2012

CLAY MATHEMATICS INSTITUTE

annual report

mission

The primary objectives and purposes of the Clay Mathematics Institute are:

- to increase and disseminate mathematical knowledge
- to educate mathematicians and other scientists about new discoveries in the field of mathematics
- to encourage gifted students to pursue mathematical careers
- to recognize extraordinary achievements and advances in mathematical research

The CMI will further the beauty, power and universality of mathematical thought.

The Clay Mathematics Institute is governed by its Board of Directors, Scientific Advisory Board and President. Board meetings are held to consider nominations and research proposals and to conduct other business. The Scientific Advisory Board is responsible for the approval of all proposals and the selection of all nominees.

CLAY MATHEMATICS INSTITUTE

annual report 2012

contents

letter from the president

PHOTO COURTESY OF WADHAM COLLEGE

Dear Friends of Mathematics,

The summer of 2012 saw a significant shift in the operation of the Clay Mathematics Institute. In July, Jim Carlson passed on his presidential files, and his accumulated wisdom, and stepped down after nine years of creative and inspired leadership. A new President's office was opened in Oxford, and the CMI headquarters in America moved from Cambridge to Providence.

The move reflects a strengthening international focus for the CMI, but not a change of direction. Landon Clay's original vision of supporting mathematical research and its dissemination at the very highest level remains the central guiding principle. The core activities and programs through which CMI is known in the mathematical community will continue in the USA and elsewhere: the offer of the Clay Millennium Prizes, the appointment of research fellows, the annual research awards, the support for summer schools, the organization of workshops, and other initiatives. Indeed, the new location, a buoyant endowment, and the very generous terms on which the Oxford Mathematical Institute has offered space and support have together created an opportunity to expand the most successful existing programs and to establish new ones.

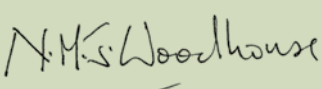
The first steps have already been taken. In September, the Scientific Advisory Board agreed a plan to increase the number of Clay Research Fellows over the next five years. It also set up a new 'Enhancement and Partnership Program', under which it can gather together proposals to build on the work of other mathematical organizations by, for example, providing funding to enable distinguished speakers to join local or regional meetings or to encourage international participation in national events.

The CMI has a flexibility in considering proposals that is now rare in the world of academic funding. It is unconstrained by the regulations and geographical restrictions under which government funding bodies must operate. That, and its focus on supporting work of the highest quality on the deep problems at the heart of contemporary research, give it a value to the mathematical community that is out of proportion to its size.

In October 2013, the new Mathematics building in Oxford will be opened, and the CMI office will move to more established quarters. The event will be celebrated by holding the Clay Research Conference on 2 October in the new building, along with a number of workshops on areas connected with the Millennium Prize Problems. We hope to see many of our friends then.

In this issue, you will find articles on the new Oxford building by Sam Howison, the departmental Chair; a report on the 2012 Clay Research Conference, which was held in Oxford; an account of the remarkable record of the Clay Research Awards; an interview with Peter Scholze, a Clay Research Fellow based in Germany; and summaries of the many activities undertaken over the year, before and after the shift across the Atlantic.

Sincerely,



N. M. J. Woodhouse

new CMI offices

St. Giles' building

In the summer of 2013, Oxford's dispersed mathematical community will come together with the move to Rafael Viñoly's new building on the city-centre site of Oxford's historic former hospital. The appointment of a New York architect for this spectacular project is just one indication of our engagement with the mathematical world beyond our shores. Another is the arrival in Oxford last summer of the Clay Mathematics Institute, which will have offices in the new building and run many of its activities in the superb new facilities.

The CMI established its reputation and built up its activities in the outstanding mathematical environment of Cambridge, MA, in close proximity to Harvard, and with MIT and other great centres of mathematical excellence nearby. The move of its president's office to Oxford provides an opportunity for CMI to extend its international reach. It also provides a wonderful opportunity for Oxford mathematicians, as well as those from further afield in the UK and Europe, to take part in CMI activities and to benefit from Landon Clay's visionary creation.

Mathematics has had a prominent place in the intellectual life of Oxford for more than 800 years. Our thirteenth century Chancellor, Robert Grosseteste, advocated the study of geometry, writing: "The usefulness of considering lines, angles and figures is the greatest, because it is impossible to understand natural

philosophy without these." In the sixteenth century, an enlightened statutory provision (now inexplicably lapsed) required all members of the University to attend mathematics lectures every day.

Then, as now, the Oxford mathematical community was scattered throughout the city. Its international connections were strong, but built on individual links. There was no central departmental building large enough to house everyone and to provide adequate welcoming space for visitors. Its first "mathematical institute", the seventeenth century "Schola Geometricae et Arithmeticae", and a succession of temporary early 20th century locations, provided basic facilities, but little more.

Many mathematical friends of Oxford will know the building in St Giles': comfortable, well conceived by a mathematician (Charles Coulson) in the 1960s, but wholly inadequate (indeed, overfull from its very opening) for a department now spread over three buildings and many colleges across the city, and with more than 1,000 students coming through its doors.

Our new building will be transformative, providing offices for everyone, ample space for visitors, and, of course, for the CMI, all in the centre of Oxford. The teaching spaces, which include a 360-seat lecture theatre, and the public spaces around them, will be a wonderful location for conferences, workshops and summer schools.

The new building, made possible in part by a hugely generous donation from the Clay family, represents an endorsement of our discipline by Oxford University's current leadership as strong as that made by their predecessor eight centuries ago. It will be shown off to the world during the Clay Research Conference on 2 October 2013, and by the formal opening by Sir Michael Atiyah on the following day.

Sam Howison, Chairman
Mathematical Institute, University of Oxford

annual meeting

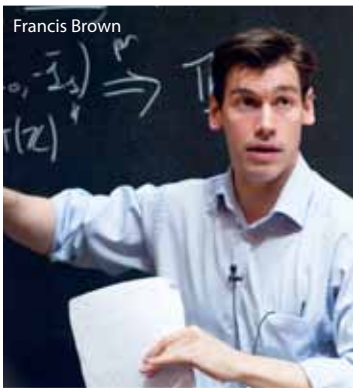
Clay Research Conference 2012, University of Oxford, Oxford, UK

The sixth annual Clay Research Conference, an event devoted to recent advances in mathematical research, was held at the University of Oxford on June 18 and 19, 2012 in the Martin Wood Lecture Theatre, Department of Physics.

Conference speakers were Francis Brown (Institut de Mathématiques de Jussieu, CNRS) Stavros Garoufalidis (Georgia Institute of Technology), Vladimir Markovic (California Institute of Technology), Jeremy Kahn (Brown University), Peter Scholze (University of Bonn), Marc Lackenby (University of Oxford) and Artur Avila (IMPA, Université Paris VI). Abstracts appear on the following pages and videos of the talks are available on the Clay Mathematics Institute website at www.claymath.org/research_conference/2012/.

On the afternoon of June 18, the Clay Research Award was presented to Jeremy Kahn and Vladimir Markovic for their work in hyperbolic geometry: (1) their proof that a closed hyperbolic three-manifold has an essential immersed hyperbolic Riemann surface, i.e., the map on fundamental groups is injective; (2) their solution of the Ehrenpreis conjecture: that given any two compact hyperbolic Riemann surfaces, there are finite covers of the two surfaces which are arbitrarily close in the Teichmüller metric.


Artur Avila


Francis Brown


Stavros Garoufalidis

Abstracts

Artur Avila, IMPA, Université Paris VI
Global Theory of One-Frequency Schrödinger Operators

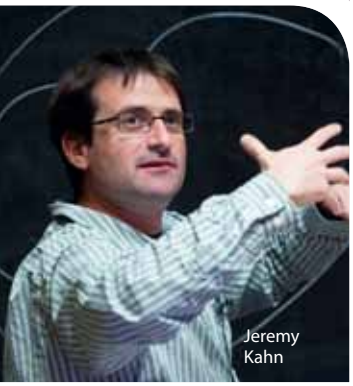
One-Frequency Schrödinger operators give one of the simplest models where fast transport and localization phenomena are possible. From a dynamical perspective, they can be studied in terms of certain one-parameter families of quasi-periodic co-cycles, which are similarly distinguished as the simplest classes of dynamical systems compatible with both KAM phenomena and non-uniform hyperbolicity (NUH). While much studied since the 1970's, until recently the analysis was mostly confined to "local theories" describing the KAM and the NUH regime in detail. In this talk we describe some main aspects of the global theory that has been developed in the last few years.

Francis Brown, Institut de Mathématiques de Jussieu, CNRS
Multiple Zeta Values

In this talk I give an overview of some recent results in the theory of multiple zeta values, which were first defined by Euler. After giving an informal introduction to motivic multiple zeta values, which can be viewed as a prototype for a Galois theory of certain transcendental numbers, I then explain how they were used to prove the Deligne-Ihara conjecture, which states that the motivic fundamental group of the projective line minus 3 points spans the Tannakian category of mixed Tate motives over Z, and a conjecture on multiple zeta values due to Hoffman.

