CLAY MATHEMATICS INSTITUTE

annual report 2010

The Resolution of the Poincaré Conjecture

$$\pi_1(M) = 0 \implies M \cong S^3$$

Nominations, Proposals, and Applications

Nominations for Senior and Research Scholars are considered four times a year at our Scientific Advisory Board (SAB) meetings. Principal funding decisions for Senior Scholars are made at the September SAB meeting. Additional nominations will be considered at other times as funds permit. Clay Research Fellow nominations are considered once a year and must be submitted according to the schedule below:

Nomination Deadlines

Senior Scholars: **August 1**
Research Fellows: **October 30**
Research Scholars: **August 1***

Address all nominations to the attention of the assistant to the president at nominations@claymath.org.

Nominations may also be mailed to:

Clay Mathematics Institute
One Bow Street
Cambridge, MA 02138

Workshops at One Bow Street

Workshops at One Bow Street on "hot topics" can be organized on fairly short notice. It is, however, best to submit a proposal three months prior to the proposed workshop date. Please write James Carlson (jcarlson@claymath.org) with a copy to nominations@claymath.org.

(*) Most funding decisions are made by the Scientific Advisory Board at its fall meeting. For the indicated programs, occasional appointments are made at later meetings. However, since most funds are allocated at the fall board meeting, application/nomination by the August date is advisable.

CLAY MATHEMATICS INSTITUTE

CMI's One Bow Street library holdings include two collections of mathematical books that were acquired as gifts to the Institute:

Raoul Bott Library, gift received from the Bott family in 2005. 701 volumes consisting of books, journals, and preprints on topology, geometry, and theoretical physics.

George Mackey Library, gift received from the Mackey family in 2007. 1,310 volumes consisting of books and periodicals related to quantum mechanics, group representations, and physics, in addition to titles on a wide range of historical, philosophical, and scientific topics.

CMI's Digital Library includes the following facsimiles of significant historical mathematical books and manuscripts that are accessible online at www.claymath.org/library/historical:

Euclid's Elements, Constantinople, 888 AD (Greek). MS at the Bodleian Library. The oldest extant manuscript and printed editions of Euclid's Elements, in Greek (888 AD) and Latin (1482 AD), respectively. High-resolution copies of the manuscript are available for study at the Bodleian Library, Oxford University and at the Clay Mathematics Institute, Cambridge, Massachusetts. Full online editions are available at CMI.

Riemann's 1859 Manuscript. The manuscript in which Riemann formulated his famous conjecture about the zeroes of the zeta function.

Felix Klein Protokolle. The Klein Protokolle, comprising 8,600 pages in twenty-nine volumes, records the activity of Felix Klein's seminar in Goettingen for the years 1872-1912.

Contents

James A. Carlson, President

Dear Friends of Mathematics

On May 24, 2000, the Clay Mathematics Institute announced in Paris the establishment of seven Millennium Prize Problems: the Birch and Swinnerton-Dyer conjecture, the Hodge conjecture, the Navier-Stokes problem, the Poincaré conjecture, the P versus NP problem, the Quantum Yang-Mills problem, and the Riemann Hypothesis. After only ten years, the first award was announced: to Grigoriy Perelman, for his resolution of the century-old Poincaré conjecture.

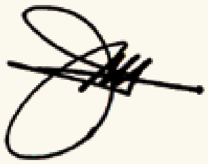
The remarkable saga of Perelman's work entered the public consciousness with a short article he posted November 11, 2002 on arXiv.org. Its first words were:

> *We present a monotonic expression for the Ricci flow, valid in all dimensions and without curvature assumptions. It is interpreted as an entropy for a certain canonical ensemble. Several geometric applications are given.*

Because Perelman did not take the usual route of submission of his work to a refereed journal, verification of his proof did not follow the standard, well-formalized path. Instead, the community of mathematicians responded with a largely self-organized effort to understand what he had done and to determine whether the proof was correct. Seminars were held. A handful of dedicated mathematicians put their own research programs aside to spend a year, two years, three, or more, going through the proof line by line, discussing one paragraph, then another with a colleague, posting notes on the Internet, writing a book, or journal article to give a detailed account of the proof. Most of what was written by way of exegesis was later reviewed in the standard way. Thus it was that Perelman's articles received one of the most thorough refereeing jobs in history. Award of the Millennium Prize to Grigoriy Perelman was announced on March 18, 2010, and celebrated at a conference in Paris on June 8 and 9, 2010.

What has been the sequel? For one, Perelman refused the prize fund of $1,000,000 several weeks after the Paris conference; the funds have since been dedicated to establishing a limited-term fellowship at the Institut Henri Poincaré (see p. 7). For another, the story of Perelman's work has become folklore, the subject of newspaper articles and films. It has raised the public consciousness of mathematics as a living discipline in a way not seen since Andrew Wiles' proof of Fermat's last theorem. This, indeed, is one of the aims of the Millennium Prizes: certainly to record some of the major problems with which mathematicians grappled at the turn of the millennium, to reward intellectual achievement of the highest level, and to emphasize the importance of attacking the hardest problems—but also to say to all that in mathematics there is a vast frontier beyond which lies the unknown, and through which hardy and adventurous souls may journey to bring back new knowledge. As for when the next Millennium Prize Problem will be solved, no one knows. The first sign, like Perelman's first Internet posting, may come tomorrow, or in a decade, or in a century.

Sincerely,



James A. Carlson
President

Clay Research Conference 2010 • Paris

Abstracts of the talks

The Institute held its annual Research Conference on June 8 and 9, 2010 at the Institut Henri Poincaré in Paris. The conference celebrated the resolution by Grigoriy Perelman of the Poincaré and geometrization conjectures.

Talks were given by Michael Atiyah (University of Edinburgh), Gérard Besson (Institut Fourier), Simon Donaldson (Imperial College), David Gabai (Princeton University), Mikhail Gromov (IHES and Courant Institute of Mathematical Sciences (NYU)), Bruce Kleiner (Courant Institute of Mathematical Sciences (NYU)) Curtis McMullen (Harvard University), John Morgan (Simons Center for Geometry and Physics, Stony Brook University), Stephen Smale (City University of Hong Kong), William Thurston (Cornell University), and Gang Tian (Beijing University and Princeton University). Abstracts follow in the next few pages. Videos of the talks are available on the Clay Mathematics Institute website, at www.claymath.org/video.

On the evening of June 7, 2010, Étienne Ghys delivered a public lecture on Henri Poincaré's work as a young man entitled, "Mathematics is just a tale about groups" at the Institut Océanographique in Paris. He spoke

Landon T. Clay and Ministre Valérie Pécresse

James Carlson shows Landon and Lavinia Clay CMI's Millenium Prize Award

Abstracts of the talks

to an enthusiastic overflow crowd of 500 people. Hundreds of students came from local high schools.

Michael Atiyah, University of Edinburgh

Geometry in 2, 3, and 4 Dimensions

The nineteenth century saw the development of geometry in two dimensions from the pioneering work of Abel and Riemann to its full flowering in the hands of Klein and Poincaré. The first half of the twentieth century moved geometry into higher dimensions, with the emphasis on topology initiated by Poincaré and developed by Lefschetz and Hodge. Towards the end of the century, interest focused on the lower dimensions of three and four, stimulated in great part by ideas from physics.

The great achievement of Perelman, following from the Thurston program, closes a chapter in three dimensions with the affirmative answer to the fundamental problem identified by Poincaré. Nonetheless, there is still much to learn about three dimensions in connection with quantum physics, which has been unearthed.

There is even more to challenge us in four dimensions emerging from the great discoveries of Donaldson. The role of topology and the links with physics have yet to be fully explored and understood.

A general survey was provided putting Perelman's work in perspective and focusing on the future as well as the past.

John Morgan, Simons Center for Geometry and Physics, Stony Brook University

History of the Poincaré Conjecture

Poincaré posed his famous question at the end of a long article, published in 1904, devoted to studying the topology of 3-dimensional manifolds. Many consider this paper to mark the founding of topology as an independent area of mathematics. It is certainly true that in this paper Poincaré introduces many of the ideas that lie at the heart of the study of topology of 3-manifolds and, indeed, of manifolds of all dimensions. This talk reviewed what Poincaré did in his seminal paper and how the topological ideas he introduced changed and evolved over the next 100 years in the hands of successive generations of topologists.

Curtis McMullen, Harvard University

The Evolution of Geometric Structures on 3-Manifolds

A survey of the geometrization conjecture: its impact, methods for solution, and problems still open.

William Thurston, Cornell University

The Mystery of 3-Manifolds

The geometrization conjecture crystallized our picture of individual 3-manifolds more than 30 years ago, and Perelman's beautiful proof established it in generality about seven years ago. But mystery is still abundant.

Stephen Smale, City University of Hong Kong

Problems in Topology, Post-Perelman

Representations of manifolds, algorithms, complexity theoretic issues, and considerations of scale were discussed.

Simon Donaldson, Imperial College, London

Invariants of Manifolds and the Classification Problem

A fundamental problem is to construct equivalences between mani-

Scientific Advisory Board members: James Carlson, Gregory Margulis, Richard Melrose, Yum-Tong Siu, and Andrew Wiles (missing: Simon Donaldson)

Lavinia D. Clay and Nicholas Woodhouse (Oxford Univ.), Mairie de Paris folds; another is to construct invariants. Of course these complementary problems should be related. We discussed some developments in these directions. Special properties of the curvature tensor of a Riemannian 3-manifold, which underpins the geometrisation theorem, were recalled. Then after an account of Riemannian and other structures in higher dimensions, we discussed the work of Gromov, Taubes, and others, using holomorphic spheres in symplectic 4-manifolds. Results of Friedl, Vidussi, and others, on fibred 3-manifolds and symplectic structures, were also treated.

David Gabai, Princeton University

Volumes of Hyperbolic 3-Manifolds

As part of his revolutionary work on hyperbolic geometry in the 1970s, Thurston, generalizing work of Jorgensen and Gromov, showed that the set of volumes of complete finite volume hyperbolic 3-manifolds is closed and well ordered. Recently, Robert Meyerhoff, Peter Milley, and the speaker showed that the Weeks manifold is the unique lowest volume closed orientable manifold. This result culminates a 30+ year effort by many mathematicians using a wide variety of techniques. In particular, we made use of the work of Agol-Dunfiled, which relies on Perelman's work on Ricci flow. This lecture surveyed these developments and discussed various open problems.

Mikhail Gromov, IHES and Courant Institute of Mathematical Sciences (NYU)

What is a manifold?

Manifolds that arise in geometry come with particular structures, e.g., with metrics satisfying certain relations or equations. If we think of manifolds themselves as solutions of such equations, we need to reconsider the concept of a manifold. Possibilities in this direction were discussed.

Bruce Kleiner, Courant Institute of Mathematical Sciences (NYU)

Collapsing with Lower Curvature Bounds

Aspects of the theory of collapsing related to Perelman's proof of the geometrization conjecture were presented.

Gérard Besson, Institut Fourier, Grenoble

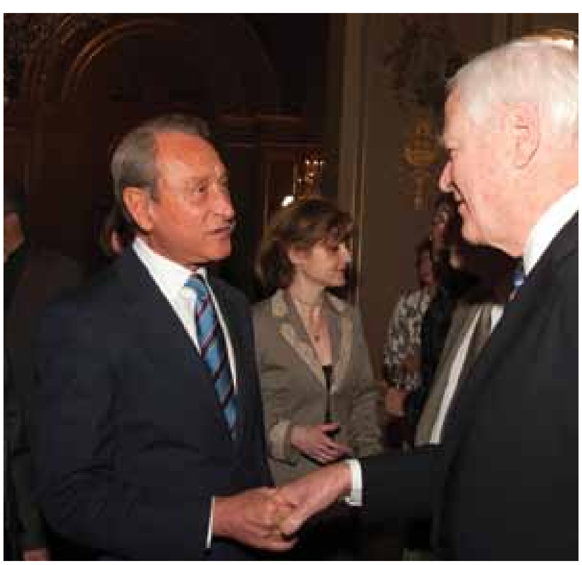
Collapsing Irreducible 3-Manifolds with Nontrivial Fundamental Group

We described, without technicalities, the main ideas of an alternative approach for the last step in Perelman's proof of Thurston's geometrization conjecture. This is joint work with L. Bessières, M. Boileau, S. Maillot, and J. Porti. Using two covering arguments we reduced the problem to Thurston's proof of his conjecture for Haken manifolds. One of the arguments made use of Gromov's simplicial volume.

