Julia Robinson

And Hilbert's Tenth Problem

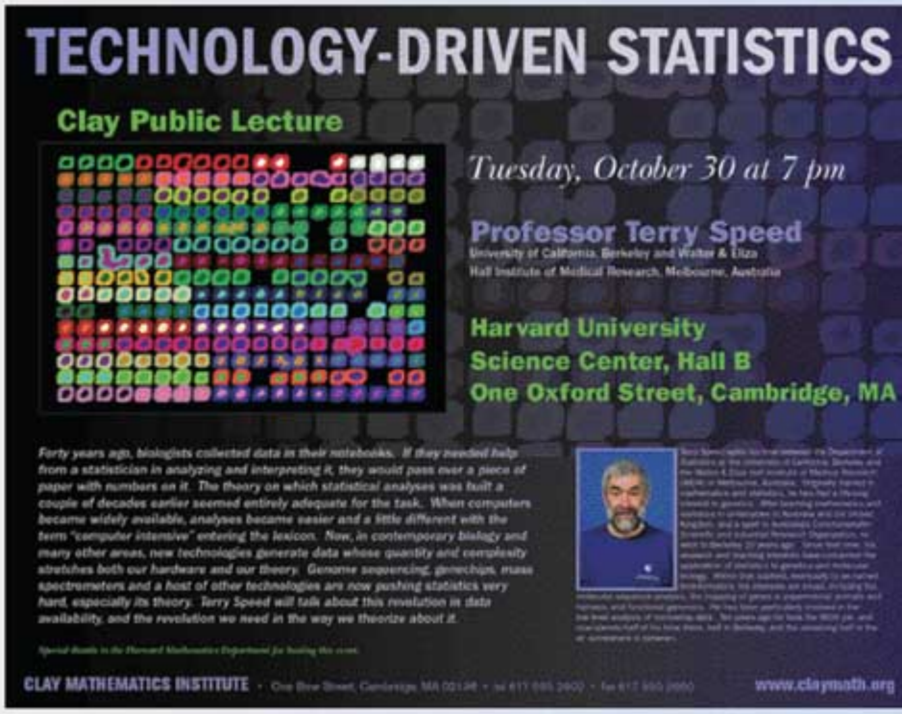
Annual Report 2007

Interview with Research Fellow Mircea Mustata

Julia Robinson and Hilbert's Tenth Problem

Clay Lectures on Mathematics at TIFR, Mumbai

CLAY MATHEMATICS INSTITUTE



Each year CMI presents two public lectures in Cambridge. The lectures are intended for a broad audience: not just mathematicians and students of mathematics, but anyone with an interest in what mathematics is and the role it plays in our intellectual life, in science, and in technology. Lectures in previous years have been *Is there such a thing as infinity*, by Tim Gowers, of Cambridge University; *Are there unsolved problems about numbers*, by Barry Mazur, of Harvard University; *Four thousand years of mathematics in images*, by Bill Casselman, of the University of British Columbia; *Escher and the Droste Effect*, by Hendrik Lenstra, of Leiden University; *Beyond computation*, by Michael Sipser, of MIT; *Mathematics and Magic Tricks*, by Persi Diaconis, of Stanford University.

In 2007–08, the first public lecture, *Surfing with Wavelets*, was delivered by Ingrid Daubechies of Princeton. In her talk, which took place at the MIT Ray and Maria Stata Center on April 10, Daubechies explained how wavelets deconstruct sounds and images into a mathematical analog of a musical score, and how they can be used in a multitude of ways, from the restoration of old recordings to the study of birdsong, fingerprints, and earthquakes. The second lecture, *Technology-driven Statistics*, by Australian statistician Terry Speed of UC Berkeley and WEHI, was held on October 30 at Harvard University. In his talk, Speed spoke about how advances in biotechnology is pushing statistical theory to its limits, and why he thinks a new paradigm is needed.

