Clay Research Conference

The second Clay Research Conference, an event devoted to recent advances in mathematical research, was held at MIT on May 12 and 13 at the MIT media lab (Bartos Theatre). The lectures, listed on the right, covered a wide range of fields: algebraic geometry, symplectic geometry, dynamical systems, geometric analysis, and probability theory.

Conference speakers were Kevin Costello (Northwestern University), Helmut Hofer (Courant Institute, NYU), János Kollár (Princeton University), Tom Mrowka (MIT), Assaf Naor (Courant Institute, NYU), Rahul Pandharipande (Princeton University), Scott Sheffield (Courant Institute, NYU), and Claire Voisin (CNRS, IHÉS, and Inst. Math. Jussieu). Abstracts of their talks are given below. Videos of the talks are available on the Clay Mathematics Institute web site, at **www.claymath.org/publications/videos.**

On the afternoon of May 12, the Clay Research Awards were presented to Claire Voisin and to Cliff Taubes. The citations read:

Cliff Taubes (Harvard University) for his proof of the Weinstein conjecture in dimension three.

Claire Voisin (CNRS, IHÉS, and Inst. Math. Jussieu) for her disproof of the Kodaira conjecture.

The Clay Research Award is presented annually to recognize major breakthroughs in mathematical research. Awardees receive the bronze sculpture "Figureight Knot Complement VII/CMI" by artist-mathematician Helaman Ferguson. They also receive flexible research support for a period of one year.

Tom Mrowka delivering his talk at the conference.

Clay Research Awards

Previous recipients of the award, in reverse chronological order are:

2007	Alex Eskin (University of Chicago) Christopher Hacon (University of Utah) and James McKernan (UC Santa Barbara) Michael Harris (Université de Paris VII) and Richard Taylor (Harvard University)
2005	Manjul Bhargava (Princeton University) Nils Dencker (Lund University, Sweden)
2004	Ben Green (Cambridge University) Gérard Laumon (Université de Paris-Sud, Orsay) Bao-Châu Ngô (Université de Paris-Sud, Orsay)
2003	Richard Hamilton (Columbia University) Terence Tao (University of California, Los Angeles)
2002	Oded Schramm (Theory Group, Microsoft Research) Manindra Agrawal (Indian Institute of Technology, Kanpur)
2001	Edward Witten (Institute for Advanced Study) Stanislav Smirnov (Royal Institute of Technology, Stockholm)
2000	Alain Connes (College de France, IHES, Vanderbilt University) Laurent Lafforgue (Institut des Hautes Études Scientifiques)
1999	Andrew Wiles (Princeton University)

The Clay Mathematics Institute presents the Clay Research Award annually to recognize major breakthroughs in mathematical research. Awardees receive the bronze sculpture "Figureight Knot Complement vii/CMI" by Helaman Ferguson and are named Clay Research Scholars for a period of one year. As such they receive substantial, flexible research support. Awardees have used their research support to organize a conference or workshop, to bring in one or more collaborators, to travel to work with a collaborator, and for other endeavors.

Clay Research Conference

Abstracts of Talks

Kevin Costello (Northwestern University)
A Wilsonian point of view on renormalization of quantum field theories

A conceptual proof of renormalizability of pure Yang-Mills theory in dimension four was given based on an approach to Wilson's effective action and the Batalin-Vilkovisky formalism.

Helmut Hofer (Courant Institute, NYU)
A generalized Fredholm theory and some new ideas in nonlinear analysis and geometry

The usual notion of differentiability in infinite-dimensional Banach spaces is Fréchet differentiability. It can be viewed as a straightforward generalization of the finite-dimensional notion. The important feature of Fréchet differentiability is the validity of the chain rule. However, there are different generalizations, and a new one is sc-differentiability. This notion also has a chain rule. Sc-smoothness requires additional structure on the Banach spaces, so-called sc-structure.

The striking difference between "Fréchet-smooth" and "sc-smooth" can be seen when studying maps $r : U \to U$, satisfying $r \circ r = r$, i.e., retractions, where U is an open subset of a Banach space. If r is Fréchet-smooth, then $r(U)$ is necessarily a submanifold of U. However, there are sc-smooth examples where $r(U)$ is finite-dimensional, but has locally varying dimension. There are also examples where a connected $r(U)$ has finite-dimensional as well as infinite-dimensional parts. If we consider pairs (O, E), where O is a subset of the sc-Banach space E and is also the image of an sc-smooth retraction, we obtain new local models for smooth spaces. We even can define the tangent of $T(O, E)$ by (TO, TE), where $TO = T r(T, U)$. Noting that by the chain rule $T r \circ Tr = Tr$ we see that TO is again an sc-smooth retraction. As it turns out, the definition does not depend on r as long as O is the image of r. We also can define the sc-smooth maps between local sc-models. Evidently, many constructions known from differential geometry can be carried over to a new "sc-retraction based differential geometry". Manifolds become M-polyfolds and orbifolds become polyfolds.