Stavros Garoufalidis, Georgia Institute of Technology
The Quantum Content of the Gluing Equations

The gluing equations are a zero-dimensional system of polynomial equations which describe the complete hyperbolic structure (and its deformations) for a 3-manifold with torus boundary. The


Jeremy Kahn

Neumann-Zagier datum consists of the solutions together with the equations themselves. Using the Neumann-Zagier datum we construct a formal power series which conjecturally (a) captures the asymptotics of the Kashaev invariant to all orders and (b) whose first order agrees with the Reidemeister-Ray-Singer torsion. We show topological invariance of the first term of our formal power series, we expand it to a series of rational function on the PSL(2,*C*) moduli space, and give a computer implementation of the first three terms confirming numerically our conjectures above (joint with T. Dimofte). In addition, we extend our results to SL(N,*C*) representations of 3-manifolds (joint with M. Goerner and C. Zickert).

Jeremy Kahn, Brown University
The Good Pants Homology and the Ehrenpreis Conjecture

Both the Surface Subgroup Theorem and the Ehrenpreis conjecture are proven by building immersed surfaces out of immersed pairs of pants. In the latter case there can be an imbalance of pants: there may be more pants on one side of a closed geodesic than the other. I explain the theory, the "good pants homology" of good closed geodesics modulo boundaries of good pants, that we use to construct the correction for this imbalance.


Marc Lackenby

Marc Lackenby, University of Oxford
The Dehn Surgery Problem

Dehn filling is the process of attaching a solid torus to a 3-manifold *M* with toral boundary. There are infinitely many ways to make this attachment, which are parameterised by the essential simple closed curves *s* (or 'slopes') on the boundary of *M*. The resulting manifold is denoted *M(s)*. When *M* is hyperbolic, then so too is *M(s)*, as long as *s* avoids finitely many possible slopes, known as 'exceptional' slopes. This is Thurston's hyperbolic Dehn surgery theorem, and it has been highly influential. How many exceptional slopes can there be? It is a famous conjecture of Gordon that the maximal number of exceptional slopes is ten, which is realised by the exterior of the figure-eight knot. In joint work with Rob Meyerhoff, we proved this conjecture. I give an overview of the proof, which includes some new geometric techniques combined with a rigorous computer-assisted calculation.


Vladimir Markovic

Vladimir Markovic, California Institute of Technology
Virtual Geometry of Riemann Surfaces and Hyperbolic 3-Manifolds

This talk is a survey of my recent work with Jeremy Kahn. I will focus on the Surface Subgroup Theorem and explain its applications in low dimensional topology.


Peter Scholze

Peter Scholze, University of Bonn
Perfectoid Spaces

An old theorem of Fontaine-Wintenberger states that the absolute Galois groups of deeply ramified extensions of Q_p are canonically isomorphic to the absolute Galois groups of fields of characteristic *p*. The notion of perfectoid spaces generalizes this isomorphism to a comparison of geometric objects over fields of characteristic zero and fields of characteristic *p*, thereby reducing certain deep questions about mixed characteristic rings to questions in equal characteristic. We discuss applications to the weight-monodromy conjecture, and to *p*-adic Hodge theory.

recognizing achievement

Clay Research Awards

The Clay Millennium Prize problems have given the Clay Mathematics Institute a high public profile. Less well known outside of the mathematical world, but widely appreciated within it, are the annual Clay Research Awards. These celebrate the outstanding achievements of the world's most gifted mathematicians. Since its inception in 1999, thirty individuals have been recognized for the major contemporary breakthroughs they have made in mathematical research.

The standard has been very high. Four of the ten Fields Medals awarded since the foundation of the CMI have gone to people who had previously received Clay Research Awards. Indeed, six of the thirty Clay awardees are also Fields Medallists.

The work of four recent Clay Award winners is highlighted in the following pages in articles by Gregory Margulis.

Stationary Measures and Orbit for Actions of Non-abelian Groups on Homogeneous Spaces

by Gregory Margulis

Yves Benoist and Jean-François Quint received the Clay Research Award for their spectacular work on stationary measures and orbit closure for actions of non-abelian groups on homogeneous spaces and, more precisely, on quotient space G/Γ where G is a "nice" group and Γ a discrete subgroup. This work is a major breakthrough in homogeneous dynamics and related areas of mathematics.

The study of dynamics of group actions on the homogeneous quotient spaces G/Γ has a long history. One of the most notable successes to date has been the fairly complete qualitative understanding of the action of unipotent groups on such quotients, achieved in the work by several researchers that spans roughly the period 1970-1995. The results and the technique developed to obtain them have had a remarkable impact on many subjects, and in particular, have had numerous number theoretic applications, including applications to the distribution of values of indefinite irrational quadratic forms at integral points, counting of integral points on homogeneous varieties, and metric diophantine approximation.

Much less understood has been the action of other types of subgroups $H \subset G$ on the quotients G/Γ. When H is a higher rank abelian group this behavior is a major open problem; there are many deep results about H-invariant measures and closures of H-orbits obtained mostly after 1990. Though these results are deep, they are rather partial; nevertheless they have had many applications, including applications to number theory.

An interesting in-between case, highlighted mostly by H. Furstenberg, is that of big non-abelian subgroups Λ in G, explicitly, the case of Zariski dense subgroups of semisimple groups $H \subset G$. If Λ is generated by unipotents, it is possible to apply results about unipotent actions. However, in the general case of Λ that contains no unipotents, for a long time the situation was quite analogous to that of higher rank abelian groups where essentially the only nontrivial thing that was known has been the classification of orbit closures of semigroups of toral automorphisms. Furstenberg has suggested that for such actions much insight can be gained by studying the stationary measures under Λ: i.e., one fixes a probability measure μ on Λ whose support generates Λ and studies all the measures ν on G/Γ for which $\mu \times \nu = \nu$.

J. Bourgain, A. Furman, E. Lindenstrauss and S. Mozes gave a classification of such stationary measures on T^d using the techniques of additive combinatorics and some harmonic analysis. These techniques have an advantage that they are quite quantitative, but also have a significant disadvantage that it is not clear (at least now) how to adapt them in the general homogeneous spaces context.

Benoist and Quint stunned experts in the field not long after the work by Bourgain, Furman, Lindenstrauss and Mozes was announced by their purely dynamical approach to the classification of stationary measures which allowed them to classify such measures on homogeneous spaces of simple Lie groups G/Γ for the action of a Zariski dense subgroup $\Lambda \subset G$. Using this result, Benoist and Quint were able to classify orbit closures of Zariski dense subgroups in G/Γ. The technique developed by Benoist and Quint has had remarkable applications in a recent work by A. Eskin and M. Mirzakhani on dynamics of Teichmüller flows.

Hyperbolic Geometry

by Gregory Margulis

Jeremy Kahn and Vladimir Markovic received the Clay Research Award for their work in hyperbolic geometry: (1) their proof that a closed hyperbolic three-manifold has an essential immersed hyperbolic Riemann surface, i.e. the map on fundamental groups is injective; (2) their solution of the Ehrenpreis conjecture: that given any two compact hyperbolic Riemann surfaces, there are finite covers of the two surfaces which are arbitrarily closed in the Teichmüller metric. More formally, they proved the following two theorems:

> Theorem 1. *Let M be a closed hyperbolic 3-manifold. Then M contains an immersed, incompressible (i.e. injective on π_1) closed surface of genus $g \geq 2$.*
>
> Theorem 2. *Let X, Y be two hyperbolic surfaces. Then for any $\varepsilon > 0$ there exist finite covers X', Y' and a $(1+\varepsilon)$-quasiconformal map $X' \to Y'$.*

Both of these theorems are (at least 30-year-old) famous conjectures that many mathematicians have worked hard on to no avail. It should also be mentioned that Theorem 1 played an important role in the recent proof by Ian Agol of Waldhausen's virtual Haken conjecture and Thurston's virtual fibering conjecture.

The proofs of Theorem 1 and 2 use mixing properties of various geometric flows (e.g. the frame flow) on the hyperbolic 3-manifold (resp. surface) to construct huge numbers of almost-isometrically immersed "pairs of pants" (i.e. 2-spheres with 3-open disks removed), which can (under appropriate circumstances) be glued together to give a desired surface (in a finite cover). This general strategy was introduced about ten years ago by Lewis Bowen. In order to make this strategy work, Kahn and Markovic introduced a number of original ideas and had to overcome truly formidable technical difficulties.

Gregory Margulis joined the CMI's Scientific Advisory Board in 2002 and retired from it in 2012. Over the years, he has given enormously valuable service and advice, greatly valued by successive presidents.

Margulis is an outstanding Russian mathematician, who studied at Moscow State University under Yakov Sinai. He has made major contributions in combinatorics, differential geometry, ergodic theory, and dynamical systems. He was awarded a Fields Medal at the Helsinki International Congress in 1978 for his work on discrete subgroups of Lie groups, but was prevented from collecting it in person by the Soviet authorities. Since 1991, he has been Professor of Mathematics at Yale. In 2005 he was awarded the Wolf Prize for "his monumental contributions to algebra".