Nominations, Proposals and Applications

Nominations for Senior and Research Scholars are considered four times a year at our Scientific Advisory Board (SAB) meetings. Principal funding decisions for Senior Scholars are made at the September SAB meeting. Additional nominations will be considered at other times as funds permit. Clay Liftoff and Clay Research Fellow nominations are considered once a year and must be submitted according to the schedule below:

Nomination Deadlines

Senior Scholars: August 1
Research Fellows: October 30
Liftoff Scholars: February 15

Address all nominations to the attention of the CMI Program Manager at nominations @claymath.org.

Nominations can also be mailed to:

Clay Mathematics Institute
One Bow Street
Cambridge, MA 02138

The Clay Mathematics Institute invites proposals for conferences and workshops. Proposals, which need not be long, will be judged on their scientific merit, probable impact, and potential to advance mathematical knowledge.

Our budget is often committed well in advance, so please submit applications at your earliest convenience.

A budget and standard cover sheet should be sent well in advance to the attention of the CMI Program Manager at proposals@claymath.org.

Proposal Deadlines

Workshops & Conferences: August 1, February 15
Bow Street Workshops: 6 months prior

Proposals may also be mailed to:

Clay Mathematics Institute
One Bow Street
Cambridge, MA 02138

Please find more information and the standard cover sheet at

www.claymath.org/proposals.

Noteworthy proposals will be considered at other times. However, most funding decisions will be made with respect to the deadlines above.

Clay Mathematics Institute

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2007

James Carlson

Dear Friends of Mathematics,

In 2007 the Clay Mathematics Institute made major changes in the format and content of its Annual Meeting. The date was changed from November to May, and the program was expanded to a two-day series of lectures on recent research developments. Presentation of the Clay Research Awards, which had always been the focus of the Annual Meeting, took place on the afternoon of the first day. The 2007 Research Conference was held at Harvard on May 14 and 15. The 2008 Conference will be held at MIT on May 12 and 13. As in the past, the conference will alternate between MIT and Harvard. Videos of the 2007 lectures are posted on the CMI website, and future talks will appear there as well.

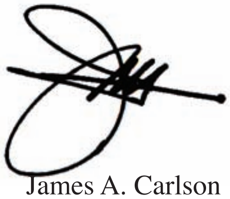
The inaugural Clay Research Conference marked an exceptional year for mathematics. Christopher Hacon and James McKernan received the Clay Research Award for their inductive proof of the existence of flips for algebraic varieties in dimension greater than three. This is a crucial step towards realization of the Mori minimal model program in all dimensions, which is one of the great outstanding problems in algebraic geometry. Michael Harris and Richard Taylor received the award for their proof of the Sato–Tate conjecture for elliptic curves with non-integral j-invariants. And Alex Eskin received the award for his proof of the quasi-isometric rigidity of sol, a longstanding problem in geometric group theory. These results follow closely other recent exceptional events in mathematics, e.g., Perelman's solution of the Poincaré conjecture and the proof by Ben Green and Terry Tao that there exist arbitrarily long arithmetic progressions in the primes. Our era is indeed a golden one for mathematics!

In 2006, CMI inaugurated the Clay Lectures in Mathematics, a series of talks by former Clay Research Fellows on topics of current interest. Ben Green and Akshay Venkatesh delivered the first series at Cambridge University in November of 2006. Elon Lindenstrauss and Mircea Mustata delivered the second series in December 2007 at the Tata Institute for Fundamental Research in Mumbai.

I am pleased to announce the formation of an editorial board for the Clay Mathematics Institute Monograph series, published with the American Mathematical Society. The Editors in Chief for the series are Simon Donaldson and Andrew Wiles. I will serve as managing editor, and there is a distinguished board of associate editors (see www.claymath.org/monographs). The third volume in the series, *Ricci Flow and the Poincaré Conjecture,* by John Morgan and Gang Tian, appeared in August, 2007. The series publishes selected expositions of recent developments, both in emerging areas and in older subjects transformed by new insights or unifying ideas. CMI takes great care in the editing and presentation of the final manuscript and in supporting a well-produced book. Authors with a project in mind are encouraged to contact an editor.