Gang Tian, Beijing University and Princeton University

Metric Geometry and Analysis of 4-Manifolds

A brief introduction to Perelman's main advances on Ricci flow in his solution was given, followed by a discussion of some counterparts in dimension four and their topological and geometric consequences. There was also a discussion of some open problems that are important for studying 4-manifolds by geometric and analytic methods.



Mayor Betrand Delanoë and Landon T. Clay, Mairie de Paris

Michael Atiyah, François Poincaré, and Jean-Pierre Serre, Mairie de Paris

The Poincaré Conjecture and the Thurston Geometrization Conjecture

by James Carlson

One of the great mathematical stories of the last decade was the breakthrough work of Grigoriy Perelman. In a series of three brief articles posted on ArXiv.org in 2002 and 2003, Perelman presented a solution to two of the deepest, most difficult problems in topology and indeed in mathematics as a whole: the Poincaré conjecture, which is one of the seven Clay Millennium Prize Problems, and Thurston's geometrization conjecture, a bold and visionary conjecture which structures the entire field of three-dimensional topology and which includes the Poincaré conjecture as a special case.

Although Perelman's result is a theorem in topology, its proof came from other areas of mathematics. It was based on Richard Hamilton's theory of the Ricci flow equation, a partial differential equation akin to Fourier's equation governing heat conduction, but which instead determines the way the shape of a manifold changes under the influence of curvature. Perelman's proof drew on other ideas as well: Alexandrov theory, Cheeger-Gromov collapsing theory, and the notion of limits of metric spaces. In a tour de force of imagination and technical mastery, Perelman brought to a close a century of efforts to demonstrate (or contradict) the Poincaré conjecture.

As is usually the case in the solution of a fundamental problem, the quest brought rich rewards long before the final prize was in hand. There were many such, too many to enumerate here, but among them were the solution of the Poincaré conjecture in dimension five and greater by Stephen Smale (1966) and in dimension four by Michael Freedman (1986). The methods used in each of the three cases—dimension five and above, dimension four, and dimension three—were all quite different. It is noteworthy that it is "our dimension," three, that resisted solution the longest.

In April of 2003, Perelman accepted invitations to visit the Massachusetts Institute of Technology, Princeton University, the State University of New York at Stony Brook, Columbia University, and New York University to speak about his work. Articles appeared in the *New York Times*, seminars were organized, and midnight oil was burned as mathematicians worked to understand the proof—and to answer the big question: was the solution correct? There was reason to be cautious, given the many ultimately incorrect proofs proposed by outstanding mathematicians in the nearly one hundred years since Poincaré announced his conjecture.

On May 29, 2003, Bruce Kleiner and John Lott began posting their "Notes on Perelman's Papers" to their website. The "Notes" were a line-by-line exegesis of Perelman's first two ArXiv articles. They were a resource for all those working to verify Perelman's proof, and they later appeared as a refereed article in Geometry & Topology (2008). Kleiner and Lott received partial CMI support for their Notes on Perelman's second paper, posted beginning in September of 2004.

In the late summer of 2004 (August 25 - September 4), the Clay Mathematics Institute held the "Workshop on Perelman's Surgery Procedure" at Princeton University, organized by John Morgan and Gang Tian. The aim of the workshop was to undertake a detailed study of Perelman's second paper. The next year, CMI held a summer school at MSRI on "Ricci Flow, 3-Manifolds and Geometry," organized by Gang Tian, John Lott, John Morgan, Bennett Chow, and Tobias Colding. The school brought together leading experts, including Hamilton, as well as graduate students and postdoctoral fellows. Meanwhile, others worked to verify the solution—the participants of Yau's seminar at Harvard, with Huai-Dong Cao and Xi-Ping Zhu, and a group in France (Bessières, Besson, Boileau, Maillot, and Porti).

Slowly a consensus emerged: the proof was correct! There was, however, a difficulty regarding the Millennium Prize. Its award was governed by a set of rules, including a waiting period of at least two years between the date of refereed publication and the date at which CMI could begin to consider it for the prize. While several years had passed since the announcement, there was still no published proof that had been refereed!

After the Princeton workshop, John Morgan and Gang Tian began work on a monograph, *Ricci Flow and the Poincaré Conjecture*, which would contain a detailed proof. CMI supported Morgan and Tian during the writing, and Tian also received support from the Simons Foundation. The manuscript was posted on ArXiv.org on July 25, 2006. After a refereeing process managed by CMI in which five experts were consulted, the Morgan-Tian book was published as the third volume of the CMI-AMS monograph series.

Two years later, two groups in succession considered the correctness and attribution of Perelman's solution: a special ad hoc committee (Simon Donaldson, David Gabai, Mikhail Gromov, Terence Tao, and Andrew Wiles), which asked for opinions from other experts, and CMI's Scientific Advisory Board (James Carlson, Simon Donaldson, Gregory Margulis, Richard Melrose, Yum-Tong Siu, and Andrew Wiles). Their positive recommendation was endorsed by CMI's Board of Directors, Mr. Landon T. Clay, Mrs. Lavinia D. Clay, and Mr. Thomas Clay. On March 18, 2010, the award of the first Millennium Prize to Grigoriy Perelman was announced. This milestone in the history of mathematics was celebrated by a series of talks by Michael Atiyah, Gérard Besson, Simon Donaldson, David Gabai, Bruce Kleiner, Mikhail Gromov, Curtis McMullen, John Morgan, Stephen Smale, William Thurston, and Gang Tian at a two-day conference held June 8 and 9, 2010 at the Institut Henri Poincaré in Paris. Although Perelman did not attend, one of the great pleasures of the meeting was the chance to speak with Henri Poincaré's grandson, François Poincaré. Mr. Poincaré was present at the symbolic award of the Millennium Prize to Grigoriy Perelman.

> As is usually the case in the solution of a fundamental problem, the quest brought rich rewards long before the final prize was in hand.

One of the seven Clay Millennium Prize Problems in mathematics has now been solved. It is a tremendous achievement—for Grigoriy Perelman, who went where no man had gone before; for Richard Hamilton, who saw the promise fulfilled for the Ricci flow theory he created and developed; and for William Thurston, who, unlike Poincaré, saw his vision of a comprehensive theory of 3-manifolds fulfilled in his own lifetime. It is also a tremendous achievement for all men and women: a guidepost, placed very high on a steep mountain slope, that shows us what the human mind is capable of imagining, understanding, and creating.

Postlude. Several weeks after the Paris conference, Grigoriy Perelman informed me that he had decided not to accept the Millennium Prize funds. Subsequently, CMI's Scientific Advisory Board and its Board of Directors decided to use the funds to provide a limited-term fellowship for mathematicians in the early stages of their careers. The fellowship will allow them to pursue their research with a minimum of distraction. For a mathematician, time to think is the most precious resource! The result is the Poincaré Chair described in greater detail by Cédric Villani in his article, "Paris Conference on the Resolution of the Poincaré Conjecture" (see page 12).

A Short History of the Poincaré Conjecture

by Donal O'Shea

When the Scientific Advisory Board of the Clay Mathematics Institute canvassed mathematicians for potential Millennium Prize Problems, the Poincaré conjecture shared the distinction with the Riemann hypothesis of having been named by everyone consulted. Unlike the Riemann hypothesis, however, the Poincaré conjecture did not emerge fully formed. It began life in 1900, when Poincaré proposed as a theorem the statement that every compact manifold without boundary having the homology of a sphere is, in fact, homeomorphic to a sphere. Four years later, in the last of his great topological papers, Poincaré constructed a beautiful counterexample: the three- dimensional manifold now bearing his name with finite fundamental group of order 120 and the homology of the three-dimensional sphere. On the last page of that paper, Poincaré asks whether every simply connected three-dimensional manifold with the homology of a sphere is a three-dimensional sphere. Subsequent advances made it clear that simple connectivity guarantees that a compact 3-manifold has the homology of a sphere, and the statement that every compact, simply connected three-dimensional manifold is homeomorphic a 3-sphere came to be known as *the* Poincaré conjecture. The modification of the earlier statement asserting that a simply connected n-dimensional manifold with the homology of a sphere is homeomorphic to the n-dimensional sphere, expressed in terms of homotopy equivalence, became known as the *generalized* Poincaré conjecture.

The conjecture served as a sort of touchstone in Poincaré's approach to topology. It is a concrete test case for characterizing manifolds up to homeomorphism by invariants, a leitmotiv that threads through all his topological papers beginning with the first foundational paper (Poincaré 1895). That extraordinary paper essentially runs through the basics of differential topology, algebraic topology, and combinatorial topology four decades before the first textbooks on any of these subjects appeared. In it, Poincaré explicitly asks which invariants characterize a manifold up to homeomorphism, and then constructs infinitely many 3-manifolds with the same Betti numbers. He introduces the fundamental group, relating it to deck transformations, and carefully sketching the differences from the first homology group, again asking which manifolds could be distinguished by their fundamental groups. This foundational paper was followed by five papers, which he calls compléments, that immeasurably deepen the examples and techniques. The second paper (Poincaré 1899b) introduces triangulations, clarifies the definition of Betti numbers, and responds to Heegaard's criticism of the presentation of Poincaré duality in the first paper. The third paper (Poincaré 1900) introduces torsion coefficients and shows that Poincaré duality also holds for them. This new set of (presumed) invariants lures him into the optimisitic misstatement of his eponymous conjecture cited above. The fourth paper (third complément: Poincaré 1902a) studies the topology of complex algebraic surfaces of the form $z^2 = F(x, y)$ where the curve $F(x,y) = 0$ is nonsingular or possesses at most ordinary double points, and the fifth paper (Poincaré 1902b) examines general algebraic surfaces $F(x,y,z) = 0$, elaborating on work of Picard and establishing the basics of what would later be called Picard-Lefschetz theory. Poincaré's sixth (Poincaré 1904) and final topological paper introduces a technique from what we now call Morse theory, and investigates the role of surface diffeomorphisms in gluing handlebodies to create 3-manifolds. By attaching two genus two handlebodies, Poincaré constructs the manifold now called the Poincaré dodecahedral space. (Nowadays the manifold is described as a regular dodecahedron with opposite faces identified after a rotation of 1/10 of a full

Poincaré would later write that every mathematical or scientific problem he examined, no matter how remote or recondite, would lead him inexorably to topology. And central to topology was the Poincaré conjecture.

turn—a description that was not established until years later by Kneser.) He notes that this manifold has the homology of the 3-sphere but cannot be homeomorphic to the sphere because it does not have trivial fundamental group. He closes with the Poincaré conjecture, asking whether a closed simply connected three-dimensional manifold is necessarily homeomorphic to the 3-sphere, and ends abruptly with the enigmatic sentence: "But this question would take us too far afield." Essentially then, Poincaré begins his topological work with the goal of characterizing manifolds by invariants, announces a version of the Poincaré conjecture as soon as he discovers a powerful set of new homological invariants, and ends with a long paper wholly devoted to the Poincaré conjecture.

Only ten of Poincaré's more than 700 papers deal with topology and, of these, four (Poincaré 1892, 1899a, 1901b, 1901c) are research announcements of the results contained in the other six. Yet their influence on the course of twentieth-century mathematics was far greater than their miniscule proportion would suggest. The six papers established topology as an independent discipline with its own methods and of interest in its own right. They added immensely to then-existing knowledge, and would establish new fields such as differential topology and algebraic topology that became central to twentieth-century mathematics and the mathematics of today. Poincaré would later write that every mathematical or scientific problem he examined, no matter how remote or recondite, would lead him inexorably to topology. And central to topology was the Poincaré conjecture.

The conjecture attracted immediate attention. It is cited in Dehn and Heegaard's survey article on combinatorial topology that was written in 1905 and that appeared two years later (Dehn 1907) in the *Enzyklopädie der Mathematischen Wissenschaften*. Unhappily, the authors make a subtle error that Poincaré avoided in constructing Poincaré's homology sphere, and the manifold they construct is actually the 3-sphere. It appears again in 1908 in Tietze's famous paper (Tietze 1908). That same year, Dehn submitted a paper to *Mathematische Annalen* purporting to prove the conjecture, but Tietze discovered a subtle error and Dehn withdrew it—see (Volkert 1996). Dehn was, however, able to use his techniques to produce an infinite series of homology 3-spheres (Dehn 1910). Alexander established the topological invariance of the Betti numbers and the torsion coefficients (Alexander 1915) and subsequently showed that two three-dimensional manifolds (discovered by Tietze) with the same (nontrivial) fundamental group and homology could


Donal O'Shea

fail to be homeomorphic (Alexander 1919). This underscored the possibility that the conjecture might be false.