Clay Research Awardees

2012	Jeremy Kahn and Vladimir Markovic
2011	Yves Benoist and Jean-François Quint Jonathan Pila
2009	Jean-Loup Waldspurger Ian Agol, Danny Calegari and David Gabai
2008	Cliff Taubes Claire Voisin
2007	Alex Eskin Christopher Hacon and James McKernan Michael Harris and Richard Taylor
2005	Manjul Bhargava Nils Dencker
2004	Ben Green Gérard Laumon and Ngô Bao Châu
2003	Richard Hamilton Terence Tao
2002	Oded Schramm Manindra Agrawal
2001	Edward Witten Stanislav Smirnov
2000	Alain Connes Laurent Lafforgue
1999	Andrew Wiles

2012 awardees Jeremy Kahn and Vladimir Markovic (center) with Landon and Lavinia Clay

2011 awardees Yves Benoist and Jean-François Quint with Gregory Margulis

researchers, workshops and conferences

Summary of 2012 Research Activities

The activities of CMI researchers and research programs are sketched below. Researchers and programs are selected by the Scientific Advisory Board (see inside front cover).

Clay Research Fellows

Ivan Corwin

Ivan Corwin received his PhD in 2011 from the Courant Institute at New York University under the supervision of Gerard Ben Arous. His interests include probability, mathematical physics and exactly solvable systems. One part of his research has been to compute exact formulas for the statistics of the solution to the Kardar-Parisi-Zhang non-linear stochastic PDE. This stochastic PDE and its statistics describe a large universality class of systems, including growth processes, interacting particle systems, directed polymers in random environments, and less directly random matrix theory. He is currently employing representation theoretic techniques to provide more conceptual understanding of the observed integrability.

Jack Thorne

Jack Thorne was born in 1987 in Hereford, UK. He received his BA at the University of Cambridge in England. He has since studied at Harvard University and Princeton University under the direction of Richard Taylor and Benedict Gross. He received his PhD in May 2012. His primary research interests are algebraic number theory and representation theory, and the diverse connections between these two subjects. Most recently he has been interested in using automorphy lifting techniques to establish new cases of the Fontaine-Mazur conjecture.

Ivan Corwin and Jack Thorne joined CMI's 2012 group of research fellows: Mohammed Abouzaid (Massachusetts Institute of Technology), Tim Austin (New York University), Davesh Maulik (Columbia University), Sucharit Sarkar (Princeton University) and Peter Scholze (University of Bonn)

Research Scholar

Roman Travkin (Massachusetts Institute of Technology)
2012-2013
University of Chicago

Senior Scholars

Richard Kenyon (MSRI)
January 9 – May 20, 2012
Random Spacial Processes

Greg Lawler (MSRI)
January 9 – May 20, 2012
Random Spacial Processes

Xavier Buff (ICERM)
February 1 – May 4, 2012
Complex and Arithmetic Dynamics

Bill Thurston (PCMI)
July 5 – July 13, 2012
Geometric Group Theory

Alex Lubotzky (PCMI)
July 1 – July 21, 2012
Geometric Group Theory

Martin Bridson (PCMI)
July 1 – July 21, 2012
Geometric Group Theory

Karen Smith (MSRI)
August 20 – September 4, 2012
Commutative Algebra

Claudio Procesi (MSRI)
August 20 – September 30, 2012
Commutative Algebra

Research Programs organized and supported by CMI in 2012

January 1 – December 31 ..... Independent University of Moscow, Moscow, Russia

January 1 – December 31 ..... PRIMES, Massachusetts Institute of Technology, Cambridge, MA

June 3 – 30 ..... CMI Summer School on the Resolution of Singular Algebraic Varieties, Obergurgl, Austria

June 18 – 19 ..... Clay Research Conference, University of Oxford, Oxford, UK

June 18 – July 27 ..... Ross Program, Ohio State University, Columbus, OH

July 1 – August 11 ..... PROMYS, Boston University, Boston, MA

August 20 – 30 ..... International Summer School of Mathematics for Young Students, Lyon, France

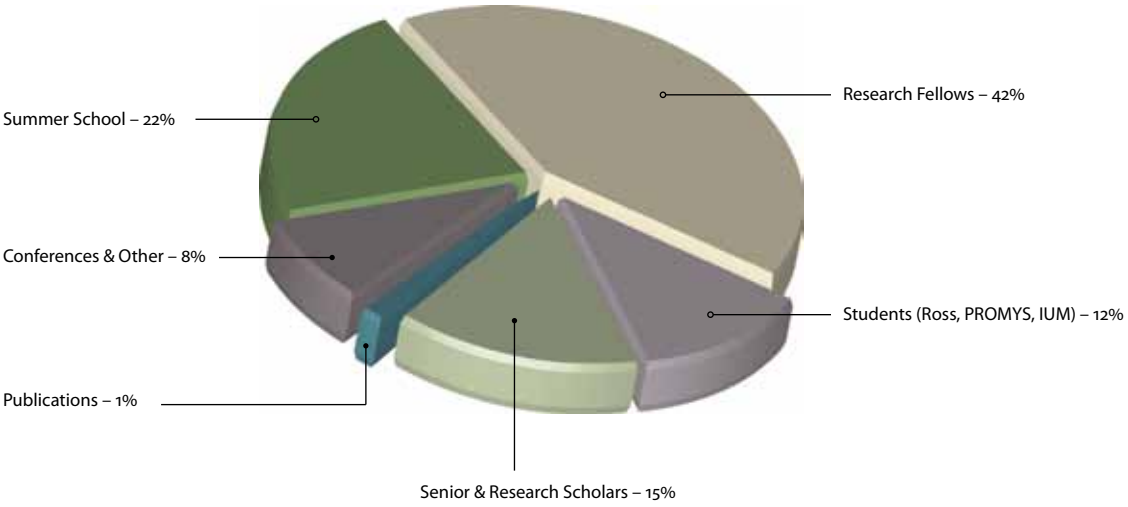
September 3 – 7 ..... SAVANT: Synergies and Vistas in Analytic Number Theory, University of Oxford, Oxford, UK

Program Allocation

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2012

Research Fellows, Research Awardees, Senior Scholars, Research Scholars: 18
PROMYS/Ross/PRIMES Faculty and Participants: 44
CMI Summer School Faculty and Participants: 90
Participants attending Conferences and Joint Programs: >200
Independent University of Moscow: 115

Research Expenses for Fiscal Year 2012



"Sometimes I have some vague intuitive idea on how things should work, and I try to reconcile this with the known theory. In some cases, the new perspective leads to new insights, simplifying or clarifying old material and maybe implying new results."

profile

Interview with Research Fellow Peter Scholze

What first drew you to mathematics? What are some of your earliest memories of mathematics?

As far back as I can remember I was fascinated by mathematics. One thing that I remember is reading the book *Fregatten-Kapitän Eins* by Wladimir Ljowschin, around the time of my first year at school. Now I can't remember much of what was in that book, but it included a lot of very good mathematics in the form of children's fairytales, and I do remember some pictures on how to catch a lion in a desert by repeatedly bisecting the desert.

Could you talk about your mathematical education? What experiences and people were especially influential?

I went to a special high school with an emphasis on mathematics and the natural sciences; this former East German school had survived the fall of the GDR. Contrary to most schools, being good at math was not something one was mocked for, and it was even mandatory to participate in the Mathematics Olympiad once. To my own surprise, I did quite well at the competition and started to learn more mathematics. My teachers were strongly supportive of me in that direction. Around the age of 16 I wanted to understand Wiles's proof of Fermat's Last Theorem, and began to read about modular forms and elliptic curves, without any previous exposure to linear

algebra. Somehow I managed to understand something and fill gaps in my knowledge, mostly by searching the internet.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

Klaus Altmann taught me algebraic geometry when I was still at school, and also helped me in choosing a place to study afterwards. He suggested that I go to Bonn to work with Michael Rapoport, who became my advisor. Rapoport taught me an incredible amount of mathematics during my time as his student, and was a wonderful mentor in general. He also shaped my mathematical interests to some extent. I was already previously drawn into arithmetic geometry, but his taste for choosing problems to think about certainly had an influence.

You were educated in Germany. Could you comment on the differences in mathematical education there and in the US?

As in most European countries, general education is finished by the end of high school and you enrol for a specific subject at the university. You learn just mathematics from your first year at the university although, usually, you are required to do a bit of a second subject, e.g. physics or computer science. Moreover, going to the university is totally free, and admission to mathematics is usually automatic; even at the best places mathematics is not very popular. Another difference is that students have to give lectures in seminars very early on, e.g. in their first year in Bonn. I think that this training is very helpful later, when you give talks about your PhD thesis.

What attracted you to the particular problems you have studied?