In closing, I would like to draw attention to CMI's program of workshops held in Cambridge, Massachusetts. Ten have been held to date, and we generally schedule four to five per year. Our workshops are intended to be small, informal, and structured in whatever way the organizers deem best. CMI initiates workshops on its own and also seeks proposals. Proposals should be short, and the same is true of the lead-time between proposal and workshop.

Sincerely,



James A. Carlson
President

Clay Research Conference

The inaugural Clay Research Conference was held at Harvard University on May 14 and 15 at the Science Center. The lectures covered a wide range of fields: topology, algebraic geometry, number theory, real and complex dynamics, and geometric group theory. The Clay Research Awards (see sidebar and following article) were presented on the afternoon of May 14. Awardees were:

Alex Eskin (University of Chicago) for his work on rational billiards and geometric group theory, in particular, his crucial contribution to joint work with David Fisher and Kevin Whyte in establishing the quasi-isometric rigidity of sol.

Christopher Hacon (University of Utah) and James McKernan (UC Santa Barbara) for their work in advancing our understanding of the birational geometry of algebraic varieties in dimension greater than three, in particular, for their inductive proof of the existence of flips.

Michael Harris (Université de Paris VII) and Richard Taylor (Harvard University) for their work on local and global Galois representations, partly in collaboration with Laurent Clozel and Nicholas Shepherd-Barron, culminating in the solution of the Sato-Tate conjecture for elliptic curves with non-integral j-invariants.

Landon Clay, joined by James Carlson, congratulating Christopher Hacon (right front) and James McKernan (right rear) after they received the Clay Research Award.

Conference speakers were Peter Ozsváth, *Holomorphic disks and knot invariants*; William Thurston, *What is the future for 3-dimensional geometry and topology?*; Shigefumi Mori, *Recent progress in higher dimensional algebraic geometry I*; Alessio Corti, *Recent progress in higher dimensional algebraic geometry II*; Mark Kisin, *Modularity of 2-dimensional Galois representations*; Richard Taylor, *The Sato-Tate conjecture*; Curtis McMullen, *Algebraic dynamics on surfaces*; Alex Eskin, *Dynamics of rational billiards*; and David Fisher, *Coarse differentiation and quasi-isometries of solvable groups*.

Clay Research Awards

Previous recipients of the award, in reverse chronological order are:

2005	Manjul Bhargava (Princeton University) Nils Dencker (Lund University, Sweden)
2004	Ben Green (Cambridge University) Gérard Laumon (Université de Paris-Sud, Orsay) Bao-Châu Ngô (Université de Paris-Sud, Orsay)
2003	Richard Hamilton (Columbia University) Terence Tao (University of California, Los Angeles)
2002	Oded Schramm (Theory Group, Microsoft Research) Manindra Agrawal (Indian Institute of Technology, Kanpur)
2001	Edward Witten (Institute for Advanced Study) Stanislav Smirnov (Royal Institute of Technology, Stockholm)
2000	Alain Connes (College de France, IHES, Vanderbilt University) Laurent Lafforgue (Institut des Hautes Études Scientifiques)
1999	Andrew Wiles (Princeton University)

The Clay Mathematics Institute presents the Clay Research Award annually to recognize major breakthroughs in mathematical research. Awardees receive the bronze sculpture "Figureight Knot Complement vii/CMI" by Helaman Ferguson and are named Clay Research Scholars for a period of one year. As such they receive substantial, flexible research support. Awardees have used their research support to organize a conference or workshop, to bring in one or more collaborators, to travel to work with a collaborator, and for other endeavors.

Clay Research Conference

The recipients of the 2007 Clay Research Awards with Landon and Lavinia Clay. From left to right: Richard Taylor, Michael Harris, Lavinia Clay, Landon Clay, Alex Eskin, Christopher Hacon, and James McKernan.

