a finite number of terms, then the derivative of the expression for $f(x)$ or, up to a part diminishing very rapidly with growing x,

$$\frac{1}{\log x} - 2\sum^{\alpha} \frac{\cos(\alpha \log x) x^{-\frac{1}{2}}}{\log x}$$

gives an approximating expression for the density of the prime number + half the density of the squares of the prime numbers + a third of the density of the cubes of the prime numbers etc. at the magnitude x.

The known approximating expression $F(x) = Li(x)$ is therefore valid up to quantities of the order $x^{\frac{1}{2}}$ and gives somewhat too large a value; because the non-periodic terms in the expression for $F(x)$ are, apart from quantities that do not grow infinite with x:

$$Li(x) - \tfrac{1}{2}Li(x^{\frac{1}{2}}) - \tfrac{1}{3}Li(x^{\frac{1}{3}}) - \tfrac{1}{5}Li(x^{\frac{1}{5}}) + \tfrac{1}{6}Li(x^{\frac{1}{6}}) - \tfrac{1}{7}Li(x^{\frac{1}{7}}) + \cdots$$

Indeed, in the comparison of $Li(x)$ with the number of prime numbers less than x, undertaken by *Gauss* and *Goldschmidt* and carried through up to $x =$ three million, this number has shown itself out to be, in the first hundred thousand, always less than $Li(x)$; in fact the difference grows, with many fluctuations, gradually with x. But also the increase and decrease in the density of the primes from place to place that is dependent on the periodic terms has already excited attention, without however any law governing this behaviour having been observed. In any future count it would be interesting to keep track of the influence of the individual periodic terms in the expression for the density of the prime numbers. A more regular behaviour than that of $F(x)$ would be exhibited by the function $f(x)$, which already in the first hundred is seen very distinctly to agree on average with $Li(x) + \log \xi(0)$.