There are many problems that I think about from time to time, anything that I feel is interesting. In many cases, my desire is just to thoroughly understand some difficult theory. Sometimes I have some vague intuitive idea on how things should work, and I try to reconcile this with the known theory. In some cases, the new perspective leads to new insights, simplifying or clarifying old material and maybe implying new results.

Can you describe your research in accessible terms?

Much of my research is related to geometry over the field of p-adic numbers. In number theory, the field of real numbers is just one of infinitely many possible completions of the rational numbers, the others being given by the p-adic numbers for a prime number p. Now, questions over p-adic numbers have a strong number-theoretic flavour, but still many results over the real numbers turn out to be true almost verbatim over p-adic fields, yet with very different proofs and very different applications. One example is p-adic Hodge theory, which compares different cohomology theories of "compact p-adic manifolds" and is an analogue of classical Hodge theory, which compares different cohomology theories of compact complex manifolds. One key difficulty in p-adic questions lies exactly in the fact that p-adic numbers have a highly number-theoretic flavour. The theory of perfectoid spaces developed in my thesis gives a way to get around this and reduce many problems to a more geometric situation instead.

What research problems and areas are you likely to explore in the future?

There is one important open conjecture about algebraic varieties over p-adic fields, the weight-monodromy conjecture. I could settle this for a large class of varieties, and I hope to say more

about the general case. Apart from that, there are many things that I think about, and I am not sure where I will be led.

How has the Clay Fellowship made a difference for you?

The Clay Fellowship gives a lot of freedom as it provides safe funding for a very long period of time, without any obligations. Moreover, although it is US-based, it allows me to stay in Germany with my wife, yet makes it possible to travel and see any person around the world that I would like to talk to or work with, or even invite that person to come to Germany. I use these options a lot, just recently I made a two-month long trip to North America and, during the time in Boston, I was able to do some very concentrated work with Jared Weinstein to finish our joint paper, which was enormously helpful.

"Mathematicians are constantly testing the boundaries of what one can meaningfully think about and they need to develop a very good sense for when a question is meaningful and can possibly be answered."

What advice would you give lay persons who would like to know more about mathematics—what it is, what its role in our society has been and is, etc.? What should they read? How should they proceed?

There are some excellent books and documentary movies showing how fascinating mathematics is. Simon Singh's book *Fermat's Last Theorem* and the corresponding documentary movie, for example, and there is a wealth of material on the internet. Yet, although this sometimes brilliantly conveys the fascination of mathematics, it can almost never explain much of the actual mathematics behind. The book *Proofs from the BOOK* by Aigner and Ziegler is very good, but it does require previous exposure to mathematics. One difficult thing about mathematics is that it has developed its own language, and it is often impossible to say anything nontrivial about mathematics without using part of that language. I find it interesting that at pretty much every mathematical institute there is one elderly person, fascinated by mathematics, attending seminars and lecture courses—maybe that is the only way of learning what mathematics is about, really.

How do you think mathematics benefits culture and society?

Many say that mathematics is important because it is used to build computers, say. I wouldn't agree with that view and would argue that its real value is more indirect and hidden. After all, people working in the most abstract mathematical fields, devoid of any practical applications, are still sought after for the skills they developed in thinking about these questions. In a sense, mathematicians are constantly testing the boundaries of what one can meaningfully think about and they need to develop a very good sense for when a question is meaningful and can possibly be answered.

Please tell us about things you enjoy when not doing mathematics.

I like hiking, skiing (cross-country even more than downhill), listening to music, and drinking a beer with friends. Back in school, I played the bass guitar in a rock band, and I would love to do that again, but until now I haven't had enough time.

2012 CMI Summer School participants

program overview

CMI Summer School

Educating mathematicians

The Resolution of Singularities of Algebraic Varieties | June 3 - 30, 2012

by Herwig Hauser

The beautiful Tyrolean setting of the Universitätszentrum Obergurgl of the University of Innsbruck in Austria offered the ideal location for the 2012 Clay Mathematics Institute Summer School. The scientific aim of the school was to introduce ambitious graduate students to the main techniques and results of resolution of singularities so as to enable them to undertake independent and genuine research. The resolution of singularities is one of the major topics in algebraic geometry. Due to its difficulty and complexity, as well as for certain historical reasons, research to date in the field has been pursued by a relatively small group of mathematicians. However, the field has begun a renaissance over the last twenty years, boosted by many small conferences and schools, with the discovery of more conceptual proofs of the characteristic zero case, as well as several brilliant attempts at the still unresolved prime characteristic case.

Scientific Committee
David Ellwood (Clay Mathematics Institute)
Herwig Hauser (University of Innsbruck and Vienna)
Shigefumi Mori (RIMS, Kyoto University)
Josef Schicho (RICAM, University of Linz)

Lecturers
Ana Bravo (Universidad Autónoma de Madrid)
Steven Cutkosky (University of Missouri)
Santiago Encinas (Universidad de Valladolid)
Herwig Hauser (University of Innsbruck and Vienna)
Heisuke Hironaka (Tokyo University)
Hiraku Kawanoue (Kyoto University)
Daniel Panazzolo (Université de Haute-Alsace)
Josef Schicho (RICAM, University of Linz)
Orlando Villamayor (Universidad Autónoma de Madrid)
Takehiko Yasuda (Osaka University)

The school was organized around three foundational courses, each of which was supplemented by exercise and problem sessions designed to provide the participant with a comprehensive framework for research in the field:

- *Resolution Techniques* by Herwig Hauser
- *Resolution of Singularities: Games and Computations* by Josef Schicho
- *Commutative Algebra for Singular Algebraic Varieties* by Orlando Villamayor

These courses presented, in very different manners, three different approaches to the same subject, that being resolution of singularities. The students gradually realized that seemingly disconnected methods and goals merge in a common objective and the multiplicity of viewpoints was reflected by the quite complementary types of presentations of the three lecturers.

Hauser highlighted the origins and motivations of the various constructions and then developed from this the logical structure of the proof of resolution in zero characteristic. From this understanding, the obstructions of the still open case of positive characteristic were discussed and the students encouraged to begin working on it. Schicho proposed a more computational and game-theoretic approach to resolution, opposing combinatorial with algebraic methods. This enabled students to attack from the beginning relevant problems and puzzles (i.e., finding winning strategies). Villamayor followed a very systematic presentation based on tools from commutative algebra. This allowed the students to become acquainted with basic techniques relevant for the resolution of singularities.

The three foundational courses were supplemented by several mini-courses aimed at providing participants with state-of-the-art techniques, as well as a survey of some of the main open problems and the most promising approaches now under investigation:

- *Rees Algebras, Elimination, and Singularities of Varieties over Perfect Fields* by Ana Bravo
- *Resolution of Singularities in Positive Characteristic* by Steven Cutkosky
- *Embedded Desingularization of Toric Varieties* by Santiago Encinas
- *Resolution of Singularities* by Heisuke Hironaka
- *Idealistic Filtration and its Properties* by Hiraku Kawanoue
- *Resolution of Singularities for Foliations* by Daniel Panazzolo
- *Higher Nash Blowups and F-Blowups* by Takehiko Yasuda

A particular highlight was Hironaka's series of lectures in which he presented his extended program of how to solve resolution in positive characteristic. Though not yet finished, he was able to distribute an advanced manuscript explaining the main steps. Cutkosky surveyed the state of the art in local uniformization, in embedded and non-embedded resolution and in the monomialization problem. He exemplified these theories by giving an outline of the proof of resolution of surfaces in arbitrary characteristic.

Bravo completed Villamayor's lectures by giving the proof of resolution in zero characteristic via projections and elimination algebras. Encinas reported on the important case of toric varieties and toric resolution (theory of binomial ideals) and Kawanoue gave an introduction to the theory of idealistic filtrations, useful to approach the case of positive characteristic. Panazzolo discussed the resolution of vector fields and foliations (only proven up to dimension three) and Yasuda presented his theory of higher Nash modification and *F*-blowups, a more geometrically inspired technique.

The schedule over the four weeks included two 90 minute lectures in the morning, independent study time following lunch, and an additional lecture and/or discussion session in the afternoon with some advanced lectures in the afternoon and evening. From the outset, students were encouraged to attend regularly all classes, to attempt the exercises and problems, and to share their knowledge and to form small working groups. In addition to the mathematical training, special emphasis was placed on the social aspects of the schools and the 80 students (from 27 nations) were reminded to use their four-week stay to enhance friendships and engage in cultural exchange. The remote location of Obergurgl and the absence of distractions, apart from nature, fostered internal communication while the permanent availability of the speakers and the informal contact between lecturers and students made for a relaxed, yet hugely productive, summer school.

PHOTO COURTESY OF JONATHAN PILA

SAVANT Conference

CMI Supported Conference

Disseminating mathematical knowledge

SAVANT: Synergies and Vistas in Analytic Number Theory | September 3 – 7, 2012

by Tim Browning

This week-long conference on analytic number theory was organized in honor of Professor Roger Heath-Brown's 60th birthday. Heath-Brown has had a tremendous impact on analytic number theory, as well as on many subjects that share a common border with it. The last decade has proved a tremendously exciting time for analytic number theory with a number of remarkable breakthroughs across the board. Many of the topics currently enjoying the limelight have received momentum through Heath-Brown's investigations.