Alessio Corti was unable to attend the conference because of security difficulties in London. Shigefumi Mori, on very short notice, graciously agreed to give Corti's lecture. David Fisher presented the work of Eskin, Fisher, and Whyte on the quasi-isometric rigidity of sol, a major problem in geometric group theory. Alex Eskin and Curtis McMullen spoke about problems in dynamics: rational billiards and the dynamics on complex algebraic surfaces, respectively. These are just two of the many active areas in the field of dynamics, which has important points of contact with number theory, representation theory, and other areas. Peter Ozsváth and William Thurston spoke about the past, present, and future of topology: Ozsváth on new knot invariants that have resolved many long-standing problems in the field; Thurston on the geometrization conjecture and what remains to be understood after the groundbreaking work of Perelman.

Videos of the lectures are available at
www.claymath.org/publications/videos

Abstracts of Talks

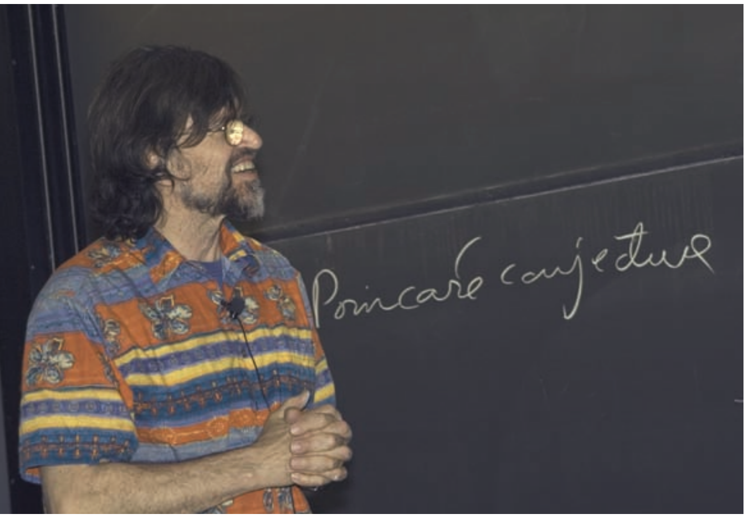
Peter Ozsváth (Columbia University)
Holomorphic disks and knot invariants

Heegaard Floer homology is an invariant for three- and four-manifolds defined using techniques from symplectic geometry. More specifically, a Heegaard diagram is used to set up an associated symplectic manifold equipped with a pair of Lagrangian submanifolds. The Heegaard Floer homology groups of the three-manifold are then defined as the homology groups of a chain complex whose differential counts pseudo-holomorphic disks in the symplectic manifold. These methods also lead to invariants for knots, links, and four-manifolds.

Ozsváth discussed applications of this theory, along with some recent calculational advances that have rendered the knot Floer homology groups purely combinatorial. Heegaard Floer homology was defined in joint work with Zoltan Szabo, in addition to further work with other collaborators, including Ciprian Manolescu, Sucharit Sarkar, and Dylan Thurston.

William Thurston (Cornell University)
What is the future for 3-dimensional geometry and topology?

Though the geometrization conjecture is now proved, there is still much to be understood in achieving a simple big picture of three-manifold topology, e.g., how is the set of hyperbolic manifolds, ordered by volume, organized; what are the deeper connections with number theory via the field of traces associated with the length of geodesic loops?



William Thurston delivering his talk at the conference.

Shigefumi Mori (RIMS, Kyoto)
Recent progress in higher dimensional algebraic geometry I

Higher dimensional algebraic geometry has recently undergone major developments related to the minimal model program. Mori reviewed the basic definitions of the minimal model program and surveyed some of the recent achievements and their applications.

Alessio Corti (Imperial College, London)
Recent progress in higher dimensional algebraic geometry II

Corti explained some of the key ideas in the recent work of Hacon and McKernan on the higher dimensional minimal model program for algebraic varieties.

Mark Kisin (University of Chicago)
Modularity of 2-dimensional Galois representations

Kisin discussed the recently proved conjecture of Serre on 2-dimensional mod p Galois representations, and its implications for modularity of 2-dimensional motives and p-adic Galois representations.