By the early 1930s, the field of topology had matured and the conjecture was well-known. Alexander talked about it in his address to the International Congress of Mathematicians in 1932. Seifert and Threlfall discuss it in their influential *Lehrbuch der Topologie* that appeared in 1934. A year later, Aleksandrov and Hopf mention it in the introduction to their magisterial text *Topologie*. By this time, too, the conjecture's treacherous reputation had solidified. Whitehead published a proof (Whitehead 1934) that he subsequently withdrew (Whitehead 1935). As was the case with Poincaré and Dehn, the mistake resulted in a significant discovery, and Whitehead's famous example of a contractible 3-manifold that is not $\mathbf{R}^3$ greatly clarified the type of behavior that one could expect of open 3-manifolds. The conjecture would continue to inspire hard work and elicit subtle errors throughout the century. Students in RH Bing's topology class tell the story of Bing entering the classroom in the early sixties waving two thick manuscripts, one purporting to prove the conjecture, the other to disprove it (M. Olinick, personal communication, 2008). Bing told the class that they could be absolutely certain that one was false. The conjecture even inspired a paper entitled "How Not to Prove the Poincaré Conjecture" (Stallings 1966).

Although no one in the sixties had any idea of whether or not the Poincaré conjecture was true, by that time topologists had sorted out the relations between the topological, differentiable, and piecewise-linear categories in dimension three. It is not the case that every manifold can be given the structure of a differentiable manifold (Kervaire 1960), but it is the case in dimension three. This follows from the fact that every three-dimensional manifold has an essentially unique piecewise-linear (Moise 1952), thus differentiable (Munkres 1960), structure. This cleared up some of the nagging uncertainties about whether results using differentiable, say, methods would carry over to the topological (or piecewise-linear) case. One could settle the Poincaré conjecture by determining, for example, whether any simply connected, compact differentiable 3-manifold is diffeomorphic to the 3-sphere. The sixties also brought spectacular progress in the higher dimensional case. The generalized Poincaré conjecture was settled in dimension five and more by Smale in 1960 using handle body methods (Smale 1960) and by Stallings and Zeeman independently using the idea of engulfing ((Stallings 1960), (Zeeman 1962)). The dimension four case was settled by very different methods two decades later by Freedman (Freedman 1982).

In the late seventies and early eighties, Thurston amassed significant evidence for his geometrization conjecture (Thurston 1982) that three-dimensional manifolds could be carved up in a natural way into manifolds that each possessed one of a small number of canonical geometries. Thurston's conjecture implied the Poincaré conjecture, providing an attractive, albeit conjectural, framework explaining why it might be true. As the twentieth century drew to a close, however, a proof of the geometrization conjecture seemed even further off than a proof of the Poincaré conjecture. Hamilton's Ricci flow methods initially offered great promise for getting at the geometrization conjecture, but only under very restrictive conditions that seemed necessary to control the analysis (Hamilton 1982 and 1995). Meanwhile, geometric topologists were still working on purely topological proofs and some very solid mathematicians (such as Rourke, Rego, Poénaru, and Dunwoody) produced putative proofs that turned out to have subtle errors. Techniques to algorithmically recognize a 3-sphere (some of which grew out of other failed attempts at the Poincaré conjecture) had advanced far enough so that one could write a computer program that would check for counterexamples to the Poincaré conjecture and stop in finite time if such existed (Rourke 1997).

Two years into the new century, Perelman would publish the first of his three preprints (Perelman 2002, 2003a, 2003b) exploiting the geometry inherent in the Ricci flow to prove both the Poincaré conjecture and the geometrization conjecture. The initial skepticism stemming from the Poincaré conjecture's tortuous history turned into cautious optimisim as others fleshed out Perelman's work, making it accessible to those outside the Ricci flow community. Morgan and Tian published an expository book giving a complete self-contained account of the part of Perelman's work needed to establish the Poincaré conjecture (Morgan and Tian, 2007) that stilled any lingering doubts. Since that time, Perelman's proof of the full geometrization conjecture has been accepted as correct and many simplifications and extensions have been found. A gala ceremony in Paris in June 2010, hosted jointly by the Clay Mathematics Institute and the Institut Henri Poincaré, marked the successful resolution of the Poincaré conjecture and awarded the first Millennium Prize to Perelman. As with the Fields Medal in 2006, Perelman declined the award.

There is a curious and inspiring irony in the ultimate solution of the Poincaré conjecture. Riemann and Poincaré drew their inspiration from their profound geometric intuition. Yet it is to Riemann that we owe the notion that one should distinguish between a space and the geometry it carries (Riemann 1854). And, despite having made his name for, among other things, his discovery of the intimate connection between geometry and topology of surfaces, it was Poincaré who carried out Riemann's vision of establishing topology as a discipline with its own methods distinct from analysis and geometry. The statement of the Poincaré conjecture is purely topological, and until relatively late in the twentieth century, no one, Poincaré included, would have imagined that it might be true for geometric reasons. The proofs of the higher-dimensional analogues of the Poincaré conjecture are purely topological. But the work of Thurston, Hamilton, and Perelman uncovered unexpected and deep connections between the geometry and topology of 3-manifolds that were as rich and beautiful as those for 2-manifolds. Poincaré would have been delighted.

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Paris Conference on the Resolution of the Poincaré Conjecture

by Cédric Villani

In 2010, the Clay Mathematics Institute (CMI) and the Institut Henri Poincaré (IHP) organized a conference to celebrate the solution of the Poincaré conjecture by Grigoriy Perelman. Together with the International Congress of Mathematicians in Hyderabad, India, this was one of the two major mathematical events of 2010.

Let me recall the context. Henri Poincaré (1854-1912) was one of the greatest mathematicians of all time, famous for his contributions to mathematics, physics, and the philosophy of sciences. He was considered to be the most prominent mathematician of his time. His conjecture, formulated in 1904, marked the rise of topology as a major field of active mathematical research. His research initiated the dream of classifying three-dimensional manifolds, a subject which is now of fundamental importance in both mathematics and theoretical physics. In the twentieth century, three Fields Medals were awarded for progress related to this conjecture. These were the awards to Stephen Smale, Michael Freedman, and William Thurston.


Stephen Smale


Bruce Kleiner


Sir Michael Atiyah


Landon T. Clay and Sir Andrew Wiles

The Poincaré conjecture was also one of the seven Millennium Prize Problems established in the year 2000 by the Clay Mathematics Institute. Thus, it came as a surprise when, only three years later, the Russian mathematician, Grigoriy Perelman, claimed the solution of this conjecture, on which he had worked quietly for seven years. Building on previous work by Richard Hamilton on the Ricci flow, Perelman solved not only the Poincaré conjecture, but also Thurston's geometrization conjecture. The geometrization conjecture proposed a bold classification of all three-dimensional manifolds. Perelman's proof was the most important mathematical advance of the past ten years, comparable in terms of impact only to the 1994 solution of Fermat's last theorem by Andrew Wiles. More details can be found in the public statement of the Clay Mathematics Institute, which was relayed in France by the IHP.

It took several years of work for the mathematical community to check Perelman's proof, and still longer for the conditions of the Millennium Prize to be satisfied, principally a two-year waiting period after refereed publication of the proof. On March 18, 2010, CMI declared Perelman the first winner of the Millennium Prize and announced the organization of the conference in Paris.

The choice of the Institut Henri Poincaré for the conference was a natural one. Founded in 1928 by the joint efforts of George David Birkhoff and Émile Borel with the support of the Rockefeller and Rothschild foundations, the IHP is an institution that embodies the scientific legacy of Poincaré. It is also a symbol of French-US scientific collaboration, having hosted numerous thematic programs, lectures, and conferences, and it has welcomed thousands of mathematicians from around the globe.

The program of the conference, prepared by the Scientific Advisory Board of CMI, included a number of winners of the Fields Medal and the Abel Prize. The founder of CMI, Mr. Landon T. Clay, CMI board members Lavinia D. Clay and Thomas Clay, as well as other members of the Clay family were in attendance.

Complementing the main program were several events prepared locally by IHP and its partners. These parallel events attracted a large number of non-professionals: high school students, amateurs, and members of the interested public.

June 7th, Institut Henri Poincaré. The day began with the inauguration of the exhibit *Mathematics and Art*, as well as the installation of a bronze plaque commemorating the life and work of Henri Poincaré. The plaque is now located at the entrance of the IHP. At 11:30 a.m., a press conference on the Millennium Prize was hosted by James Carlson, president of CMI, in the Amphithéâtre Hermite hall of IHP. The press conference featured a panel of distinguished mathematicians, each of whom made a statement and answered questions. The panelists were: Sir Andrew Wiles, William Thurston, Stephen Smale, John Morgan, Curtis McMullen, Donal O'Shea, Marcus du Sautoy, Mikhail Gromov, and Étienne Ghys**.** Later that day, James Carlson gave an informal lecture in IHP's Amphithéâtre Hermite for 150 high school students brought together by Animath. The day closed with an extraordinary public lecture on the early work of Poincaré at the Institut Océanographique's Grand Amphithéâtre by Étienne Ghys. The Institut Océanographique, established in 1906 by Albert I, Prince of Monaco, who was devoted to scientific investigation, is located just a few meters away from IHP. The audience of 500, of which about 200 were high school students, filled the lecture hall to capacity.

June 8th, Institut Océanographique. The program featured lectures in the Grand Amphithéâtre by Atiyah, Morgan, Donaldson, McMullen, Smale, and Thurston. The prize was symbolically awarded by Mr. Landon T. Clay, in the presence of François Poincaré, the grandson of Henri Poincaré. About 500 mathematicians were in attendance.

During the lunch at École Normale Supérieure (ENS), which was hosted by Ms. Monique Canto-Sperber in the "Salon de la Direction," Ms. Valérie Pécresse, the French Minister of Research and Higher Education, gave a short presentation. That evening, the ENS hosted a reception for all participants in its bibliotheque. The party featured a piano recital in the Salon des Acts by Karol Beffa (ENSE Math graduate and Ph.D. in Musicology in 2003), who improvised pieces upon request.

June 9th, IHP. The second and final day of the conference featured lectures by Besson, Gabai, Gromov, Kleiner, and Tian, with 200 attending, filling to capacity both the Darboux and Hermite Halls. That same day, the association Animath, working to foster contact between mathematics researchers and high school students, organized a short workshop for high school students (competition prize winners) in collaboration with Bill Thurston. This day closed with a dinner at the Mairie de Paris' Le Salon Georges Bertrand. Short remarks were given by the Paris Deputy Mayor, Mr. Jean-Louis Missika, Dr. James Carlson, Sir Andrew Wiles, and Mr. Landon T. Clay. The Mayor of Paris, Mr. Bertrand Delanoë, made a visit during the cocktail preceding the dinner.

Despite the fact that Perelman had already been the center of attraction prior to the conference, the conference had good press coverage, including, in particular, an excellent article in *die Zeit*, and mention on the second page of *Le Monde*.

Several weeks after the conference in Paris, Dr. Perelman notified Dr. James Carlson that he had decided not to accept the one million dollars provided for in the Millennium Prize for resolution of the Poincaré conjecture. Subsequently, the Clay Mathematics Institute moved to use these funds to establish a fellowship program for mathematicians in the early stages of their careers. Named the Poincaré Chair, the program will be operated independently of CMI.

On September 21, 2011, the Institut Henri Poincaré announced that it will offer the Poincaré Chair each year, beginning in the fall of 2012, for a period of up to seven years. Modelled on the Miller Fellowship at the University of California, Berkeley, the Fellowship honors both the work of Henri Poincaré and the solution of the Poincaré conjecture by Grigoriy Perelman. As Perelman was himself a Miller Fellow in 1993-95, such a use to the benefit of young mathematicians' research seems most fitting.

The selection of Poincaré Fellows will be made by a committee of mathematicians of world repute, whose interests span the main branches of mathematics. The committee, appointed by the IHP, consists of Artur Avila, Simon Donaldson, Ingrid Daubechies, Làszlo Lovàsz, Claire Voisin, and two other mathematicians to be named. The director of IHP will collect applications received before October of each year

Ghys answers journalists, along with fellow press panelists: (left) Thurston, Gromov, McMullen; (right) Sautoy and O'Shea.