The focus of the conference was on the many interactions of analytic number theory with related fields, such as Diophantine geometry, additive combinatorics and dynamical systems. The speakers were chosen to reflect the fact that the most exciting recent developments in analytic number theory have come through synergy with neighboring fields.

The level of talks was extremely high, as to be expected from such a distinguished group of mathematicians, and occasioned much discussion and interaction. Sarnak's discussion of the affine sieve and Green's talk on an inverse large sieve problem were among the scientific highlights of the conference. Time was also set aside for several contributed talks, most notably Pankaj Vishe's talk on cubic hypersurfaces over number fields and Frank Thorne's talk on Shintani zeta functions.

The meeting was both a celebration of Heath-Brown's contributions to analytic number theory as well as a platform from which to marshal progress and chart the course ahead.

Organizers
Tim Browning (Bristol University)
David Ellwood (CMI)
Jonathan Pila (University of Oxford)

Speakers
Régis de la Bretèche (Université Paris 7)
Jörg Brüdern (Göttingen University)
Alina Cojocaru (University of Illinois at Chicago)
Brian Conrey (AIM, Bristol University)
Chantal David (Concordia University)
Alex Gorodnik (Bristol University)
Andrew Granville (Université de Montréal)
Ben Green (University of Cambridge)
Gergely Harcos (Renyi Institute)
Henryk Iwaniec (Rutgers University)
Alex Kontorovich (Yale University)
Emmanuel Kowalski (ETH Zurich)
David Masser (Universität Basel)
Philippe Michel (EPFL)
Peter Sarnak (IAS, Princeton University)
Kannan Soundararajan (Stanford University)
Trevor Wooley (Bristol University)
Matthew Young (Texas A&M University)

CMI Supported Programs

Encouraging gifted students

Ross, PROMYS, PRIMES

In 2012 the Clay Mathematics Institute continued its support of programs for talented high school students who excel in math by sponsoring, in part, the Ross Mathematics Program at Ohio State University, PROMYS at Boston University and PRIMES at Massachusetts Institute of Technology. All three programs are distinguished for offering the best pre-college learning experiences available to students with a special aptitude for mathematics by immersing them in the creative world of mathematical discovery.

The Ross Mathematics Program at Ohio State University is a six-week intensive summer course in Number Theory. Founded by Dr. Arnold Ross in 1957, the central goal of the program has always been to instruct bright young students in the art of mathematical thinking and to inspire them to discover for themselves that abstract ideas are valuable and important.

Elementary, but fast paced, the course for first year students (mostly 14 to 18 years old) starts with modular arithmetic, the Euclidean algorithm, and prime factorizations and then moves to quadratic reciprocity, structure of the unit group (mod m), extensions to polynomials, Gaussian integers and quadratic number fields, geometry of numbers, Möbius inversion, and more. Students work through those ideas (with proofs) guided by the extensive problem sets and by discussions with the counselors.

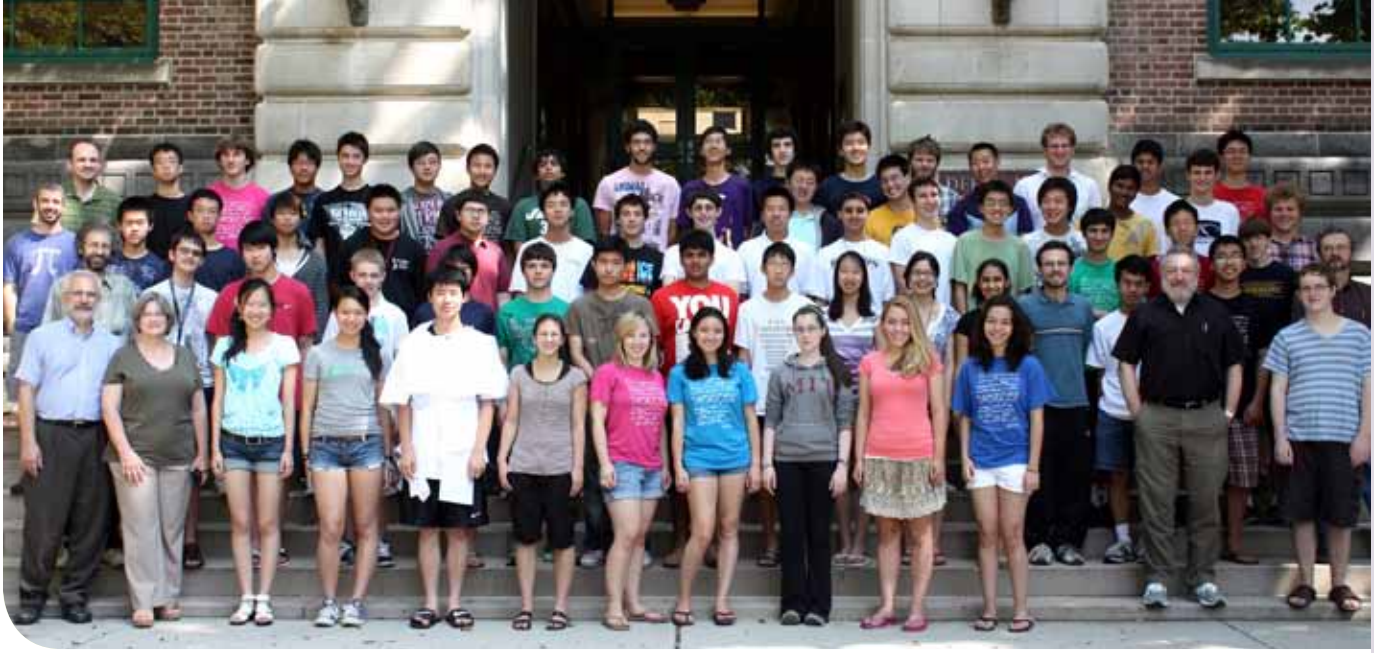


PHOTO COURTESY OF DAWN JONES

2011 Ross participants

These daily lectures by program director Daniel Shapiro (professor at Ohio State University) were supported by problem seminars taught by Jim Fowler (postdoc at Ohio State University), John Maharry (associate professor at Ohio State University) and Robert Mendris (professor at Shawnee State University).

The Ross Program offers advanced courses attended by counselors and junior counselors. The 2012 courses were *Projective Geometry* by Warren Sinnott (professor at Ohio State University) and *Linear Algebra and Its Connections* by Vitaly Bergelson (professor at Ohio State University).

Two visiting professors each presented short courses which proved to be very popular with the students. Professor Ezra Getzler (Northwestern University) taught *Bernoulli Numbers and their properties* and Professor Ray Pierrehumbert (University of Chicago) taught *The 3n + 1 Conjecture: A mathematical and computational exploration*.

In addition to the six-week courses and seminars, there were two colloquium-style lectures: *The Geometry of Quaternions* by Daniel Shapiro and *Juggling Patterns* by John Maharry and Jon Stadler, with the latter also including a juggling training session!

PROMYS (Program in Mathematics for Young Scientists) is a six-week summer program at Boston University that was developed by Professor Glenn Stevens (Boston University) to encourage strongly motivated high school students to explore in-depth the world of mathematics in a supportive community of peers, counselors, research mathematicians and visiting scientists.

Since 1999, the PROMYS/CMI partnership has run research labs where participants form teams of four to engage in open-ended exploratory mathematics projects with the guidance of research mathematicians, mentors, and seminar leaders. Mentors pose new research problems at the start of the summer; the materials include problem statements, hints for getting started, and references to the pertinent literature.

In 2012, topics for these projects were *Points on Parabolas*, proposed by Amanda Beeson (University of Rochester), *Fibonacci Number Projects*, proposed by Ira Gessel (Brandeis University), *Deformations of Symmetric Functions and Character Formulas* and *Hyperplane Arrangements*, proposed by Paul Gunnells (University of Massachusetts at Amherst), *Root Subsystems and Weyl Group Representations*, proposed by Ben Harris (Louisiana State University) and *Random Involutions and the Number of Prime Factors of an Integer*, proposed by Kirsten Wickelgren (Harvard University). As participants worked on these team projects, they developed habits of thought associated with creative scientific research. At the end of the summer program, the teams gave presentations to the PROMYS faculty, counselors, and all of their fellow PROMYS participants.

PROMYS participants

PHOTO COURTESY OF THOMAS LI

Students who find the PROMYS experience especially worthwhile may be invited to return for a second summer to participate in the advanced PROMYS/CMI activities. To ensure that returning students and counselors find their experience intellectually stimulating, PROMYS, in partnership with the CMI, offers a variety of advanced seminars and mentored research projects each summer. In 2012 returning students participated in the following seminars: *The Analytic Class Number Formula* by Jared Weinstein (Boston University), *Algebra* by Professor Marjory Baruch (Syracuse University) and *Geometry and Symmetry* by Professor Steven Rosenberg (Boston University).