Richard Taylor (Harvard University)
The Sato-Tate conjecture

A fixed elliptic curve over the rational numbers is known to have approximately p points modulo p for any prime number p. In about 1960, Sato and Tate gave a conjectural distribution for the error term. Laurent Clozel, Michael Harris, Nicholas Shepherd-Barron and Taylor recently proved this conjecture in the case that the elliptic curve has somewhere multiplicative reduction.

Curtis McMullen (Harvard University)
Algebraic dynamics on surfaces

McMullen discussed the role of Hodge theory, Salem numbers, and Coxeter groups in the construction of new dynamical systems on compact complex surfaces.

Clay Research Conference

Alex Eskin, (University of Chicago)
Dynamics of rational billiards

Eskin called his talk "a short and extremely biased survey of recent developments in the study of rational billiards and Teichmüller dynamics."

David Fisher (Indiana University)
Coarse differentiation and quasi-isometries of solvable groups

In the early 1980s Gromov initiated a program to study finitely generated groups up to quasi-isometry. This program was motivated by rigidity properties of lattices in Lie groups. A lattice T in a group G is a discrete subgroup where the quotient G/T has finite volume. Gromov's own major theorem in this direction is a rigidity result for lattices in nilpotent Lie groups.

In the 1990s, a series of dramatic results led to the completion of the Gromov program for lattices in semisimple Lie groups. The next natural class of examples to consider are lattices in solvable Lie groups, and even results for the simplest examples were elusive for a considerable time. Fisher's joint work with Eskin and Whyte in which they proved the first results on quasi-isometric classification of lattices in solvable Lie groups was discussed. The results were proven by a method of coarse differentiation, which was outlined.

Some interesting results concerning groups quasi-isometric to homogeneous graphs that follow from the same methods will also be described.

Satellite Workshop at the Clay Mathematics Insitute May 16–17

A satellite workshop held at the Clay Mathematics Institute in the days following the Conference consisted of more detailed talks on recent progress in higher dimensional algebraic geometry. On this occasion, Christopher Hacon and James McKernan spoke on the existence of flips and MMP scaling to an audience of advanced graduate students in the field.

Clay Research Awards

The Clay Research Awards

Below is a brief account of the mathematics of the work for which each of the three Clay Research Awards were given. – jc

1. Minimal Models in Algebraic Geometry

Let X be projective algebraic variety over the complex numbers, that is, the set of common zeroes of a system of homogeneous polynomial equations. The meromorphic functions on X form a field, the *function field* of X. For the Riemann sphere (the projective line $\mathbf{CP}^1$) this field is $\mathbf{C}(t)$, the field of rational functions in one variable. For an elliptic curve $y^2 = x^3 + ax + b$, it is the field obtained by adjoining the algebraic function $y = \sqrt{x^3 + ax + b}$ to $\mathbf{C}(x)$. Two varieties are *birationally equivalent* if they have isomorphic function fields.

The *birational equivalence problem* is a fundamental one in algebraic geometry. Given two varieties X and Y, how do we recognize whether they are birationally equivalent? In the case of elliptic curves, there is an easy answer: the fields are isomorphic if and only if the quantity b^2/a^3 is the same in both cases. What can we say about other varieties? On what data does the birational equivalence class of a variety depend?

Consider first the case of complex dimension one. Every algebraic curve is birational to a smooth one, its normalization. Thus two curves are birational if and only if their smooth models are isomorphic. Consequently, the birational equivalence problem is the same as the moduli problem. Take, for example, the algebraic curves defined by the affine equations $x+y = 1$, $x^2+y^2 = 1$ and $x^2+y^2+x^3+y^3 = 0$.

The first two curves are smooth and isomorphic to the Riemann sphere, as one sees by stereographic projection. The last curve has one singular point, but its smooth model is the Riemann sphere, as we see by the parametrization

$$x = -(1 + t^2)/(1 + t^3),\ y = -t(1 + t^2)/(1 + t^3).$$

Thus all three varieties are birationally equivalent, with function field $\mathbf{C}(t)$.