(Above) June 8th, Institute Océanographique
(Left) Cédric Villani, IHP Director

and then transmit them to the committee. The committee will examine applications, then make proposals and transmit a short report to the scientific advisory board of IHP. Selection will be based on both excellence of past research and potential for future breakthroughs. Final approval of the chosen fellow or fellows lies with the administrative board of the IHP, and will take place in December, with the Fellowship to commence in September of the next year. There will be either one fellow for the whole academic year, or two fellows for up to a total of twelve months, typically for the periods September–February, and March–August. The precise dates can be negotiated, and an overlap is possible; the exact duration can also be discussed, but will not be less than four months for a given fellow.

The Poincaré Fellow will be given all facilities needed to work in IHP: an office, personal grant, and help finding housing. The Fellows are normally expected to remain in the neighborhood of IHP, but mobility within France and Europe more generally is encouraged. Short trips to other parts of the world are possible whenever scientifically motivated. In general, the organization of the stay of the Poincaré Fellow should be discussed with IHP; the latter will be careful to make the position sufficiently flexible that the Fellow does not feel constrained, but at the same time will ensure that the time of the fellowship is spent on focused research and profitable interactions.

Although the Poincaré Fellow will have no teaching obligations**,** interaction with local mathematicians will be encouraged.

The salary for the Poincaré Fellows will come in its entirety from the one million dollar Millennium Prize Fund. Disbursements from the fund will be for the exclusive support of the Fellows. When the fund is exhausted, after a period of six or so years, it is hoped that a new sponsor can be found to perpetuate the fellowship.

I am one of the many mathematicians whose research has greatly benefitted from the Miller Fellowship; some others are Phillip Griffiths, Robert Langlands, Grigoriy Perelman, and Dennis Sullivan. I look forward, with great anticipation, to seeing the mathematical work of the Poincaré Fellows in the coming years.

Cédric Villani
Professeur de l'Université de Lyon
Directeur de l'Institut Henri Poincaré
11 rue Pierre et Marie Curie
75231 Paris Cedex 05
villani@ihp.jussieu.fr

Summary of 2010 Research Activities

The activities of CMI researchers and research programs are sketched below. Researchers and programs are selected by the Scientific Advisory Board (see inside front cover).


Tim Austin

Clay Research Fellows

Tim Austin received his Ph.D. in 2010 from the University of California, Los Angeles, under the supervision of Terence Tao. His interests cover ergodic theory, metric geometry, and geometric group theory. In his recent research he has developed new techniques for the analysis of certain nonconventional ergodic averages associated with the phenomenon of multiple recurrence, and has shown how to construct examples of infinite discrete groups with various novel geometric properties.

Tim received his B.A. from Trinity College, Cambridge University in 2005. While at UCLA, he held the positions of research assistant and teaching assistant; during the summers he has frequently held a visiting position at Microsoft Research. He is currently based at Brown University for the 2011 and 2012 academic years.

Tim Austin joined CMI's group of research fellows: Mohammed Abouzaid (MIT), Artur Avila (IMPA Brazil), Roman Bezrukavnikov (MIT), Manjul Bhargava (Princeton University), Daniel Biss, Maria Chudnovsky (Columbia University), Dennis Gaitsgory (Harvard University), Soren Galatius (Stanford University), Daniel Gottesman (Perimeter Inst.), Ben Green (University of Cambridge), Sergei Gukov (UC Santa Barbara), Adrian Ioana (UCSD), Bo'az Klartag (Tel-Aviv), Elon Lindenstrauss (Jerusalem), Ciprian Manolescu (UCLA), Davesh Maulik (Columbia University), Maryam Mirzakhani (Princeton University), Sophie Morel (Harvard), Mircea Mustata (University of Michigan), Sam Payne (Yale), Igor Rodnianski (Princeton University), Sucharit Sarkar (Columbia University), Peter Scholze (Universität Bonn), David Speyer (Michigan), Terence Tao (UCLA), András Vasy (Stanford University), Akshay Venkatesh (Stanford University), and Teruyoshi Yoshida (University of Cambridge).

Research Scholars

Ian Agol (University of California, Berkeley)
April 19-23, 2010
Virtual Properties of 3-Manfifolds at UQAM

Senior Scholars

Pierre-Louis Lions (Isaac Newton Institute)
January 18-22, 2010 and March 8-12, 2010
Stochastic Partial Differential Equations (SPD)

Tomasz Mrowka (MSRI)
January 1 - May 31, 2010
Homology Theories of Knots and Links

Peter Ozsvath (MSRI)
January 1 - May 31, 2010
Homology Theories of Knots and Links

Ingrid Daubechies (PCMI, Utah)
June 27 - July 17, 2010
Mathematics of Image Processing

Jean-Michel Morel (PCMI, Utah)
June 27 - July 17, 2010
Mathematics of Image Processing

Percy Deift (MSRI)
August 16 - December 17, 2010
Program on Inverse Problems and Applications

Gunther Uhlmann (MSRI)
August 16 - December 17, 2010
Program on Inverse Problems and Applications

Haruzo Hida (Kyoto University) at RIMS
September 21 - December 20, 2010
Hida Theory Lecture Series

Research Programs organized and supported by CMI

January 1 - December 31. Independent University of Moscow, Moscow, Russia

January 4 - March 31. Galois Trimester at The Institut Henri Poincaré (IHP), Paris, France

March 7 - 11. Macdonald Polynomials and Geometry, CMI

June 2 - 6. Number Theory and Representation Theory, Harvard University, Cambridge, MA

June 7 - 25. Structure of Local Quantum Fields, Les Houches, France

June 8 - 9. Clay Research Conference at The Institut Henri Poincaré (IHP), Paris, France

June 14 - July 3. ICTP Summer School on Hodge Theory, ICTP, Trieste, Italy

June 21 - August 13. ROSS Program, Ohio State University, Columbus, OH

June 27 - August 7. PROMYS Program, Boston University, Boston, MA

July 11 - August 2. CMI Summer School on Probability and Statistical Physics in Two and More Dimensions, Buzios, Brazil

July 20 - 30. Pacific Rim Workshop on Geometric Analysis, University of British Columbia and PIMS, Vancouver, Canada

July 26 - August 6. Winter School on Topics in Noncommutative Geometry, Universidad de Buenos Aires, Buenos Aires, Argentina

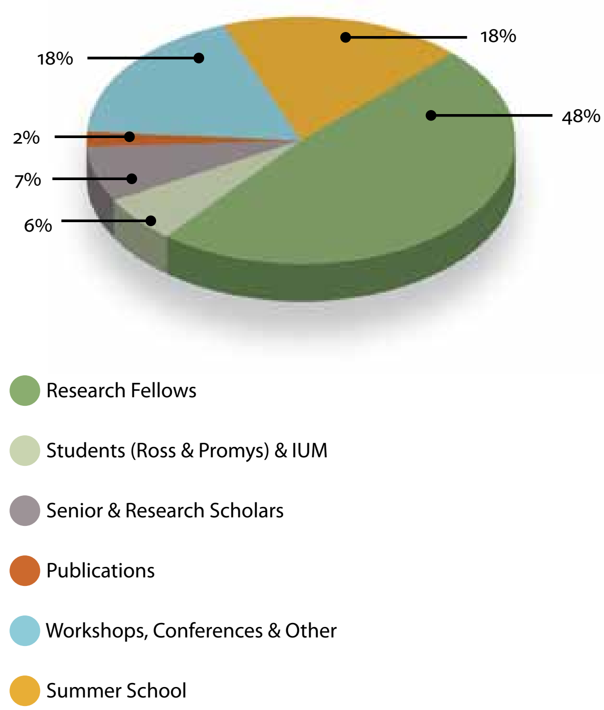
August 2 - 9. Conference in honor of the 70th birthday of Endre Szemerédi, Budapest, Hungary

Program Allocation

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2010:

- Research Fellows, Research Awardees, Senior Scholars, Research Scholars 17
- Summer School Participants and Faculty 120
- PROMYS/Ross Participants and Faculty 14
- CMI Workshops. 93
- Participants attending Conferences and Joint Programs .... >5000
- Independent University of Moscow (IUM) 95

Research Expenses for Fiscal Year 2010



Profile | Interview with Research Fellow Soren Galatius

Soren Galatius (b. 1976), a native of Denmark, recieved his Ph.D. from the University of Aarhus in 2004 under the direction of Ib Madsen. The focus of his research is in algebraic topology, especially the interplay between stable homotopy theory and geometry. A recent finding involves automorphism groups of free groups; he proved that the stable rational homology is trivial.

Soren Galatius

What first drew you to mathematics? What are some of your earliest memories of mathematics?

As far back as I remember, I was fascinated by numbers and mathematical concepts. My parents once brought home a pocket calculator with buttons for the four basic arithmetic operations, a button for square root, and one for percent. I knew about the arithmetic operations, but I wanted to know what the two remaining buttons were for so I asked my parents. I didn't understand the percent button, but I did understand the square root, and I proudly told my teacher the next day.

Could you talk about your mathematical education? What experiences and people were especially influential?

My first six years of education were in a small school in the countryside of Denmark. I liked my math teacher, and she was always willing to answer my questions or direct me to someone else if she didn't know. Then, when I was in seventh grade, my grandfather gave me a small math encyclopedia, which I think was influential. I always brought it on family vacations.

In my first year of high school, I took the qualifying exam for the Danish International Mathematical Olympiad team. I didn't make it onto the team, but I was invited to a one-week training camp with around twenty other high school students. That experience was also quite influential. It was much more intense and fast-paced than any other math training I had encountered, and I liked that. I also liked the more challenging problems and I liked meeting the other students.

My first encounter with a more rigorous and abstract approach to math was when I went to college to study math and physics. My first year, I was drawn to math especially by my linear algebra class, which was taught quite abstractly.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

I didn't really have a mentor until I started working with Ib Madsen, who became my Ph.D. advisor. He influenced me in many ways. He taught me a lot of mathematics of course, and he introduced me to an area that I still think about. He also shaped me as a mathematician in more indirect ways; for example I think my mathematical taste is very influenced by his.

> When I was in seventh grade, my grandfather gave me a small math encyclopedia, which I think was influential. I always brought it on family vacations.

You were educated in Denmark. Could you comment on the differences in mathematical education there and in the US?

College education in the US is broader than in Denmark. At Stanford for example, new students are admitted to the whole university instead of a specific department; they take classes in many different subjects and only later choose a major. The Danish college education is much more specialized, and students focus from the beginning on one or two subjects only. For example, when I studied math and physics as an undergraduate in Aarhus, I took classes in those two departments only. Admission also depended on the subject (math and science were not very popular, so anyone who applied was admitted).

What attracted you to the particular problems you have studied?

It has been a random process. In graduate school, I chose an advisor more than I chose a subject or a specific problem to work on.

Can you describe your research in accessible terms? Does it have applications to other areas?

Much of my research has been about various moduli spaces. An important example is the moduli space of Riemann surfaces, which parametrizes families of Riemann surfaces. Another important example is the moduli space of graphs. These objects appear in many different areas of math, and have been studied from many points of view.

I have mainly studied the homotopy theoretical properties of such moduli spaces. The moduli spaces themselves show up in many areas of mathematics and even physics, and I hope the homotopy theoretic methods can provide useful insights.

What research problems and areas are you likely to explore in the future?

I don't know. If I get an idea about something, I'll follow it where it leads me.

Could you comment on collaboration versus solo work as a research style? Are certain kinds of problems better suited to collaboration?

There are obvious advantages to collaborations, both in the beginning, when trying to get ideas and see the big picture, and later, when working out details. Sometimes a single conversation can lead to new ideas that neither person would have had alone—ideas which might become the outline of a project. The process of turning the outline into a detailed argument is much less frustrating in a collaboration than in solo work, where I find it easy to get stuck. In a collaboration, it's likely that one person will figure out the details that the other person gets stuck on.

Finally, I find it psychologically easier to work in collaboration. In solo work that takes a long time, I tend to get depressed and think that I'm never going to finish.

Regarding individual work versus collaboration, what do you find most rewarding or productive?

I think collaborations are more rewarding and productive. When I finished graduate school, I was more interested in solo work, probably because I wanted to test myself, but generally I find collaborations much more fun.

