$$F(x) = \frac{F(x+0) + F(x-0)}{2}.$$

If in the identity

$$\log\zeta(s) = -\sum \log(1 - p^{-s}) = \sum p^{-s} + \tfrac{1}{2}\sum p^{-2s} + \tfrac{1}{3}\sum p^{-3s} + \cdots$$

one now replaces

$$p^{-s} \text{ by } s\int_p^\infty x^{-s-1}\,ds, \quad p^{-2s} \text{ by } s\int_{p^2}^\infty x^{-s-1}\,ds, \ldots,$$

one obtains

$$\frac{\log\zeta(s)}{s} = \int_1^\infty f(x)x^{-s-1}\,dx,$$

if one denotes

$$F(x) + \tfrac{1}{2}F(x^{\frac{1}{2}}) + \tfrac{1}{3}F(x^{\frac{1}{3}}) + \cdots$$

by $f(x)$.

This equation is valid for each complex value $a + bi$ of s for which $a > 1$. If, though, the equation

$$g(s) = \int_0^\infty h(x)x^{-s}\,d\log x$$

holds within this range, then, by making use of *Fourier*'s theorem, one can express the function h in terms of the function g. The equation decomposes, if $h(x)$ is real and

$$g(a+bi) = g_1(b) + ig_2(b),$$

into the two following:

$$g_1(b) = \int_0^\infty h(x)x^{-a}\cos(b\log x)\,d\log x,$$

$$ig_2(b) = -i\int_0^\infty h(x)x^{-a}\sin(b\log x)\,d\log x.$$

If one multiplies both equations with

$$(\cos(b\log y) + i\sin(b\log y))\,db$$

and integrates them from $-\infty$ to $+\infty$, then one obtains $\pi h(y) y^{-\alpha}$ on the right hand side in both, on account of *Fourier*'s theorems; thus, if one adds both equations and multiplies them by iy^α, one obtains

$$2\pi i h(y) = \int_{a-\infty i}^{a+\infty i} g(s) y^s \, ds,$$

where the integration is carried out so that the real part of s remains constant.

For a value of y at which there is a jump in the value of $h(y)$, the integral takes on the mean of the values of the function h on either side of the jump. From the manner in which the function f was defined, we see that it has the same property, and hence in full generality

$$f(y) = \frac{1}{2\pi i} \int_{a-\infty i}^{a+\infty i} \frac{\log \zeta(s)}{s} y^s \, ds.$$

One can substitute for $\log \zeta$ the expression

$$\frac{s}{2} \log \pi - \log(s-1) - \log \Pi \left(\frac{s}{2}\right) + \textstyle\sum^\alpha \log \left(1 + \dfrac{(s-\frac{1}{2})^2}{\alpha\alpha}\right) + \log \xi(0)$$

found earlier; however the integrals of the individual terms of this expression do not converge, when extended to infinity, for which reason it is appropriate to convert the previous equation by means of integration by parts into

$$f(x) = -\frac{1}{2\pi i} \frac{1}{\log x} \int_{a-\infty i}^{a+\infty i} \frac{d \dfrac{\log \zeta(s)}{s}}{ds} x^s \, ds$$

Since

$$-\log \Pi \left(\frac{s}{2}\right) = \lim \left(\sum_{n=1}^{n=m} \log \left(1 + \frac{s}{2n}\right) - \frac{s}{2} \log m \right),$$

for $m = \infty$ and therefore

$$-\frac{d \frac{1}{s} \log \Pi \left(\frac{s}{2}\right)}{ds} = \sum_1^\infty \frac{d \frac{1}{s} \log \left(1 + \frac{s}{2n}\right)}{ds},$$

it then follows that all the terms of the expression for $f(x)$, with the exception of

$$\frac{1}{2\pi i} \frac{1}{\log x} \int_{a-\infty i}^{a+\infty i} \frac{1}{ss} \log \xi(0) x^s \, ds = \log \xi(0),$$