Varieties of higher dimension are birationally equivalent to a smooth one by Hironaka's resolution of singularities theorem. Nonetheless, this powerful result does not answer the birational equivalence problem. To see why, consider a smooth algebraic

continued on page 17

surface X and a point p on it. One may replace the point by the set of tangent lines through p to obtain a new surface Y. The set of tangent lines is an algebraic curve E isomophic to one-dimensional projective space $\mathbf{CP}^1$. Since $X - \{p\}$ and $Y - E$ are isomorphic dense open sets in X and Y, respectively, the latter two varieties have isomorphic function fields. In algebraic geometry we say that Y is obtained from X by blowing up p. In more topological language, we say that Y is obtained from X by surgery: cut out the point p, and glue in the projective line E. What is important here is that the surgery is an operation on algebraic varieties.

More generally, we can (and will) consider surgeries of the form "cut out a subvariety A and paste in a variety B." More formally, we have varieties X and Y such that $X - A$ is isomorphic to $Y - B$, where we say that Y is obtained from X by surgery. Since $X - A$ and $Y - B$ are dense open sets, the function fields of X and Y are isomorphic. The partially defined map $X \dashrightarrow Y$ induces the isomorphism of function fields.

The curve E obtained by blowing up p is a projective line with self-intersection number -1. Such curves are known in the trade as "(-1) curves." Any time one finds a (-1) curve on a surface Y, one can construct a smooth surface X and a map $f : Y \longrightarrow X$ that maps E to a point. This operation is called "blowing down," or "contracting E." By successively contracting all the (-1) curves in sight, one can construct from any algebraic surface S a smooth variety S_{min} devoid of such curves. Let us call S_{min} a *classical minimal model* for S. Existence of classical minimal models was proved by Castelnuovo and Enriques in 1901. They also showed that as long as S and S' are not *uniruled*, they are birational if and only if S_{min} and S'_{min} are isomorphic. In the non-uniruled case a classical minimal model X_{min} is topologically the simplest: its second Betti number is smaller than that of any smooth surface birationally equivalent to it.

A variety X is uniruled if there is a map $\mathbf{CP}^1 \times Y \longrightarrow X$ whose image contains an open dense set. Thus, there is a curve birational to a projective line passing through almost every point of X.

A ruled surface, that is, a $\mathbf{CP}^1$ bundle over a curve, is uniruled. So is $\mathbf{CP}^2$. For uniruled surfaces, the minimal model is not unique. For example, blow up two points a and b on $\mathbf{CP}^2$. The proper transform of the line joining them is a (-1) curve. Blow it down to obtain a new surface. It is isomorphic to $\mathbf{CP}^1 \times \mathbf{CP}^1$. Both $\mathbf{CP}^2$ and $\mathbf{CP}^1 \times \mathbf{CP}^1$ are classically minimal, and both represent the purely transcendental function field $\mathbf{C}(x, y)$.

What can one say in dimension greater than two? The conjecture of Mori-Reid (see [4]) states the following:

> $(*)$ *Let X be an algebraic variety of dimension n which is not uniruled. Then (a) it has a minimal model X_{min} and (b) it has a Kähler metric whose Ricci curvature is ≤ 0.*

In the Mori-Reid conjecture, minimality is defined in a different way, as a kind of algebro-geometric positivity condition. We will discuss this notion in greater detail below. For surfaces, it coincides with the classical one: there are no (-1) curves. For higher dimensional varieties, minimality as positivity signaled a major change in the way mathematicians viewed the birational equivalance problem. The new line of investigation, initiated by Shigefumi Mori, developed further by Kawamata, Kollár, Mori, Reid, and Shokurov, culminated in 1988 with Mori's proof of (a) for varieties of dimension three [8]. For this result, the goal of the "minimal model program," Mori received a Fields Medal in 1990. Although refereeing is still in process, it now appears that (a) is also a theorem for all dimensions. An algebraic approach has been given by Birkar, Cascini, Hacon, and M^cKernan [1] and an analytic approach has been give by Siu [9].