How has the Clay Fellowship made a difference for you?

The most important difference was that it gave me more flexibility. I used that to spend some time at MIT and at the University of Copenhagen.

What advice would you give to young people starting out in mathematics?

Follow your heart.

What advice would you give lay persons who would like to know more about mathematics—what it is, what its role in our society has been and is, etc.? What should they read? How should they proceed?

That is more difficult. There is a lot of stuff on the web now, some of which could be interesting for lay people. For example, many mathematicians have blogs about mathematics. In an area with a university, the mathematics department might have public lectures from time to time, although unfortunately that's not as common as it should be.

What advice would you give to young people starting out in mathematics?

Follow your heart.

How do you think mathematics benefits culture and society?

Mathematics is behind most of our understanding of the world. Most laws of physics require some amount of mathematics to even state. Most technological advances could not have been made without mathematics.

Contemporary pure math is perhaps less directly applicable, but our students are in high demand, so we must be teaching them some useful skills.

Please tell us about things you enjoy when not doing mathematics.

I enjoy hiking, camping, motorcycling, exercising, and taking advantage of the many goings-on in San Francisco where I live.


Étienne Ghys

Clay Public Lecture

"Mathematics is just a tale about groups" by Étienne Ghys

(CNRS, École Normale Supérieure de Lyon)
June 7, 2010, Institut Océanographique, Paris

In recent years, it has become commonplace to begin mathematical meetings with "public lectures" open to the general public, as kind of *apéritif* before the main course! It probably fits in with the general feeling among professional mathematicians that mathematics is moving away from society at large and is becoming more and more technical and incomprehensible to the average person. Some mathematicians do not mind, and consider that mathematics can develop very well on its own. But most of my mathematics colleagues today think that there is a need for better engagement with society. After all, mathematics is part of culture and should not remain isolated from the rest of the world. More pragmatically, young students are less and less attracted to the sciences as a whole and it is our duty to show them how beautiful mathematics can be!

The public lecture is one of the many ways to link current mathematics to the public. This is not an easy task. Quite often, we have the feeling that it is simply impossible to explain contemporary mathematical ideas to non-mathematicians. In the introduction to his famous address "On mathematical problems," in 1900, Hilbert wrote the following:

> *A mathematical theory is not to be considered complete until you have made it so clear that you can explain it to the first man whom you meet on the street.*

This is more easily said than done (but, admittedly, Hilbert did!), but I was asked by Dr. Jim Carlson to prepare such a public lecture for the Clay Research Conference 2010 in Paris, celebrating the resolution of the Poincaré conjecture.

The main questions I had to solve while preparing the talk were: "Who is my audience?" and "Which story should I tell?"

Getting an answer to the first question was fundamental because I had already given public lectures in the past, which were totally frustrating. They were frustrating for me because I was speaking in front of experts and there were no laymen in the audience at all. It was also frustrating for the audience because the content of my discussion did not meet their expectations.

Because of those bad experiences, I learned that it is useful to insist as much as possible, sometimes very strongly, to get as much information as possible about the attending audience from the organizers. I consider that the words "general public" have no meaning by themselves. One does not deliver a talk to teenagers in the same way one would to a more mature or technical audience, for example.

Young students are less and less attracted to the sciences as a whole and it is our duty to show them how beautiful mathematics can be!

There should be some kind of an arrangement between the speakers and the organizers of a meeting. For example: "I will tell you what I am going to discuss and you will tell me who is going to listen to me!" In my case, I was extraordinarily lucky since I had incredible help from the Institut Henri Poincaré and Animath—the latter, an organization promoting mathematics among teenagers. Therefore, I had a very clear idea about who my public would be—gifted pupils from nearby high schools in Paris, among the very best in France. I am very familiar with this kind of "general public." I know what they should know and not know in mathematics. For instance, they should know what a complex number is, even if their vision of complex numbers is still somehow abstract.

The second question was: "Which story should I tell?" This was not an easier task. First, since I was giving the opening introduction for a Poincaré event, I had to speak about Poincaré himself. Fortunately, Poincaré is my scientific hero. So, it seemed only logical that I would speak about him as a young man, since I was speaking to young people. My plan was to explain how it is still possible to produce excellent and original mathematics, even without knowing a huge quantity of mathematics (if you are talented like Poincaré).

Here is an abstract of my talk:

"On May 28, 1880, Henri Poincaré submitted an extraordinary paper to the French Academy of Sciences. He had just turned twenty-six and did not know the work of his predecessors well, in particular those from Germany. But he had visionary ideas. On June 12, 1881, he began an impassioned correspondence—a mixture of competition and collaboration—with Felix Klein, the reigning master of German mathematics. On August 8, 1881, he announced that he had proven the uniformization theorem, which no one would have ever dreamt of formulating a few months earlier. In this elementary talk, intended primarily for high school students, I would like to evoke this wonderful year for Poincaré."

When I prepared my talk, I tried to tell a story. I did my best to prepare it in such a way that it was not necessary to follow each mathematical statement in order to enjoy the story. Poincaré was competing for a prize from the Academy of Sciences. Would he win the competition? (No, as a matter of fact.) I tried to insert as many relevant asides as possible, such as historical or cultural comments. My purpose was to speak on several levels at the same time. Of course, it is not easy, and I am not sure I fully succeeded.

I am very lucky to have a friend and collaborator, Jos Leys, who is an engineer, and who is very gifted in producing pictures and animations. We worked together to produce quite a few animations, making my overall discussion more lively. I had fun playing with the portrait of Poincaré, showing his image under the square root map, for instance. The result was a very visual pdf file, including facsimiles of Poincaré's handwriting, animations, and photos, and other material.

The talk was held in an incredibly beautiful amphitheater, in the Institut Océanographique, with beautiful marine paintings on the walls. It was inaugurated in the early 1900s, so one can guess that Poincaré himself visited this place. I must say that I was amazed to see the over 200 seats occupied by seventeen- and eighteen-year-old boys and girls—a rare honor for a mathematician! Of course, some of the seats were actually occupied by older colleagues, but I did my best to ignore them.

As a matter of fact, I am old enough not to take into account four Fields Medalists in the front row (or I should say three, since Cédric Villani was not yet a Medalist)! What mattered more for me is the slight possibility that twenty future Fields Medalists were in the room… Well, I may be optimistic, but before these boys and girls reach forty, there will be five International Congresses and, therefore, twenty Medalists.

It is not so often in the life of a mathematician that one can communicate with such an enthusiastic audience and I thank the Clay Mathematics Institute and the Institut Henri Poincaré for giving me this opportunity.

Last but not least, the talk was filmed. I have been filmed many times but never in such a professional way. The recording was done by a team led by François Tisseyre, from "Atelier: EcoutezVoir." The result is a high-quality film that one can find here: http://www.poincare.fr/evenements/item/19-les-maths-ne-sont-quune-histoire-de-groupes.html.

I have never seen a mathematics talk filmed so well, with opinions from the public, slides, the speaker, zoom shots, and more. I think this is important, since this talk might have a second life and may be downloaded.

Was my talk a success?

The only thing I can say about that is that I got a telephone call last month from my brother, who is a physician and knows nothing about mathematics. He found the film accidentally on the web and he told me that even if he did not understand all of it, he enjoyed it a lot! I would say that pleasing my elder brother is a personal success.

June 7th, about 200 high school students were brought together by Animath at IHP. Here they listen to James Carlson's informal lecture before proceeding on to the main event with Étienne Ghys next door at the Institute Océanographique.

CMI Workshops

Macdonald Polynomials and Geometry by Fernando Rodriguez Villegas

March 8 - 11, 2010

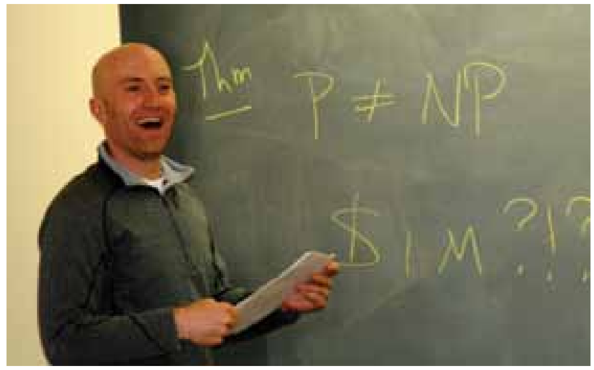
In his influential paper, "The self-duality equations on a Riemann surface" of 1986, Hitchin describes certain spaces parametrizing solutions of a two-dimensional reduction of the self-dual Yang-Mills equations of mathematical physics. He writes "…the moduli space of all solutions turns out to be a manifold with an extremely rich geometric structure." Quite an understatement! Since the inception of this theory, the Hitchin spaces have continued to play a fundamental role in several areas of mathematics and physics; notably, they were used by Ngô to complete the proof of the fundamental lemma of the Langlands program in number theory.

In one of its incarnations (to be precise, in one of its possible complex structures), a Hitchin space is isomorphic to a certain moduli space of representations of the fundamental group of a Riemann surface into the complex general linear group. This affine algebraic variety has, by the work of Deligne, an associated mixed Hodge structure. It was conjectured by Hausel, Letellier, and Rodriguez Villegas that the dimension of the graded pieces of this mixed Hodge structure can be determined by means of a generating function involving the Macdonald polynomials, which appear in combinatorics. The conjecture is a non-trivial, though natural, extension of the calculation of the number of points of the character variety over finite fields.

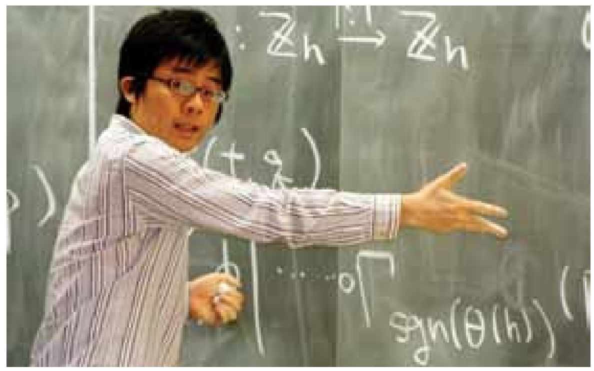
The Macdonald polynomials are symmetric functions in infinitely many variables depending on two parameters q and t. They were originally defined by Macdonald as a q-deformation of the Hall-Littlewood symmetric functions, themselves a t-deformation of the classical Schur functions. They proved to be central objects related to numerous aspects of representation theory and geometry. For example, by work of Garsia, Haiman, Nakajima, and others, the Macdonald polynomials are intimately tied to the geometry of the Hilbert scheme of n points in $C \times C$.

The connection with the geometry of character varieties, emphasized in this workshop, is intriguing and still rather mysterious. However, very recent work of Diaconescu and his collaborators suggests a direct link via the Hilbert scheme just mentioned.

The participants of the workshop were evenly divided among the three main areas of the theory of Macdonald polynomials: geometry, representation theory, and combinatorics. The goal was to bring them together to learn from each other and to foster the chance encounters and discussions that are crucial to the development of mathematics.



David Nadler


Nagao Kentaro

Organizers

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Olivier Schiffmann (Jussieu University)
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Selected CMI-Supported Conferences

Number Theory and Representation Theory: A conference in honor of the 60th birthday of Benedict Gross, Harvard University, Cambridge, Massachusetts by Richard Taylor

June 2 - 5, 2010

Number theory, which is concerned with the properties of whole numbers, is one of the oldest branches of mathematics, going back at least to the ancient Greeks, and quite possibly to the Babylonians. It combines old, simply stated problems and some of the most sophisticated modern mathematics. An archetypal example is Andrew Wiles' beautiful and sophisticated proof of Fermat's last theorem, over 350 years after Fermat first raised the problem. (This theorem asserts that no nth power of a non-zero integer is the sum of the nth powers of two other non-zero integers if n is at least 3.) Another example is the progress in the last thirty years on the congruent number problem, a 1,000-year-old question from Arabic mathematics that asks: which whole numbers are the areas of right-angled-triangles the length of all whose sides are rational numbers?

