years, Newstead introduced the study of Brill-Noether theory for rank greater than one and the related problem of coherent systems on algebraic curves. A coherent system is defined as a pair consisting of a vector bundle and a linear subspace of its space of sections. Coherent systems appear naturally as solutions of certain types of differential equations associated with problems in physics. King and Newstead gave an algebro-geometric construction of their moduli spaces in 27.

Brill-Noether theory allows us (as in the rank one case) to define theta divisors inside the moduli spaces of vector bundles. The dimension of the spaces of sections of multiples of these theta divisors are given by the Verlinde formulae while the divisors themselves generate the Picard group of the moduli space of vector bundles of fixed rank and determinant. Moreover, some more general Brill-Noether loci define special subvarieties of the moduli space of curves that characterize, for example, curves contained in a K3 surface and have also been used to produce counterexamples to the slope conjecture.

A more surprising result in this theory is contained in 30. Newstead and his collaborators showed that, unlike in the rank one case, generic curves do not behave as expected and they were able to give a full description of the situation in the case when the degree is small compared with the rank. Other collaborations led to less complete results in the case of higher degree (and more sections) in 32, 33, 39. In 37 and 43 a further collaboration has been introducing new techniques to deal with the problem using several moduli spaces at once and the rational transformations (or"flips") between them.

Current research is concentrated mainly on coherent systems and in particular the problem of determining more precisely when the moduli spaces are non-empty. This work is mainly concerned with genus $g \geq 2$; however, strong but incomplete results for $g = 0$ are given in 38 and 44, and a complete solution for $g = 1$ in 41.

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