In the remainder of this article we explain the modern notion of minimality and how it relates to the classical one. We then touch on just one of the crucial parts of the proof. This is the existence of *flips* and *flops*. These are surgeries that alter a variety in codimension two. Flops leave the positivity of the canonical bundle unchanged, whereas flips make it it more positive, transforming the variety to one that is closer to minimal. One cost of introducing a flip is that certain mild singularities must be admitted. These are the so-called "terminal singularities." A consequence of working with singular varieties is that the natural intersection numbers, while well-defined, can be rational numbers.

Summary of 2007 Research Activities


Clay Research Fellow Mohammed Abouzaid

The activities of CMI researchers and research programs are sketched below. Researchers and programs are selected by the Scientific Advisory Board (see inside back cover).

Clay Research Fellows

Mohammed Abouzaid began his five-year appointment in July 2007. He received his Ph.D. from the University of Chicago where he worked under the direction of Paul Seidel. Abouzaid is currently a postdoctoral fellow at MIT.

Soren Galatius, a native of Denmark, graduated from the University of Aarhus in 2004, where he worked under Ib Madsen. He is currently Assistant Professor of mathematics at Stanford University. He has a three-year appointment that began in September 2007.

Davesh Maulik recently completed his doctorate in mathematics at Princeton University. He began his five-year appointment in July 2007 at Columbia University.

Teruyoshi Yoshida is a graduate of Harvard University, where he is currently conducting his work in algebraic number theory as a member of the Society of Fellows. Yoshida began his three-year appointment in December 2007.

Abouzaid, Galatius, Maulik and Yoshida joined CMI's current group of research fellows Artur Avila (IMPA Brazil), Daniel Biss (University of Chicago), Maria Chudnovsky (Columbia University), Bo'az Klartag (Princeton University), Ciprian Manolescu (Columbia University), Maryam Mirzakhani (Princeton University), Sophie Morel (Institute for Advanced Study), Samuel Payne (Stanford University), and David Speyer (MIT).

Research Scholars

Mihalis Dafermos (DPMMS, University of Cambridge). December 31, 2006–December 30, 2007 at University of Cambridge.

Dipendra Prasad (TIFR, Mumbai). September 24, 2007–June 13, 2008 at University of California, San Diego.

Andras Stipsicz (Alfréd Rényi Institute of Mathematics, Budapest). September 1–December 31, 2007 at Columbia University.

Senior Scholars

Alex Eskin (University of Chicago). August 20–December 14, 2007. MSRI Program on Teichmüller Theory and Kleinian Groups.

Gerhard Huisken (Max Planck Institute, Potsdam, Germany). March 12–22, 2007. MSRI Program on Geometric Evolution Equations.

Peng Lu (University of Oregon). January 8–March 30, 2007. MSRI Program on Geometric Evolution Equations.

Andrei Okounkov (Princeton University). July 1–21, 2007. PCMI Program on Statistical Mechanics.


Clay Research Fellow Soren Galatius

Gang Tian (Princeton University). January 1– March 31, 2007. MSRI Program on Geometric Evolution Equations.

Srinivasa Varadhan (Courant Institute at NYU). July 1–21, 2007. PCMI Program on Statistical Mechanics.

Liftoff Fellows

CMI appointed seventeen Liftoff Fellows for the summer of 2007. Clay Liftoff Fellows are recent Ph.D. recipients who receive one month of summer salary and travel funds before their first academic position. See www.claymath.org/liftoff

Dmitriy Boyarchenko
Hans Christianson
Jason DeBlois
Adrian Ioana
Anthony Licata
Grace Lyo
Elizabeth Meckes
Yi Ni
Jeehoon Park
Pavlo Pylyavskyy
Shahab Shahibi
Sug Woo Shin
Jeffrey Streets
Junecue Suh
Joshua Sussan
Ben Webster
Josephine Yu

Research Programs Organized and Supported by CMI

January, Spring Semester. Semester-Long Program in Symplectic Topology at MIT.