PRIMES (Program for Research in Mathematics, Engineering and Science for High School Students) is a year-long after-school research program for high school students at the Massachusetts Institute of Technology, founded by Professor Pavel Etingof and Dr. Slava Gerovitch in 2011. Program participants work with mentors at MIT on exciting, unsolved problems in mathematics, computer science and computational biology. Students gain experience reading mathematical literature, discussing mathematics, writing mathematical texts and presenting talks. The program allows students to do their research at a natural pace, facilitated by weekly meetings with their mentors.

In January 2012, 30 students were each assigned a research project and a mentor. The mentors recommended background reading necessary for the 24 individual and joint projects, which the students conducted independently in consultation with the mentor. This reading phase was followed by an active research period (February – May) during which students met with their mentors weekly at MIT, attended lectures and discussed their projects with faculty members. At the end of May, the students presented their results at a PRIMES conference at MIT. The presentations were arranged in nine sessions in the fields of *Discrete Mathematics*, *Number Theory*, *Representation Theory*, *Algorithms and Complexity*, *Programming Languages and Robotics*, *Computational Medicine* and *Computational and Physical Biology*.

The students continued working on their projects independently over the summer, staying in e-mail contact with their mentors. From September through December the students, with the help of their mentors, finalized their results and prepared them for submission to national science competitions and for publication. As a result, eight students were invited to present at the MAA Undergraduate Student Poster Session of the 2013 Joint Mathematics Meeting in San Diego; two students took fifth prize at the 2012 Siemens Competition in Math, Science and Technology; one student won tenth prize in the 2013 Intel Science Talent Search; and 14 research papers were completed and eight published on arXiv.org.

PRIMES participants

PHOTO COURTESY OF SLAVA GEROVITCH

publications

Selected Articles by Research Fellows

Mohammed Abouzaid

Exact Lagrangians in plumbings, with Ivan Smith, *Geometric and Functional Analysis*, 22 (2012), no. 4, 785–831.

Lagrangian fibrations on blowups of toric varieties and mirror symmetry for hypersurfaces, with Denis Auroux and Ludmil Katzarkov, submitted. arXiv:1205.0053

Tim Austin

Exchangeable random measures, submitted. arXiv:1302.2116.

Continuity properties of measurable group cohomology, with Calvin C. Moore, *Mathematische Annalen*. arXiv:1004.4937

Ivan Corwin

Macdonald processes, with Alexei Borodin, to appear in *Probability Theory and Related Fields*. arXiv:1111.4408

Brownian Gibbs property for Airy line ensembles, with Alan Hammond, to appear in *Inventiones Mathematicae*. arXiv:1108.2291

Davesh Maulik

Stable pairs and the HOMFLY polynomial, submitted. arXiv:12106323

Quantum groups and quantum cohomology, with Andrei Okounkov, submitted. arXiv:1211.1287

Sucharit Sarkar

A refinement of Rasmussen's *s*-invariant, with Robert Lipshitz, submitted. arXiv:1206.3532

A Steenrod square on Khovanov homology, with Robert Lipshitz, submitted. arXiv:1204.5776

Peter Scholze

p-adic Hodge theory for rigid-analytic varieties, submitted. arXiv:1205.3463

Moduli of *p*-divisible groups, with Jared Weinstein, submitted. arXiv:1211.6357

Jack Thorne

Automorphy lifting for residually reducible *l*-adic Galois representations. www.math.harvard.edu/~thorne/

Some canonical constructions in arithmetic invariant theory, submitted. www.math.harvard.edu/~thorne/

Books

Topics in Noncommutative Geometry

Editor: Guillermo Cortiñas

CMI/AMS, 2012, 276 pp., softcover, ISBN: 0-8218-6864-0. List Price: $79. AMS Members: $63.20. Order Code: CMIP/16. www.claymath.org/publications/Cortinas/

This volume contains the proceedings of the third Luis Santaló Winter School held at FCEN in 2010. Topics included in this volume concern noncommutative geometry in a broad sense, encompassing various mathematical and physical theories that incorporate geometric ideas to the study of noncommutative phenomena. It explores connections with several areas, including algebra, analysis, geometry, topology and mathematical physics.

Probability and Statistical Physics in Two and More Dimensions

Editors: David Ellwood, Charles Newman,Vladas Sidoravicius and Wendelin Werner

CMI/AMS, 2012, 467 pp., softcover, ISBN: 0-8218-6863-2. List Price: $114. AMS Members: $91.20. Order Code: CMIP/15. www.claymath.org/publications/Newman/

This volume is a collection of lecture notes for six of the ten courses given in Búzios, Brazil by prominent probabilists at the 2010 CMI Summer School, "Probability and Statistical Physics in Two and More Dimensions" and at the XIV Brazilian School of Probability. Together, these notes provide a panoramic, state-of-the-art view of probability theory areas related to statistical physics, disordered systems and combinatorics.

Grassmannians, Moduli Spaces and Vector Bundles

Editors: David A. Ellwood, Emma Previato

CMI/AMS, 2011, 180 pp., softcover, ISBN: 0-8218-5205-1. List Price: $55. AMS Members: $44. Order Code: CMIP/14. www.claymath.org/publications/VectorBundles/

This collection of cutting-edge articles on vector bundles and related topics originated from a CMI workshop, held in October 2006, that brought together a community indebted to the pioneering work of P. E. Newstead, visiting the United States for the first time since the 1960s. Moduli spaces of vector bundles were then in their infancy, but are now, as demonstrated by this volume, a powerful tool in symplectic geometry, number theory, mathematical physics, and algebraic geometry. This volume offers a sample of the vital convergence of techniques and fundamental progress taking place in moduli spaces at the outset of the twenty-first century.

On Certain *L*-Functions

Editors: James Arthur, James W. Cogdell, Steve Gelbart, David Goldberg, Dinakar Ramakrishnan, Jiu-Kang Yu

CMI/AMS, 2011, 647 pp., softcover, ISBN: 0-8218-5204-3. List Price: $129. AMS Members: $103.20. Order Code: CMIP/13. www.claymath.org/publications/Shahidi/

This volume constitutes the proceedings of the conference organized in honor of the 60th birthday of Freydoon Shahidi, who is widely recognized as having made groundbreaking contributions to the Langlands program. The articles in this volume represent a snapshot of the state of the field from several viewpoints. Contributions illuminate various areas of the study of geometric, analytic, and number theoretic aspects of automorphic forms and their *L*-functions, and both local and global theory are addressed.

Motives, Quantum Field Theory, and Pseudodifferential Operators

Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg

CMI/AMS, 2010, 349 pp., softcover. ISBN: 0-8218-5199-3. List price: $89. AMS Members: $71.20. Order Code: CMIP/12. www.claymath.org/publications/Motives_Quantum/

This volume contains articles related to the conference "Motives, Quantum Field Theory, and Pseudodifferential Operators" held at Boston University in June 2008, with partial support from the Clay Mathematics Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

Quanta of Maths; Proceedings of the Conference in honor of Alain Connes

Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa

CMI/AMS, 2010, 675 pp., softcover, ISBN: 0-8218-5203-5. List price: $129. AMS Members: $103.20. Order Code: CMIP/11. www.claymath.org/publications/Quanta_Maths/

The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory and physics.

Homogeneous Flows, Moduli Spaces and Arithmetic

Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Klein, Elon Lindenstrauss, Gregory Margulis, Stefano Marmi, Jean-Christophe Yoccoz

CMI/AMS, 2010, 438 pp., softcover, ISBN: 0-8218-4742-2. List price: $99. AMS Members: $79.20. Order Code: CMIP/10.
www.claymath.org/publications/Homogeneous_Flows/

This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.

The Geometry of Algebraic Cycles

Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua

CMI/AMS, 2010, 187 pp., softcover, ISBN: 0-8218-5191-8. List Price: $52. AMS Members: $41.60. Order Code: CMIP/9.
www.claymath.org/publications/Algebraic_Cycles/

The subject of algebraic cycles has its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.

Arithmetic Geometry

Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel.

CMI/AMS 2009, 562 pp., softcover. ISBN:0-8218-4476-8. List price: $119. AMS Members: $95.20. Order Code: CMIP/8.
www.claymath.org/publications/Arithmetic_Geometry/

This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Göttingen. It introduces readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

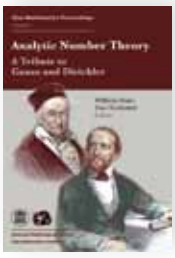
Dirichlet Branes and Mirror Symmetry

Editors: Michael Douglas, Mark Gross.

CMI/AMS 2009, 681 pp., hardcover. ISBN: 0-8218-3848-2. List price: $109. AMS Members: $87.20. Order Code: CMIM/4.
www.claymath.org/publications/Dirichlet_Branes/

The book first introduces the notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics and homological mirror symmetry, and discusses more recent physical developments.