(Left to right) Mazur, Serre, Stark, Gross, Birch, Tate, Wallach, and Katz. (Photo by Jeff Mozzochi)

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Noam Elkies (Harvard)
Wee Teck Gan (UCSD)
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Richard Taylor (Harvard)
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Dipendra Prasad (Tata)
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Noam Elkies (Harvard)
Joseph Harris (Harvard)
Don Zagier (MPIM and Collège de France)

Representation theory, on the other hand, is a much younger subject with its origins in the work of Frobenius at the end of the nineteenth century. It is the study of symmetries of various sorts and the different ways the same abstract group of symmetries can be realized, often in quite different settings. For example, the Galois group of the polynomial $x^4 - 2$ (i.e., the symmetries or permutations of the roots of this polynomial that preserve all the rational polynomial relations between them) is the same as the usual (geometric) group of symmetries of a square. Or again, the group of even permutations of five letters is the same as the group of rotational symmetries of a regular dodecahedron. (This isomorphism is connected to the classical fact that five cubes can be inscribed in a dodecahedron: any symmetry of the dodecahedron induces a permutation of the five cubes.)

Representation theory became one of the great themes of twentieth-century mathematics (and of twentieth-century physics). In the latter half of the twentieth-century, extraordinary connections began to become apparent between number theory (or more particularly Galois theory) and the representation theory of various groups of geometric symmetries (for instance, groups of symmetries of hyperbolic space). This web of interrelations is often loosely referred

Compound of five cubes.

to as "The Langlands Program." It has greatly enriched both number theory and representation theory. Things that are fairly clear in one of these domains are often reflected by deep and unexpected results in the other. Much progress has been made in studying these connections, but even more still remains to be made.

In this conference we brought together experts in both number theory and representation theory to review recent developments and to try and build closer links between practitioners of the two subjects. We were very pleased that most of the speakers made a large effort to be comprehensible to a wide audience.

We also took the opportunity to celebrate Dick Gross' 60th birthday. He is one of the few mathematicians really at home in both these areas. We emphasized topics on which Dick has worked, including recent generalizations of the Gross-Zagier formula (which had important implications for the congruent number problem mentioned above) and questions in the representation theory of p-adic reductive groups inspired by the Galois theory of local fields.

Summer school on structures of local quantum field theory, Les Houches, France by Dirk Kreimer

June 7 - 25, 2010

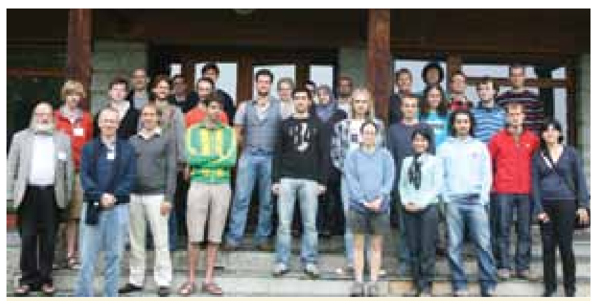
The three-week workshop/summer school on structures in local quantum field theory at Les Houches in June 2010 was a truly interdisciplinary effort, with speakers split between physics and mathematicians in equal terms, and similarly for participants. It focused on topics as diverse as:

- Perturbative quantum field theory, Hopf algebras, and renormalization;
- AdS/CFT correspondence and Britto-Cachazo-Feng-Witten recursions; and
- QFT and Dyson-Schwinger equations in gauge theory and quantum gravity.

In recent years, we have seen many new insights into the mathematical structure of renormalizable quantum field theories. Such theories still form the core of theoretical physics underwritten by their ability to predict the outcome of physics experiments. Comparison is made through tedious computational efforts by theoretical physicists. As an empirical fact, such efforts revealed mathematical structures that are at the center of contemporary mathematics research.

As a result, we have now a direct bridge between contemporary practice in theoretical physics, and research in algebraic geometry and the theory of mixed Hodge structures, with all of its motivic and number-theoretic flavor.

It was the task of this workshop to bring the relevant communities of physicists and mathematicians together, on a student as well as lecturer and researcher level, to build new bridges to the benefit of both. This was achieved through thirty-eight lectures, given by twelve speakers—six physicists and six mathematicians. We had forty-one participants, including the speakers.



Organizers

Dirk Kreimer (IHES)
Spencer Bloch (University of Chicago)
Francis Brown (Jussieu)

Speakers

Louis F. Alday (IAS, Princeton)
Spencer Bloch (Chicago University)
Johannes Bluemlein (DESY Theory, Zeuthen)
David Broadhurst (Open University)
Ruth Britto (CEA Saclay)
Francis Brown (CNRS, Math., Jussieu)
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Mathias Staudacher (AEI, Potsdam)
Matt Szczesny (Boston University)
Walter van Suijlekom (Radboud University Nijmegen)
Karen Yeats (Simon Fraser University)

Pacific Rim Workshop on Geometric Analysis, University of British Columbia and PIMS, Vancouver, Canada by Karen Manders

July 20 - 30, 2010

Participants of the Pacific Rim Workshop on Geometric Analysis at the University of British Columbia, Vancouver, BC, Canada.

More than seventy participants from all over the world were drawn to Vancouver, BC for this summer program, consisting of mini-courses and workshops. The first week explored topics such as the interplay between positive curvature, minimal surfaces, and the Ricci flow (Richard Schoen, Stanford University); a Kähler Ricci flow approach to the minimal model conjecture in algebraic geometry; and a new curvature flow on Hermitian manifolds (Gang Tian, Beijing University and Princeton University). Warner Ballmann (Bonn University and Max Planck Institute) lectured on Dirac operators on non-compact manifolds. Recent developments in the field were presented with overviews of the relevant background given for graduate students at the beginning of each lecture.

The second week featured a wide range of topics in geometric analysis, including geometric evolution, Willmore surfaces, conformal geometry, compactness of manifolds with a lower Ricci curvature bound, the Yamabe problem on orbifolds, and manifolds with boundary. The topics covered were among the most active and important areas in geometric analysis.

The workshop was organized by the Pacific Institute for the Mathematical Sciences (PIMS) Collaborative Research Group in Differential Geometry and Analysis. It was supported by PIMS, the National Science Foundation, and the Clay Mathematics Institute.

Organizers

Jingyi Chen (University of British Columbia)
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Jeff Viaclovsky (University of Wisconsin-Madison)
Yu Yuan (University of Washington)

Scientific Committee

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John Lott (UC Berkeley)
Toshiki Mabuchi (Osaka University)
Richard Schoen (Stanford)
Gang Tian (Beijing University and Princeton)

Mini-course speakers

Werner Ballmann (Max Planck Institute for Mathematics and University of Bonn)
Richard Schoen (Stanford)
Gang Tian (Beijing University and Princeton)

Workshop speakers

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Zheng-Chao Han (Rutgers)
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A conference in honor of the 70th birthday of Endre Szemerédi, Budapest, Hungary by Imre Bárány

August 2 - 7, 2010

This international conference, with twenty-five invited speakers and more than 250 participants from all over the world-celebrated Endre Szemerédi's 70th birthday.

Szemerédi is a mathematician with exceptional research power. His influence on today's mathematics is enormous. He solved several fundamental problems that had been raised decades earlier. Many of his results have generated research for the future, and have laid the foundation for new directions in mathematics. Some of his main achievements were born prematurely; their full power and significance became evident only decades later. Although Szemerédi's research interest is in combinatorics, number theory, and computer science, his influence on other fields of mathematics, such as ergodic theory and analysis, is remarkable.

One of his key results is a lemma, now called Szemerédi's regularity lemma, whose influence cannot be overestimated. It asserts that every graph can be partitioned into equal parts, whose number only depends on an error bound, so that the bipartite graph between any two such parts is "essentially random" (with a small number of exceptional parts). This statement is counterintuitive since the graph is completely deterministic, and not random. It shows that randomness is everywhere and inevitably present. Through the genius of Szemerédi, the mathematical community (and humankind) have had the opportunity to discover, appreciate, and put to use this ubiquitous and unavoidable presence of randomness.

The conference was a celebration of Szemerédi's achievements and personality. It exemplified his extraordinary vision and unique way of thinking. Topics included, among others, extensions and applications of the regularity lemma, the existence of k-term arithmetic progressions in various subsets of the integers, problems in graph and hypergraph theory, random graphs, additive combinatorics, and discrete geometry.

Endre Szemerédi (Photo by Boris Bukh)

Organizers

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Zoltán Füredi (UIUC)
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József Beck (Rutgers)
Béla Bollobás (University of Cambridge)
Mei-Chu Chang (University of California Riverside)
Zoltán Füredi (University of Illinois at Urbana-Champaign)
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Probability and Statistical Physics in Two (and More) Dimensions

July 11 - August 7, 2010

A report by Ivan Corwin (Courant, NYU), Marcelo Hilario (IMPA), and Adrien Kassel (ENS)

The sunny Brazilian peninsula of Buzios made for a perfect location for the 2010 Clay Mathematics Institute Summer School. The goal of the school was to provide a complete picture of a number of recent and groundbreaking developments in the study of probability and statistical physics in two and more dimensions. In the past ten to fifteen years, various areas of probability theory related to rigorous statistical mechanics, disordered systems, and combinatorics have enjoyed an intensive development with regards to two-dimensional random structures. Progress has come mainly in two forms: understanding large-scale properties of lattice-based models (on a periodic, deterministic lattice or in the case where the lattice is itself random), and directly constructing and manipulating continuous objects that describe these scaling limits. These themes guided the three foundational courses around which the first two weeks centered:

- Large random planar maps and their scaling limits by Jean-Francois Le Gall and Gregory Miermont
- SLE and other conformally invariant objects by Vincent Beffara
- Noise-sensitivity and percolation by Jeffrey Steif and Christophe Garban

Building on the foundations of the first two weeks, a variety of mini-courses covered very exciting and recent research:

- Random geometry and Gaussian free field by Scott Sheffield
- Conformal invariance of lattice models by Stanislav Smirnov
- Integrable combinatorics by Philippe Di Francesco
- Fractal and multifractal properties of SLE by Gregory Lawler
- The double-dimer model by Rick Kenyon

The fourth week of the school was held jointly with the XIV Brazilian School on Probability and focused on two main courses:

- Random polymers by Frank den Hollander
- Self-avoiding walks by Gordon Slade

Tutorials were organized for all courses and enabled the students to do hands-on work on the proofs of results mentioned in the lectures as well as to get familiar with numerous explicit examples. Teaching assistants were Curien, Duminil-Copin, and Freij for the fundamental courses and Alberts, Bauerschmidt, Caravenna, Goodman, Hongler, Pétrélis, and Werness for the mini-courses. Evening research talks supplemented the courses and mini-courses and were interspersed among the four weeks. The speakers included Adams, Benjamini, Biskup, Dubédat, Duplantier, Garcia, Ioffe, Koenig, Kozma, Le Jan, Maas, Mountford, Mytnik, Nolin, Peres, Sidoravicius, Turova, and van der Hofstadt. Students also organized a lunch-time seminar in which they could present their own work.

Much of the school was concerned with statistical physics models on lattices. Such models are random processes indexed by the vertices or the edges of a lattice, often considered to be a planar periodic graph (such as $\mathbf{Z}^2$). Each index point has a *spin* that takes values in a finite alphabet, typically {0,1}. The energy of a configuration of spins σ is given by a *Hamiltonian* $H(\sigma)$ and the probability of seeing σ is proportional to a *Gibbs factor* $e^{-\beta H(\sigma)}$ where β is called the *inverse temperature*. Different Hamiltonians give rise to different Gibbs measures and in particular to different behaviors for various natural *observables* such as interfaces between regions of 0's

Scientific Committee

David Ellwood (CMI)
Charles Newman (Courant, NYU)
Vladas Sidoravicius (IMPA & CWI)
Wendelin Werner (Université Paris-Sud 11)

Speakers

Vincent Beffara (ENS Lyon)
Philippe Di Francesco (CEA Saclay)
Christophe Garban (ENS Lyon)
Frank den Hollander (Leiden University)
Rick Kenyon (Brown University)
Jean-François Le Gall (Universite Paris-Sud)
Gregory Lawler (University of Chicago)
Gregory Miermont (Universite Paris-Sud)
Scott Sheffield (MIT)
Gordon Slade (University of British Columbia)
Stanislav Smirnov (Université de Genève)
Jeffrey Steif (Chalmers)

and 1's. A variety of such models was introduced in the last century by physicists to study the properties of matter. The *Ising model* is perhaps the most famous lattice model (and received ample attention during the school).