Helmut Hofer delivering his talk at the conference.

A nonlinear elliptic differential operator can usually be interpreted as a Fredholm section of a Banach space bundle and, given enough compactness, can be studied topologically. Many interesting problems in geometry are related to elliptic problems that show a lack of compactness, like bubbling-off. However, these problems usually have fancy compactifications. Two such problems of interest are Gromov-Witten theory and the more general symplectic field theory. Due to serious compactness and transversality issues, it is difficult to study them in a classical Banach manifold set-up. However, it turns out that they are much more easily described in sc-retraction-based differential geometry.

Finally, there is a generalization of the classical nonlinear Fredholm theory to the sc-world, which also has a built-in Sard-Smale-type perturbation and transversality theory. In its applications to symplectic field theory the solution spaces are the compactified moduli spaces.

János Kollár (Princeton University)
Local Integrability of holomorphic functions

Question: Let $f(z_1,\ldots,z_n)$ be a holomorphic function on an open set $U \subset \mathbb{C}^n$. For which $s \in \mathbb{R}$ is $|f|^{-s}$ locally integrable? It is not hard to see that there is a largest value s_0 (depending on f and p) such that $|f|^{-s}$ is integrable in a neighborhood of p for $s < s_0$ but not integrable for $s > s_0$. Our aim is to study this "critical value" s_0. Subtle properties of these critical values are connected with Mori's program (especially the termination of flips), with the existence of Kähler-Einstein metrics in the positive curvature case and many other topics.

Tom Mrowka (MIT)
Monopoles, closed Reeb orbits and spectral flow: Taubes' work on the Weinstein conjecture

We survey Cliff Taubes' recent proof of the Weinstein conjecture in dimension three and related topics. Taubes shows how to construct periodic orbits of Reeb vector fields on contact three manifolds from special cycles in the Seiberg-Witten Monopole Floer homology. The proof follows ideas from Taubes' work relating the Seiberg-Witten and Gromov invariants of four-manifolds but with a new twist. It hinges on new results describing the asymptotic behavior of spectral flow for Dirac type operators.

Assaf Naor (Courant Institute, NYU)
Probabilistic reasoning in quantitative geometry

Many problems of an asymptotic and quantitative nature in geometry have recently been solved using a variety of probabilistic tools. Apart from the classical use of the probabilistic method to prove existence results, it turns out thinking "probabilistically," or interpreting certain geometric invariants in a probabilistic way, is a powerful way to bound a variety of geometric quantities. This talk is devoted to surveying the ways in which probabilistic reasoning plays a sometimes unexpected role in topics such as bi-Lipschitz and uniform embedding theory, extension problems for Lipschitz maps, metric Ramsey problems, harmonic analysis, and theoretical computer science. We will show how random partitions of metric spaces can be used to embed them in normed spaces, find large Euclidean subsets, extend Lipschitz functions, and bound the weak (1,1) norm of maximal functions. We will also discuss the role of random projections, and describe the connections between the behavior of Markov chains in metric spaces and Lipschitz extension, lower bounds for bi-Lipschitz embeddings, and the computation of compression exponents for discrete groups.

Rahul Pandharipande (Princeton University)
Curve counting via stable pairs in the derived category

Let X be a projective 3-fold. We construct a moduli space of stable pairs in the derived category of X with a well-defined enumerative geometry. The enumerative invariants are conjectured to be equivalent to the Gromov-Witten theory of X. The geometry is a very natural place to study recent derived category wall-crossing formulae. Fibrations of K3 surfaces provide computable examples. Connections to work of Kawai-Yoshioka and the Yau-Zaslow formula for enumerating rational curves on K3 surfaces are made.

This is joint work with R. Thomas.

Scott Scheffield (Courant Institute, NYU)
Quantum gravity and the Schramm-Loewner evolution

Many "quantum gravity" models in mathematical physics can be interpreted as probability measures on the space of metrics on a Riemannian manifold. We describe several recently derived connections between these random metrics and certain random fractal curves called "Schramm-Loewner evolutions" (SLE).

Claire Voisin (CNRS, IHÉS and Inst. Math. Jussieu)
Hodge structures, cohomology algebras and the Kodaira problemn

We show that there exist, starting from complex dimension 4, compact Kähler manifolds whose cohomology algebra is not that of a projective complex manifold. In particular their complex structure does not deform to that of a projective manifold, while Kodaira proved that compact Kähler surfaces deform to projective ones. The argument uses the notion of Hodge structure on a cohomology algebra, and exhibits algebraic obstructions for the existence of such Hodge structure admitting a rational polarization. We will also explain further applications of this notion, e.g. the fact that cohomology algebras of compact Kähler manifolds are strongly restricted amongst cohomology algebras of compact symplectic manifolds satisfying the hard Lefschetz property.