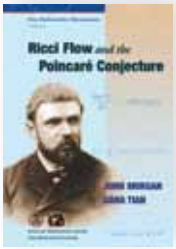
Analytic Number Theory: A Tribute to Gauss and Dirichlet



Editors: William Duke, Yuri Tschinkel.

CMI/AMS, 2007, 265 pp., softcover. ISBN: 0-8218-4307-9. List Price: $50. AMS Members: $40. Order Code: CMIP/7.
www.claymath.org/publications/Gauss_Dirichlet/

This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.



Ricci Flow and the Poincaré Conjecture

Authors: John Morgan, Gang Tian.

CMI/AMS, 2007, 521 pp., hardcover. ISBN: 0-8218-4328-1. List price: $71. AMS Members: $56.80. Order Code: CMIM/3. www.claymath.org/publications/ricciflow/

This book presents a complete and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and had remained open until the work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.

The Millennium Prize Problems

Editors: James Carlson, Arthur Jaffe, Andrew Wiles.

CMI/AMS, 2006, 165 pp., hardcover. ISBN: 0-8218-3679-X. List Price: $30. AMS Members: $24. Order Code: MPRIZE. www.claymath.org/publications/ Millennium_Problems/

This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.

Surveys in Noncommutative Geometry

Editors: Nigel Higson, John Roe.

CMI/AMS, 2006, 189 pp., softcover. ISBN: 0-8218-3846-6. List Price: $50. AMS Members: $40. Order Code: CMIP/6. www.claymath.org/publications/Noncommutative_Geometry/

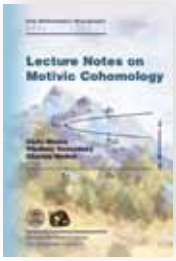
In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.

Floer Homology, Gauge Theory, and Low-Dimensional Topology

Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo.

CMI/AMS, 2006, 297 pp., softcover. ISBN: 0-8218-3845-8. List price: $66. AMS Members: $52.80. Order Code: CMIP/5. www.claymath.org/publications/Floer_Homology/

This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.



Lecture Notes on Motivic Cohomology

Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel.

CMI/AMS, 2006, 216 pp., softcover. ISBN: 0-8218-5321-X. List Price: $47. AMS Members: $37.60. Order Code: CMIM/2.S. www.claymath.org/publications/Motivic_Cohomology/

This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.

Harmonic Analysis, the Trace Formula and Shimura Varieties

Editors: James Arthur, David Ellwood, Robert Kottwitz.

CMI/AMS, 2005, 689 pp., softcover. ISBN: 0-8218-3844-X. List Price: $131. AMS Members: $104.80. Order Code: CMIP/4. www.claymath.org/publications/Harmonic_Analysis/

The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School at Fields Institute, Toronto. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

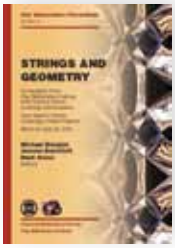
Global Theory of Minimal Surfaces

Editor: David Hoffman.

CMI/AMS, 2005, 800 pp., softcover. ISBN: 0-8218-3587-4. List Price: $131. AMS Members: $104.80. Order Code: CMIP/2
www.claymath.org/publications/Minimal_Surfaces/

This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and the double-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.

Strings and Geometry

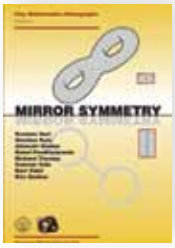


Editors: Michael Douglas, Jerome Gauntlett, Mark Gross.

CMI/AMS, 2004, 376 pp., softcover. ISBN: 0-8218-3715-X. List Price: $76. AMS Members: $60.80. Order Code: CMIP/3.
www.claymath.org/publications/Strings_Geometry/

This volume is the proceedings of the 2002 Clay Mathematics Institute Summer School held at the Isaac Newton Institute for Mathematical Sciences in Cambridge, UK. It contains a selection of expository and research articles by lecturers at the school and highlights some of the current interests of researchers working at the interface between string theory and algebraic geometry. The topics covered include manifolds of special holonomy, supergravity, supersymmetry, D-branes, the McKay correspondence and the Fourier-Mukai transform.

Mirror Symmetry



Editors: Cumrun Vafa, Eric Zaslow.

CMI/AMS, 2003, 929 pp., hardcover. ISBN: 0-8218-2955-6. List Price: $137. AMS Members: $109.60. Order Code: CMIM/1
www.claymath.org/publications/Mirror_Symmetry/

This thorough and detailed exposition develops mirror symmetry from both mathematical and physical perspectives and will be particularly useful for those wishing to advance their understanding by exploring mirror symmetry at the interface of mathematics and physics. This one-of-a-kind volume offers the first comprehensive exposition on this increasingly active area of study. It is carefully written by leading experts who explain the main concepts without assuming too much prerequisite knowledge.

Strings 2001

Editors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia.

CMI/AMS, 2002, 489 pp., softcover. ISBN: 0-8218-2981-5. List Price: $86. ASM Members: $68.80. Order Code: CMIP/1.
www.claymath.org/publications/Strings_2001/

This multi-authored book summarizes the latest results across all areas of string theory from the perspective of world-renowned experts, including Michael Green, David Gross, Stephen Hawking, John Schwarz, Edward Witten and others. The book comes out of the "Strings 2001" conference, organized by the Tata Institute of Fundamental Research (Mumbai, India), the Abdus Salam ICTP (Trieste, Italy), and the Clay Mathematics Institute (Cambridge, MA, USA). Individual articles discuss the study of D-branes, black holes, string dualities, compactifications, Calabi-Yau manifolds, conformal field theory, noncommutative field theory, string field theory, and string phenomenology. Numerous references provide a path to previous findings and results.

To order books please visit www.ams.org/bookstore.

Digital Library

CMI's Digital Library includes facsimiles of significant historical mathematical books and manuscripts, collected works and seminar notes.

Euclid's Elements, Constantinople, 888 AD (Greek).
MS at the Bodleian Library

The oldest extant manuscript and printed editions of Euclid's Elements, in Greek (888 AD) and Latin (1482 AD), respectively. High resolution copies of the manuscript are available for study at the Bodleian Library, Oxford University. Full online edition available at rarebookroom.org.
www.claymath.org/library/historical/euclid/

Riemann's 1859 Manuscript

The manuscript in which Riemann formulated his famous conjecture about the zeroes of the zeta function. **www.claymath.org/library/historical/riemann/**

Felix Klein Protokolle

The "Klein Protokolle," comprising 8600 pages in twenty-nine volumes, record the activity of Felix Klein's seminar in Göttingen for the years 1872-1912.This image on the left is from Volume I, page 113, the seminar "Über die Gruppe der Modulargleichung für Transformation p^{ter} Ordnung und specielle über die Transformation 25^{ter} de elliptischer functione." Seminar of Sunday, February 14, 1880.
www.claymath.org/library/historical/klein/

James Arthur Archive

James Arthur attended the University of Toronto as an undergraduate, and received his PhD at Yale University in 1970, where his advisor was Robert Langlands. He has been a University Professor at the University of Toronto since 1987. Almost all of Arthur's professional career has been dedicated to exploring the analogue for general reductive groups of the trace formula for SL_2 first proved by Selberg in the mid 1950s. This has proved to be enormously complex in its details, but also extraordinarily fruitful in its applications. With help from Bill Casselman at the University of British Columbia, this website presents the author's complete published work in an easily accessible set of searchable PDFs. **www.claymath.org/cw/arthur/**

Notes of Talks at the I. M. Gelfand Seminar

The notes presented here were taken by a regular participant at the celebrated Monday evening mathematical seminar conducted by Israel Moiseevich Gelfand at Moscow State University. Mikhail Aleksandrovich Shubin, who began attending in September 1964 as a fourth-year student in the mathematics department of Moscow State University, took notes over 25 years and, even more remarkably, managed to keep all his notes. With the financial support of the Clay Mathematics Institute, Shubin's notes have been scanned for all to appreciate. The entire project would not have been possible without the involvement of M. A. Shubin, S. I. Gelfand, and the assistance of the Moscow Center of Continuous Mathematical Education. **www.claymath.org/publications/GelfandNotes/**

nominations, proposals and applications

Senior and Research Scholars

Nominations for Senior and Research Scholars are considered four times a year at our Scientific Advisory Board (SAB) meetings. Principal funding decisions for Senior Scholars are made at the September SAB meeting. Additional nominations will be considered at other times as funds permit. Clay Research Fellow nominations are considered once a year and must be submitted according to the schedule below. Address all nominations to Nick Woodhouse at president@claymath.org, copied to Naomi Kraker at admin@claymath.org.

Nomination Deadlines

- Senior Scholars: August 1
- Research Fellows: October 30
- Research Scholars: August 1*

Workshops at the Mathematical Institute

The Clay Mathematics Institute invites proposals for small workshops, typically ten to twenty persons, to be held at the Mathematical Institute in Oxford, UK. The aim is to bring a small set of researchers together quickly, outside the usual grant and application cycle, when this is likely to result in significant progress. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge. For more information, or to make a proposal, contact president@claymath.org, copied to admin@claymath.org.