Bernoulli percolation is another important lattice model with a particularly simple Gibbs measure, namely, the product measure such that each index point has spin 0 or 1 independently with probability 1-p and p. All observables of this process can thus be expressed as Boolean functions of these spins. Garban and Steif's course focused on an innovative approach to the study of this model via the Fourier transform of these Boolean functions. Using results of theoretical computer scientists on the stability of Boolean functions, they studied the sensitivity of critical percolation to small perturbations. This *noise sensitivity* is measured in terms of the spectrum of the Boolean functions and gives precise estimates on the *influence* of different spins—essentially a measure of the contribution of the spin at a particular index point to the probability of an event. Key concepts of *pivotality* and *revealment* were introduced as was a dynamical version of percolation. In addition, Garban and Steif showed how randomized algorithms may be used to approximate percolation interfaces at low computation cost.

Complementing the discrete approach of Garban and Steif, much of the rest of the school focused on studying lattice models from the perspective of determining their scaling limits and deducing properties of the discrete models from these continuum limits. A long-held belief among statistical physicists is that scaling limits of critically tuned lattice models will display a great deal of universality with respect to perturbations of the lattice or model. For instance, it is believed that regardless of the lattice, the scaling limit of the interfaces between 0's and 1's in percolation with critically tuned probability p will converge (in law) to the same random collection of curves and that this limit will be *conformally invariant* (i.e., invariant in law under the action of conformal maps). Similar beliefs exist for other lattice models (like Ising). Significant progress was made about ten years ago with the introduction of the *Schramm-Loewner evolution* (SLE) which is a one-parameter family of measures on curves that should serve as the basis for the critical scaling limits of a variety of lattice models.

Beffara's course focused on rigorously defining these random curves and using them to describe the scaling limits and critical exponents (governing, for instance, correlation length and crossing probabilities) of a variety of lattice models. As an illustration of the power of these techniques, Beffara presented a proof (adapted from Smirnov's work) of Cardy's formula for the probability that there exists a connected path of 1's between two opposite sides of a large

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rectangle (in a particular lattice called the honeycomb lattice). Beffara also presented results on the geometry of the random curves, showing that the Hausdorff dimension of the SLE_κ is $1+\kappa/8$ (κ is the aforementioned parameter for this family of measures). Werner built upon this with some further techniques necessary to translate these continuum results into analogous statements about lattice models. Lawler went into more technical details about the path properties of SLE including the rigorous proof of their existence and Hölder continuity, as well as their natural time parameterization and the reverse Loewner flow.

In his mini-course, Smirnov explained the theory he has developed to prove scaling limits for a variety of models, which now includes both percolation and the Ising model (and more generally the *random-cluster* model). His approach emphasized the link between statistical physics in two dimensions and discrete complex analysis. In particular he presented a number of observables of models (such as his *parafermionic* observable) that can be shown to be discrete holomorphic (or preholomorphic). In certain cases these observables have been shown to converge to continuous holomorphic functions (as expected by the physics belief of conformal invariance of scaling limits). As Beffara had explained in his course, using this result as well as methods involving martingales, it is then possible to prove convergence of the entire interface of the lattice model to an SLE. To further emphasize the deep link between lattice models and complex analysis, Smirnov also gave a beautiful constructive proof of the Riemann mapping theorem via a discrete approximation using an appropriate measure on uniform spanning trees.

To round out the study of lattice models, Kenyon's mini-course and Dubédat's evening talk focused on the *dimer* model (related to perfect matchings in graph theory). In particular, Kenyon lectured on the *double-dimer* model, which provides a natural measure on non-intersecting loops. He gave evidence for the conformal invariance of the scaling limit of this model that is conjectured to be given by a variant of SLE called CLE_4 (the conformal loop ensemble that looks locally like SLE_4). Di Francesco studied a variety of other models in statistical physics using techniques from integrable—exactly solvable systems such as the Yang-Baxter equations, the transfer matrix approach, and formulas coming from representation theory.

Random discrete surfaces and Riemannian manifolds play essential roles in combinatorics and statistical physics, as does the study of lattice models on these surfaces. For example, significant progress in theoretical physics has been made in the last thirty years from the understanding that in string theory and gauge theory one should sum over random surfaces (as opposed to over random paths as in Feynman's formulation of quantum mechanics). Le Gall and Miermont approached this subject from the discrete side in their course on large random planar maps and their scaling limits. A *random map* provides a very natural approach to defining a discrete random surface and its accompanying metric. They explained an important line of recent progress in understanding the large limit of these random surfaces and metrics. Under an appropriate re-scaling of distances there exist (sub-sequential) limits of these discrete metrics that are called the *Brownian map*. Figuring prominently during the course was the so-called Bijective approach that emphasizes how these maps can be encoded and studied in terms of a correspondence with certain decorated plane trees.

On the continuum side, there exists another formulation (believed to be equivalent to the limit of the large planar maps) for a random geometry, which is called *Liouville quantum gravity*. This random geometry was the subject of the mini-course by Scott Sheffield and the evening talk of Bertrand Duplantier. Sheffield's course built on the foundational courses of both Le Gall and Miermont, as well as Beffara. Based on very recent work, Sheffield showed that SLE arises when gluing (via conformal welding) two random geometries together and then conformally mapping the result to the plane. Duplantier spoke of other exciting connections between statistical physics models on deterministic lattices and on random geometries. In particular he showed how to prove the KPZ formula that relates scaling exponents and fractal dimensions between the two types of geometries. This shows how, by studying lattice models in a random geometry, one can gain information about the geometry itself. In fact, in his evening talk, Benjamini emphasized this perspective by explaining how the study of two basic lattice models (percolation and random walks) on deterministic and random graphs provides a great deal of information in describing properties of the geometry itself.

The fourth week of the school (held jointly with the XIV Brazilian School on Probability) shifted focus to the study of random path measures on $\mathbf{Z}^d$. These paths are commonly called *polymers* and den Hollander's mini-course focused on the general theory of polymers, while Slade's mini-course delved deep into a particularly important polymer, called the *self-avoiding walk* (SAW). The SAW is a measure on paths of a fixed length that assigns equal probability to each nearest neighbor path starting at the origin and never intersecting itself. More generally, a random polymer is usually defined by a Gibbs measure (on the set of paths of a fixed length) with a Hamiltonian that may take into account the self-interactions, self-avoidance, and possibly interactions with a (possibly random) environment.

Many of the important questions about polymers and the SAW are expressed in terms of their asymptotic behavior, as their length goes to infinity. Important questions include studying the growth in the number of such collections of paths, the behavior of the mean square displacement, and the possibility of critical scaling limits. This last point drew us back to the subjects covered in the first three weeks of the school. In fact, Beffara had visited this subject in his course and explained that, while not rigorously established, if the scaling limit for the SAW in two dimensions exists and is conformally invariant, then it ought to be given by $SLE_{8/3}$, due to the fact that it naturally would inherit a certain *restriction property* from the discrete SAW and that this property is only verified by SLE_κ for the value $\kappa = 8/3$.

Much of Slade's course focused, however, on a large number of rigorous results about the critical behavior of the SAW in dimensions $d = 4$ and $d \geq 5$. He introduced a few critical exponents (governing how the SAW scales and behaves when very long) and explained the universality of these exponents, relations between them, and how they change according to the dimension. Slade then introduced the *Lace Expansion* and showed its convergence for sufficiently high dimension (which can be reduced to $d \geq 5$) which, in turn, implies that the number of length n SAWs grows purely exponentially, just as with the simple random walk. For $d = 4$, Slade proved exact functional integral representation formulas for the two-point function of the continuous weakly SAW and showed how, via a renormalization-group analysis, these formulas prove that the two-point function decays as the inverse square of distance.

Rather than totally excluding self-intersection, one may consider polymers whose Hamiltonian is a function of the number of self-intersections. In fact, a variety of other energetic rewards (or punishments) are important and studied. For example, the path can interact with a linear surface with energy dependent on random charges along the surface. Alternatively, the entire lattice may have charges, providing a random potential in which the path arranges itself. A critical simplification to these polymers used in den Hollander's course was to consider directed, or semi-directed, versions of polymers (thus avoiding some of the complexities of the SAW). In his course, large deviations served as a central tool to prove several recent results related to the existence of phase transitions in the behavior of polymers. These included results about collapse, localization, and pinning of polymers interacting with a linear surface, and about the diffusivity of the polymer endpoint.

Probability and statistical physics in two and more dimensions have recently benefited from the introduction of a variety of important and powerful new tools and techniques. The summer school was held at a perfect time: a few important problems in the field have recently been solved, but many other important open problems remain unsolved. It is likely that some of these will yield eventually to variants of these new tools and techniques. Thanks to the school, a new generation of mathematicians has been made aware of these problems and new approaches. Perhaps the main theme underlying this school was that there exist certain universal classes of continuum scaling limits that underlie and unite many discrete lattice models and random geometries. This theme will likely echo for many years in the work of those who participated in the 2010 Clay Mathematics Institute Summer School.

Acknowledgements: The scientific committee would especially like to thank Brazil's National Institute for Pure and Applied Mathematics (IMPA), its Director César Camacho, as well as the President of the Brazilian Academy of Sciences Jacob Palis Junior, and the President of CNPq at that time, Carlos Alberto Aragão de Carvalho Filho, for their help in organizing this large and complex school. Special mention goes to IMPA's exceptional Chief of Staff of the Department of Scientific Activities (DAC), Suely Lima, as well as her staff—Pedro Faro, Jurandira Ribas, Ana Paula Fonseca, Juliana Bressan, Sonia Maria Alves, Rafael de Souza, and Bruno Correia. The warm atmosphere they brought to the school, and the precise attention they gave to every detail, made for an enjoyable and memorable experience that we will never forget. We would also like to thank the staff at CMI, in particular, CMI's Program Manager Amanda Battese, Katherine Brack, and Vida Salahi, as well as the following organizations for their support, both direct and indirect: IMPA, CNPq, FAPERJ, CAPES, and grants Edital Universal and Prosul (V. Sidoravicius), the Fondation Cino del Duca (W. Werner), and the US National Science Foundation PIRE program grant OISE-0730136 to the Courant Institute of NYU (C. Newman). This additional support greatly enhanced the size and scope of the school, making it the largest ever undertaken by CMI.

Selected articles by Research Fellows

Mohammed Abouzaid
>> A geometric criterion for generating the Fukaya category, *Publications Mathématiques de l'IHES*, **112** (2010), no. 1, 191-240, arXiv:1001.4593v3.
>> Altering symplectic manifolds by homologous recombination, with P. Seidel, submitted, arXiv:1007.3281v3.

Spiridon Alexakis
>> The decomposition of global conformal invariants, *Annals of Mathematics Studies.*
>> Hawking's local rigidity theorem without analyticity, with A. Ionescu and S. Klainerman, *Geometric and Functional Analysis*, 2010, **20**, no. 4, 845-869.

Tim Austin
>> Sharp quantitative nonembeddability of the Heisenberg group into superreflexive Banach spaces, (2010), with Assar Naor and Romain Tessera, submitted, arXiv:1007.4238.
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>> Stable homology of automorphism groups of free groups, to appear in *Ann. of Math.* **173** (2011), no. 2, arXiv:0610.216v3.
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>> W*-superrigidity for Bernoulli actions of property (T) groups, *Journal of the AMS*, arXiv:1002.4595v3.
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>> Curves on K3 surfaces and modular forms, with R. Pandharipande and R.P. Thomas, to appear in *Journal of Topology*, arXiv:1001.2719v3.
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>> Quantum cohomology of the Springer resolution, with A. Braverman and A. Okounkov, to appear in *Advances in Mathematics*, arXiv:1001.0056v2.

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>> Note sur les polynomes de Kazhdan-Lusztig, to appear in *Mathematische Zeitschrift*, arXiv:0603.519v1.
>> Cohomologie d'intersection des varietes modulaires de Siegel, suite, *Compositio Mathematica*. Pre-publication.
>> The intersection complex as a weight truncation and an application to Shimura varieties, *Proceedings of the ICM*, 2010.

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>> Boundary complexes and weight filtrations, in submission.
>> A tropical proof of the Brill-Noether Theorem, with F. Cools, J. Draisma, and E. Robeva, submitted, arXiv:1001.2774v2.

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>> Maslov index formulas for Whitney n-gons, to appear in *Journal of Symplectic Geometry*, arXiv:0609.673v3.
>> A note on sign conventions in link Floer homology, to appear in *Quantum Topology*, arXiv:1002.0918v1.

