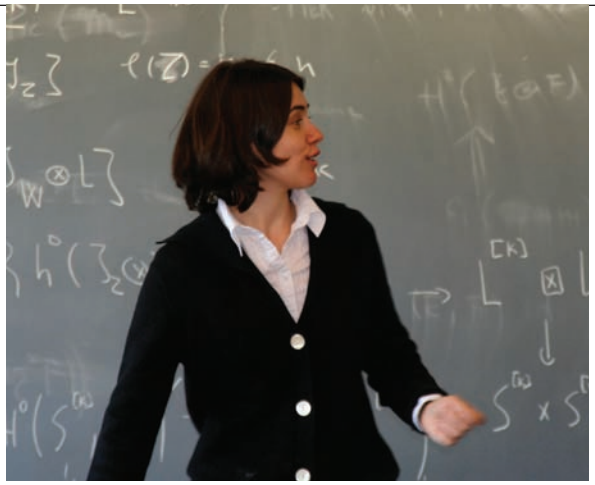
# CMI Workshops



**Organizers:** David Ellwood, Rahul Pandharipande, and Davesh Maulik

**Participants:**
Kai Behrend (University of British Columbia)
Jim Bryan (University of British Columbia)
Ron Donagi (University of Pennsylvania)
Lothar Gottsche (ICTP)
Joe Harris (Harvard University)
Brendan Hassett (Rice University)
Albrecht Klemm (University of Wisconsin)
Conan Leung (The Institute of Mathematical Sciences)
Eduard Looijenga (Utrecht University)
Davesh Maulik (CMI)
Alina Marian (Institute of Advanced Study)
Greg Moore (Rutgers University)
Rahul Pandharipande (Princeton University)
Tony Pantev (University of Pennsylvania)
Emanuel Scheidegger (University of Eastern Piedmont)
Domingo Toledo (University of Utah)
Yuri Tschinkel (New York University)
Wei Zhang (Columbia University)
Aleksey Zinger (State University of New York)

## K3s: Modular Forms, Moduli, and String Theory

### March 20 - 23, 2008

*by Rahul Pandharipande (Princeton University) and Davesh Maulik (CMI)*

The main goal of this workshop was to bring together mathematicians from various areas related to the study of K3 surfaces and their moduli. Research over the last twelve months had produced results and conjectures in the form of identities involving modular forms, hypergeometric series, and K3 moduli on the one hand, and the geometry of classical Noether-Lefschetz loci in the moduli of K3s on the other. A narrower goal was to make progress on a circle of rather concrete conjectures arising from this interaction.

The workshop consisted of fourteen research talks centered on the geometry of K3 surfaces. The group of participants included a successful mix of string theorists, complex geometers, and arithmetic geometers. Although the differences in language and backgrounds covered were large, the workshop fostered productive dialogue. The narrow goal of making progress on concrete conjectures was also met. Organizers R. Pandharipande and D. Maulik, together with A. Klemm and E. Scheidegger, succeeded to prove the Yau-Zaslow conjecture about rational curves on K3 surfaces for all (not necessarily primitive) curve classes using a combination of techniques motivated by the workshop. (See "Noether-Lefschetz theory and the Yau-Zaslow conjecture," A. Klemm, D. Maulik, R. Pandharipande, and E. Scheidegger; arXiv: 0807.2477v1 [math.AG].)

The organization and infrastructure of the CMI were instrumental in the progress made at the workshop. Through the effort of its staff, the focus of the participants was entirely mathematical; moreover, the environment was extremely conducive for collaboration.

A list of abstracts for all the talks and other information can be found at:

**www.claymath.org/workshops.**

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## Automorphic Forms in Moduli Problems of Schottky and Brill-Noether Type

### March 28-30, 2008

*by Emma Previato, Professor of Mathematics, Boston University*

This workshop addressed the dual nature of certain special functions, in their dependence on moduli as automorphic forms, and their dependence on a Fourier-Mukai dual variable, which describes a moduli space of bundles, theta functions being the prime example. A distinctive feature of the workshop was that of bringing together experts of the analytic and of the algebraic techniques, to discuss open problems in these two somewhat separate areas, rooted in classical (nineteenth-century) mathematics and bearing on cutting-edge issues such as renormalization for superstring amplitudes in mathematics physics.

The authors of the foundational "Theta constants, Riemann surfaces and the modular group" (American Mathematical Society, 2001), H.M. Farkas and I. Kra, were both present. Farkas and Kopeliovich talked about recent work on Thomae's formulas for non-hyperelliptic curves, generalizing the classical expression for the Weierstrass points of the curve in terms of thetanulls: Thomae's formulas are crucial in the study of the KZB (Knizhnik–Zamolodchikov–Bernard) connection. Kra led the "open questions" session, proposing the theme of a stratification of Teichmüller spaces by hyperbolic geodesics. Prymian theta functions and varieties also figured prominently. Krichever, whose work recently settled the Schottky problem for Jacobians and certain Prymians using linear differential equations in two variables, a startling discovery, proposed new equations for Prym theta functions, and Farrington talked about the Klein curve and her thesis project of applying to it the Prymian construction of an AgM (Arithmetic-geometric-Mean) in genus three, due to R. Donagi and R. Livné (*Ann. Scuola Norm. Sup. Pisa Cl. Sci.* (4) 28 (1999), 323-339).

**Organized by David Ellwood and Emma Previato**

**Participants**

Hershel Farkas (The Hebrew University of Jerusalem and Stony Brook)
Eleanor Farrington (Boston University)
Samuel Grushevsky (Princeton University)
Jay Jorgenson (The City College of New York)
Yaacov Kopeliovich (MEAG/Munich Reinsurance, New York, NY)
Irwin Kra (Stony Brook University)
Igor Krichever (Columbia University)
Alan Mayer (Brandeis University)
Emma Previato (Boston University)

As regards subloci of moduli space defined by thetanulls, Grushevsky gave an exposition of his recent characterization of genus-4 Jacobians with a vanishing thetanull (joint with R. Salvati Manni). Mayer presented a highly innovative and provocative set of ideas that go back to his work with A. Andreotti in the 1960s, for example, a conjectural stratification of the moduli space of Riemann matrices by rank of quadrics viewed as modular forms; ideas for a proof include degeneration and Heisenberg-group actions. Jorgenson rightly concluded the workshop celebrating the analytic origin of the theta function through kernel functions for the Laplacian on a Riemann surface, and computing modern invariants (e.g. Arakelov metric, Falting's delta function) by a hyperbolic kernel for certain modular curves.

The synergy that filled the Institute's hospitable headquarters never flagged, conjectures and proofs were reached by the interplay of geometry and analysis, and Jorgenson gave an informal presentation of his work in progress, "Zeta functions, heat kernels and spectral asymptotics on degenerating families of discrete tori" (arXiv:0806.2014 joint with G. Chinta and A. Karlsson). R.D.M. Accola, B. Harris, R.C. Gunning, I. Dolgachev, and L. Takhtajan had accepted invitations but were unable to attend.

For more information, please see:

**www.claymath.org/workshops/**