Enhancement and Partnership

The Clay Mathematics Institute invites proposals under its new Enhancement and Partnership Program. The aim is to enhance activities that are already planned, particularly by funding international participation. The program is broadly defined, but subject to general principles: CMI funding will be used in accordance with the Institute's mission and its status as an operating foundation to enhance mathematical activities organized by or planned in partnership with other organizations; it will not be used to meet expenses that could be readily covered from local or national sources; and all proposals will be judged by the CMI's Scientific Advisory Board. For more information, visit www.claymath.org/programs/enhancement/. Enquiries about eligibility should be sent to president@claymath.org and proposals should be sent to admin@claymath.org.

Nominations and proposals may also be mailed to:

Clay Mathematics Institute
Office of the President
Mathematical Institute
24-29 St. Giles'
Oxford OX1 3LB
United Kingdom

(*) Most funding decisions are made by the Scientific Advisory Board at its Fall meeting. For the indicated programs, occasional appointments are made at later meetings. However, since most funds are allocated at the Fall board meeting, application/nomination by the August date is advisable.

2013 Clay Research Conference

September 29 – October 4, 2013

University of Oxford
Mathematical Institute, Radcliffe Observatory Quarter

Clay Research Conference

Wednesday, October 2

Peter Constantin, Princeton University
Lance Fortnow, Georgia Institute of Technology
Fernando Rodriguez Villegas, University of Texas at Austin
Edward Witten, Institute for Advanced Study, Princeton

A number of workshops will be held in Oxford in association with the conference.

Conference Workshops

The Navier-Stokes Equations and Related Topics

September 29 – October 1

Organizers
Peter Constantin, Princeton
Gregory Seregin, Oxford
Edriss S. Titi, Weizmann Institute, UC Irvine

Participants
Camillo De Lellis, Zürich
Charles Fefferman, Princeton
Yoshikazu Giga, Tokyo
Thomas Yizhao Hou, Caltech
Alexander Kiselev, Wisconsin
Nader Masmoudi, NYU
Laure Saint-Raymond, ENS Paris
Vlad Vicol, Princeton
Sijue Wu, Michigan

Quantum Mathematics and Computation

September 30 – October 4

Organizers
Samson Abramsky, Oxford
Bob Coecke, Oxford
Michael Collins, Oxford
Christopher Douglas, Oxford
Kobi Kremnitzer, Oxford
Ulrike Tillmann, Oxford
Jamie Vicary, Oxford

Participants
Steve Awodey, Carnegie Mellon
John Baez, UC Riverside
Alexander Beilinson, Chicago
Lucien Hardy, Waterloo
Martin Hyland, Cambridge
Christopher Isham, Imperial
Dana Scott, Carnegie Mellon
Vladimir Voevodsky, IAS
Anton Zeilinger, Vienna

New Insights into Computational Intractability

September 30 – October 4

Organizer
Eric Allender, Rutgers

Participants
Scott Aaronson, MIT
Manindra Agrawal, IIT Kanpur
Harry Buhrman, CWI Amsterdam
Stephen Cook, Toronto
Irit Dinur, Weizmann Institute
Martin Dyer, Leeds
Lance Fortnow, Georgia Tech
Leslie Ann Goldberg, Liverpool
Paul Goldberg, Liverpool
Georg Gottlob, Oxford
Russell Impagliazzo, UC San Diego
Peter Jeavons, Oxford
Mark Jerrum, Queen Mary London
Elias Koutsoupias, Oxford
Peter Bro Miltersen, Aarhus
Ketan Mulmuley, Chicago
Toniann Pitassi, Toronto
Ran Raz, Weizmann Institute, IAS
Rahul Santhanam, Edinburgh
Salil Vadhan, Harvard
Ryan Williams, Stanford

Number Theory and Physics

September 30 – October 4

Organizers
Tamas Hausel, EPFL
Emmanuel Letellier, Caen
Fernando Rodriguez Villegas, UT Austin

Participants
Gaetan Borot, Genève
Francis Brown, Jussieu
Philip Candelas, Oxford
Brian Conrey, AIM
Duiliu Diaconescu, Rutgers
Tudor Dimofte, Caltech
Herbert Gangl, Durham
Victor Ginzburg, Chicago
Sergei Gukov, Caltech
Jon Keating, Bristol
Frances Kirwan, Oxford
Maxim Kontsevich, IHES
Dirk Kreimer, Humboldt
Gerard Laumon, Orsay, Paris
Patrick Leboeuf, Paris Saclay
Sergey Lysenko, Elie Cartan, Nancy
Marcus Reineke, Wuppertal
Zeev Rudnick, Tel Aviv
Olivier Schiffmann, Orsay, Paris
Nina Snaith, Bristol
Balazs Szendroi, Oxford
Eric Vasserot, Jussieu
Edward Witten, IAS

Registration

Registration for the Clay Research Conference is free but required.
Participation in the workshops is by invitation. A limited number of additional places is available.

To register for the Clay Research Conference and Workshops, visit **www.claymath.org/CRC13/**

Some financial assistance is available for PhD students and early career researchers. Some accommodation is available. For more information email Naomi Kraker at admin@claymath.org.

October 3, 2013

Mathematical Institute Opening Conference

These events are held in conjunction with Oxford University's Mathematical Institute Conference celebrating the opening of the Institute's new building. For more information and to register, visit www.maths.ox.ac.uk/opening

CLAY MATHEMATICS INSTITUTE

www.claymath.org

2013 institute calendar

Date	Event	Location
January 1 – December 31	PRIMES	Massachusetts Institute of Technology, Cambridge, MA
January 1 – December 31	Independent University of Moscow	Moscow, Russia
January – May	Senior Scholar Toby Stafford, "Noncommutative Algebraic Geometry and Representation Theory"	MSRI, Berkeley, CA
April 3 – 5	Simplicity: Ideals of Practice in Mathematics and the Arts	City University of New York, New York, NY
April 8 – 9	IMA Conference on Mathematics in Finance	University of Edinburgh, Edinburgh, UK
April 15 – 16	Higher Structures in Topology and Number Theory Workshop	University of Oxford, Oxford, U.K.
April 18 – 19	Women in Mathematics Days	Isaac Newton Institute, Cambridge, U.K.
May - July	Conformal Geometry and Geometric PDEs	Centre de Recerca Matemàtica, Bellaterra, Spain
June 17 – July 26	Ross Program	Ohio State University, Columbus, OH
June 30 – July 20	Senior Scholars Richard Schoen and Gerhard Huisken, "Geometric Analysis"	PCMI, Park City, UT
June 30 – August 10	PROMYS	Boston University, Boston, MA
July 1 – 13	ICMS Summer School and Workshop *Ricci curvature: limit spaces and Kähler geometry*	University of Edinburgh, Edinburgh, UK
July 2 – 12	Modern Mathematics: International Summer School for Students	Jacobs University, Bremen, Germany
July 8 – 12	LMS-EPSRC Instructional Course: Modern Nonlinear PDE Methods in Fluid Dynamics	University of Reading, Reading, UK
July 8 – 19	CIMPA Research School: Current Trends in Computational PDEs	Indian Institute of Science, Bangalore, India
July 15 – 18	4th European Set Theory Conference	Barcelona, Spain
July 29 – August 9	Summer Graduate School on Mathematical General Relativity	Cortona, Italy
August 11 – 17	Oxford Masterclasses in Combinatorics	University of Oxford, Oxford, UK
August 19 – 23	Mixed Hodge Modules and their Applications Workshop	University of Oxford, Oxford, UK
August 28 – September 2	Gelfand Centennial Conference: a view of 21st century mathematics	Cambridge, MA
August – November	Senior Scholar Robert Griffiths, "Biodiversity and Evolution"	CRM, Université de Montréal, Canada
August – December	Senior Scholar Cedric Villani, "Optimal Transport: Geometry and Dynamics"	MSRI, Berkeley, CA
August – December	Senior Scholars Igor Rodnianski and Vincent Moncrief, "Mathematical General Relativity"	MSRI, Berkeley, CA
September 11 – 13	Mathematics of CCC: Mathematical Physics with Positive Lambda Workshop	University of Oxford, Oxford, UK
September 29 – October 1	The Navier-Stokes and Related Topics Workshop	University of Oxford, Oxford, UK
September 30 – October 4	New Insights into Computational Intractability Workshop	University of Oxford, Oxford, UK
September 30 – October 4	Number Theory and Physics Workshop	University of Oxford, Oxford, UK
September 30 – October 4	Quantum Mathematics and Computation Workshop	University of Oxford, Oxford, UK
October 2	Clay Research Conference	University of Oxford, Oxford, UK
October 11 – 13	Mostowski 100	University of Warsaw, Warsaw, Poland
October – tbc	Instructional Workshop on Anabelian Geometry	University of Oxford, Oxford, UK

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