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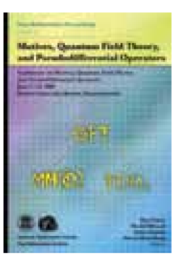
>> Calabi-Yau Theorem and Algebraic Dynamics, with Shou-wu Zhang, *Inventiones Mathematicae*. September 5, 2010.
>> Small Points and Berkovich Metrics, with Shou-wu Zhang, *Journal of Algebraic Geometry*. September 5, 2010.

Books & videos

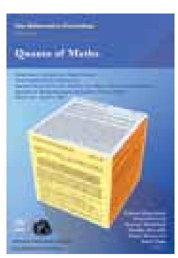
Books



Motives, Quantum Field Theory, and Pseudodifferential Operators
Editors: Alan Carey, David Ellwood, Sylvie Paycha, Steven Rosenberg. CMI/AMS, 2010, 349 pp. www.claymath.org/publications/Motives_Quantum/

This volume contains articles related to the conference "Motives, Quantum Field Theory, and Pseudodifferential Operators" held at Boston University in June 2008, with partial support from the Clay Mathematics Institute, Boston University, and the National Science Foundation. There are deep but only partially understood connections between the three conference fields, so this book is intended both to explain the known connections and to offer directions for further research.

Quanta of Maths; Proceedings of the Conference in honor of Alain Connes
Editors: Etienne Blanchard, David Ellwood, Masoud Khalkhali, Matilde Marcolli, Henri Moscovici, Sorin Popa. CMI/AMS, 2010, 675 pp. www.claymath.org/publications/Quanta_Maths/



The work of Alain Connes has cut a wide swath across several areas of mathematics and physics. Reflecting its broad spectrum and profound impact on the contemporary mathematical landscape, this collection of articles covers a wealth of topics at the forefront of research in operator algebras, analysis, noncommutative geometry, topology, number theory, and physics.

Homogeneous Flows, Moduli Spaces and Arithmetic; Proceedings of the CMI 2007 Summer School
Editors: Manfred Einsiedler, David Ellwood, Alex Eskin, Dmitry Klein, Elon Lindenstrauss, Gregory Marguli, Stefano Marmi, Jean-Christophe Yoccoz. CMI/AMS, 2010, 438 pp. www.claymath.org/publications/Homogeneous_Flows/

This book contains a wealth of material concerning two very active and interconnected directions of current research at the interface of dynamics, number theory, and geometry. Examples of the dynamics considered are the action of subgroups of SL(n,R) on the space of unit volume lattices in Rn and the action of SL (2, R) or its subgroups on moduli spaces of flat structures with prescribed singularities on a surface of genus >= 2.

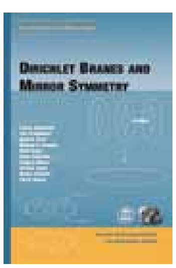
The Geometry of Algebraic Cycles; Proceedings of the Conference on Algebraic Cycles
Editors: Reza Akhtar, Patrick Brosnan, Roy Joshua. CMI/AMS, 2010, 187 pp. www.claymath.org/publications/Algebraic_Cycles/

The subject of algebraic cycles has its roots in the study of divisors, extending as far back as the nineteenth century. Since then, and in particular in recent years, algebraic cycles have made a significant impact on many fields of mathematics, among them number theory, algebraic geometry, and mathematical physics. The present volume contains articles on all of the above aspects of algebraic cycles.

Arithmetic Geometry; Proceedings of the 2006 CMI Summer School at Gottingen
Editors: Henri Darmon, David Ellwood, Brendan Hassett, Yuri Tschinkel. CMI/AMS, 2009, 562 pp. www.claymath.org/publications/Arithmetic_Geometry.

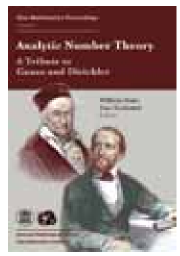
This book is based on survey lectures given at the 2006 CMI Summer School at the Mathematics Institute of the University of Gottingen. It will introduce readers to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

Dirichlet Branes and Mirror Symmetry
Editors: Michael Douglas, Mark Gross. CMI/AMS, 2009, 681 pp. www.claymath.org/publications/Dirichlet_Branes.



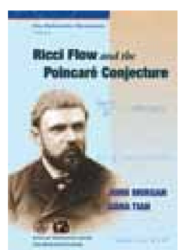
The book first introduces the notion of Dirichlet brane in the context of topological quantum field theories, and then reviews the basics of string theory. After showing how notions of branes arose in string theory, it turns to an introduction to the algebraic geometry, sheaf theory, and homological algebra needed to define and work with derived categories. The physical existence conditions for branes are then discussed, culminating in Bridgeland's definition of stability structures. The book continues with detailed treatments of the Strominger-Yau-Zaslow conjecture, Calabi-Yau metrics, and homological mirror symmetry, and discusses more recent physical developments.

Analytic Number Theory: A Tribute to Gauss and Dirichlet
Editors: William Duke, Yuri Tschinkel. CMI/AMS, 2007, 265 pp. www.claymath.org/publications/Gauss_Dirichlet.



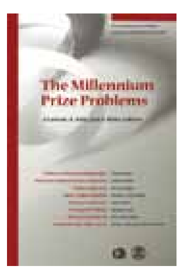
This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20-24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.

Ricci Flow and the Poincaré Conjecture
Authors: John Morgan, Gang Tian. CMI/AMS, 2007, 521 pp. www.claymath.org/publications/ricciflow.

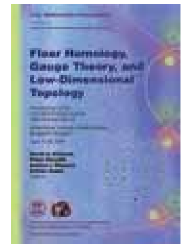


This book presents a complete and detailed proof of the Poincaré conjecture. This conjecture was formulated by Henri Poincaré in 1904 and has remained open until the recent work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three preprints.

The Millennium Prize Problems
Editors: James Carlson, Arthur Jaffe, Andrew Wiles. CMI/AMS, 2006, 165 pp. www.claymath.org/publications/Millennium_Problems.

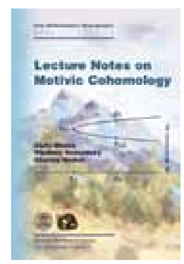


This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.



Floer Homology, Gauge Theory, and Low-Dimensional Topology; Proceedings of the CMI 2004 Summer School at Rényi Institute of Mathematics, Budapest
Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo. CMI/AMS, 2006, 297 pp. www.claymath.org/publications/Floer_Homology.

This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.



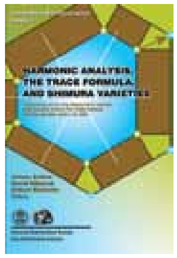
Lecture Notes on Motivic Cohomology
Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. CMI/AMS, 2006, 210 pp. www.claymath.org/publications/Motivic_Cohomology.

This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to motivic cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology, and Chow groups.

Surveys in Noncommutative Geometry
Editors: Nigel Higson, John Roe. CMI/AMS, 2006, 189 pp. www.claymath.org/publications/Noncommutative_Geometry.

In June of 2000, a summer school on noncommutative geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.

Harmonic Analysis, the Trace Formula and Shimura Varieties; Proceedings of the 2003 CMI Summer School at Fields Institute, Toronto

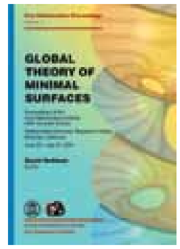


Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp. www.claymath.org/publications/Harmonic_Analysis.

The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or to the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

Global Theory of Minimal Surfaces; Proceedings of the 2001 CMI Summer School at MSRI

Editor: David Hoffman. CMI/AMS, 2005, 800 pp. www.claymath.org/publications/Minimal_Surfaces.

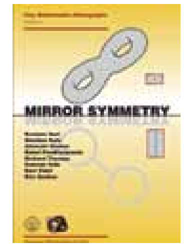


This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and the double-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.

Strings and Geometry; Proceedings of the 2002 CMI Summer School held at the Isaac Newton Institute for Mathematical Sciences, UK

Editors: Michael Douglas, Jerome Gauntlett, Mark Gross. CMI/AMS, 376 pp., paperback, ISBN 0-8218-3715-X. List: $69. AMS Members: $55. Order code: CMIP/3. To order, visit www.ams.org/bookstore.

Mirror Symmetry

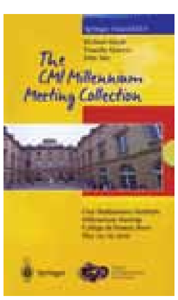


Authors: Kentaro Hori, Sheldon Katz, Albrecht Klemm, Rahul Pandharipande, Richard Thomas, Ravi Vakil. Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS, 929 pp., hardcover, ISBN 0-8218-2955-6. List: $124. AMS Members: $99. Order code: CMIM/1. To order, visit www.ams.org/bookstore.

Strings 2001

Authors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. Tata Institute of Fundamental Research. Editor: American Mathematical Society (AMS), 2002, 489 pp., paperback, ISBN 0-8218-2981-5. List: $74. AMS Members: $59. Order code: CMIP/1. To order, visit www.ams.org/bookstore.

Videos



The CMI Millennium Meeting Collection

Authors: Michael Atiyah, Timothy Gowers, John Tate, François Tisseyere. Editors: Tom Apostol, Jean-Pierre Bourguignon, Michele Emmer, Hans-Christian Hege, Konrad Polthier. Springer VideoMATH, Clay Mathematics Institute, 2002.

Box set consists of four video cassettes: *The CMI Millennium Prize Problems, a lecture by Michael Atiyah*; and *The Millennium Prize Problems, a lecture by John Tate*.

VHS/NTSC or PAL. ISBN 3-540-92657-7. List: $119, EUR 104.95. To order, visit www.springer.com (in the U.S.) or www.springer.de (in Europe).

These videos document the Paris meeting at the Collége de France where CMI announced the Millennium Prize Problems. The videos are for anyone who wants to learn more about these seven grand challenges in mathematics. Videos of the 2000 Millennium event are available online and in VHS format from Springer-Verlag. To order the box set or individual tapes, visit www.springer.com.

2011 Institute Calendar

Date	Event	Location
January 1 - December 31	Independent University of Moscow	Moscow, Russia
January 3 - 7	DRIP: Density of Rational and Integral Points	Ein Gedi Field School, Israel
January 4 - July 1	Moduli Spaces	Isaac Newton Institute for Mathematical Sciences, Cambridge, UK
January 10 - May 20	Senior Scholars Henryk Iwaniec and Barry Mazur, "Arithmetic Statistics"	MSRI
February 20 - 25	Frontiers in Complex Dynamics (Celebrating John Milnor's 80th Birthday)	BIRS and Banff Conference Center, Alberta, Canada
March 22 - 25	K-theory and Motives	UCLA, Los Angeles, CA
May 16 - 21	Conference in Honor of Elias Stein's 80th Birthday	Princeton University
May 16 - 20	Trends in Complex Dynamics	CMI, Cambridge, MA
June 20 - Aug 12	Ross Program	Ohio State University, Columbus, OH
July 2 - 12	Modern Mathematics for International Students	Bremen, Germany and France
July 3 - 23	Senior Scholars Joe Harris and Dennis Sullivan, "Moduli Spaces of Riemann Surfaces"	PCMI, Park City, Utah
July 4 - August 13	PROMYS	Boston University, Boston, MA
July 11 - 15	Geometry and Topology Down Under (Summer School)	University of Melbourne
July 18 - 22	Geometry and Topology Down Under (Conference)	University of Melbourne
August 1 - 5	Latin American School of Algebraic Geometry and Applications, Graduate Courses	Buenos Aires, Argentina
August 8 - 12	Latin American School of Algebraic Geometry and Applications, Research Workshop	Córdoba, Argentina
August 15 - 19	Research Scholar Michael Harris, workshop on "Automorphic Forms, Galois Representations, and Geometric Representation Theory"	Instituto Balseiro-Centro, Atómico Bariloche, Argentina
August 15 - December 16	Senior Scholars Keith Ball, Tobias Colding, and William Johnson, "Quantitative Geometry"	MSRI
August 25 - 28	A Celebration of Algebraic Geometry: A Conference in Honor of Joe Harris' 60th Birthday	Harvard University
August 29 - September 1	Logarithmic Geometry and Moduli Workshop	CMI, Cambridge, MA
November 19 - December 13	Algebraic vs. Analytic Geometry	ESI, Vienna, Austria
2010 - 2011	Research Scholar Daniel Allcock	Kyoto University

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One Bow Street, Cambridge, Massachusetts 02138