take the form

$$\pm\frac{1}{2\pi i}\frac{1}{\log x}\int\limits_{a-\infty i}^{a+\infty i}\frac{d\left(\frac{1}{s}\log\left(1-\frac{s}{\beta}\right)\right)}{ds}x^s\,ds.$$

But now

$$\frac{d\left(\frac{1}{s}\log\left(1-\frac{s}{\beta}\right)\right)}{d\beta}=\frac{1}{(\beta-s)\beta},$$

and, if the real part of s is larger than the real part of β,

$$-\frac{1}{2\pi i}\int\limits_{a-\infty i}^{a+\infty i}\frac{x^s\,ds}{(\beta-s)\beta}=\frac{x^\beta}{\beta}=\int\limits_{\infty}^{x}x^{\beta-1}\,dx,$$

or

$$=\int\limits_{0}^{x}x^{\beta-1}\,dx,$$

depending on whether the real part of β is negative or positive. One has as a result

$$\frac{1}{2\pi i}\frac{1}{\log x}\int\limits_{a-\infty i}^{a+\infty i}\frac{d\left(\frac{1}{s}\log\left(1-\frac{s}{\beta}\right)\right)}{ds}x^s\,ds$$

$$\begin{aligned}
&= -\frac{1}{2\pi i}\int\limits_{a-\infty i}^{a+\infty i}\frac{1}{s}\log\left(1-\frac{s}{\beta}\right)x^s\,ds\\
&= \int\limits_{\infty}^{x}\frac{x^{\beta-1}}{\log x}\,dx+\ \text{const.}
\end{aligned}$$

in the first, and

$$=\int\limits_{0}^{x}\frac{x^{\beta-1}}{\log x}\,dx+\ \text{const.}$$

in the second case.

In the first case the constant of integration is determined if one lets the real part of β become infinitely negative; in the second case the integral from 0 to x takes on values separated by $2\pi i$, depending on whether the integration is taken through complex values with positive or negative argument, and becomes infinitely small, for the former path, when the coefficient of i in the value of β becomes infinitely positive, but for the latter, when this coefficient becomes infinitely negative. From this it is seen how on the left hand side $\log\left(1-\dfrac{s}{\beta}\right)$ is to be determined in order that the constants of integration disappear.

Through the insertion of these values in the expression for $f(x)$ one obtains

$$f(x) = Li(x) - \textstyle\sum^{\alpha}\left(Li\left(x^{\frac{1}{2}+\alpha i}\right) + Li\left(x^{\frac{1}{2}-\alpha i}\right)\right) + \displaystyle\int_x^{\infty} \frac{1}{x^2-1}\frac{dx}{x\log x} + \log\xi(0),$$

if in $\sum^{\alpha}$ one substitutes for α all positive roots (or roots having a positive real part) of the equation $\xi(\alpha) = 0$, ordered by their magnitude. It may easily be shown, by means of a more thorough discussion of the function ξ, that with this ordering of terms the value of the series

$$\textstyle\sum\left(Li\left(x^{\frac{1}{2}+\alpha i}\right) + Li\left(x^{\frac{1}{2}-\alpha i}\right)\right)\log x$$

agrees with the limiting value to which

$$\frac{1}{2\pi i}\int_{a-bi}^{a+bi} \frac{d\frac{1}{s}\sum\log\left(1+\frac{(s-\frac{1}{2})^2}{\alpha\alpha}\right)}{ds} x^s\, ds$$

converges as the quantity b increases without bound; however when re-ordered it can take on any arbitrary real value.

From $f(x)$ one obtains $F(x)$ by inversion of the relation

$$f(x) = \textstyle\sum \frac{1}{n} F\left(x^{\frac{1}{n}}\right),$$

to obtain the equation

$$F(x) = \textstyle\sum(-1)^{\mu}\frac{1}{m} f\left(x^{\frac{1}{m}}\right),$$

in which one substitutes for m the series consisting of those natural numbers that are not divisible by any square other than 1, and in which μ denotes the number of prime factors of m.

If one restricts $\sum^{\alpha}$ to