January 8–27. School and Workshop in the Geometry and Topology of Singularities in Cuernavaca, Mexico.

January 29–Feb 2. Diophantine and Analytic Problems in Number Theory Conference at Moscow Lomonosov University.

January 1–April 30. Homological Mirror Symmetry and Applications Conference at IAS.

March 5–9. Workshop on Hopf Algebras and Props at CMI.

March 18–21. Dynamics Workshop in Honor of Hillel Furstenberg at the University of Maryland.
March 15–16. Conference on Hilbert's 10th Problem at CMI, including a preview screening of George Csicsery's film on Julia Robinson at the Museum of Science, Boston.

March 19–23. Motives and Algebraic Cycles: A Conference dedicated to the Mathematical Heritage of Spencer J. Bloch at the Fields Institute.

April 2–7. Noncommutative Geometry at IHES, Bur-sur-Yvette.

April 16–20. Minimal and Canonical Models at MSRI in Berkeley, California.

April 20–22. Workshop on Symplectic Topology at CMI.

April 29–May 5. Advances in Algebra and Geometry Conference at MSRI in Berkeley, California.

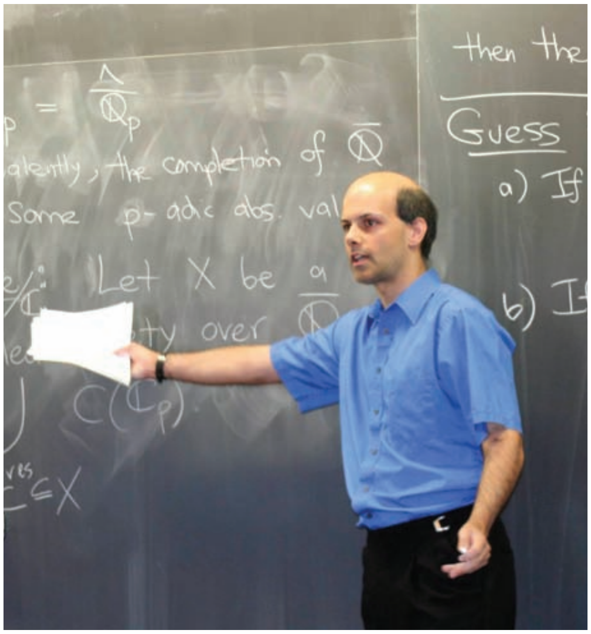
May 14–15. Clay Research Conference, Harvard University Science Center, Cambridge.

June 2. Developments in Algebraic Geometry Conference in Honor of David Mumford at Brown University.

June 7–11. Geometry and Imagination Conference at Princeton University.

June 7–20. Summer School on Serre's Modularity Conjecture at CIRM in Marseille, France.

June 11–July 6. CMI 2007 Summer School on Homogeneous Flows, Moduli Spaces and Arithmetic in Pisa, Italy.



Bjorn Poonen presenting his talk "Speculations about Rational Curves on Varieties over Countable Fields" at CMI's Rational Curves Workshop.

Summary of 2007 Research Activities

July 23–27. Infinite Dimensional Algebras and Quantum Integrable Systems II Conference at the University of the Algarve in Faro, Portugal.

July 30–August 3. Conference On Certain L-Functions at Purdue University.

August 12–17. IV IberoAmerican Conference on Complex Geometry at Centro Metalurgico, Ouro Preto in Minas Gerais, Brazil.

August 27–31. Colloque JPB at IHES and L'Ecole Polytechnique in Paris.

September 18–21. Solvability and Spectral Instability at CMI.

September 30. SAGE Days 5: Computational Arithmetic Geometry at CMI.

November 2–4. Workshop on Geometry of Moduli Spaces of Rational Curves with Applications to Deophantine Problems over Function Fields at CMI.

December 11–14. Clay Lecture Series at the Tata Institute of Fundamental Research (TIFR) in Mumbai, India.

Izzet Coskun (MIT) presenting his talk on "Vanishing of Quantum Cohomology" at CMI's Rational Curves Workshop.

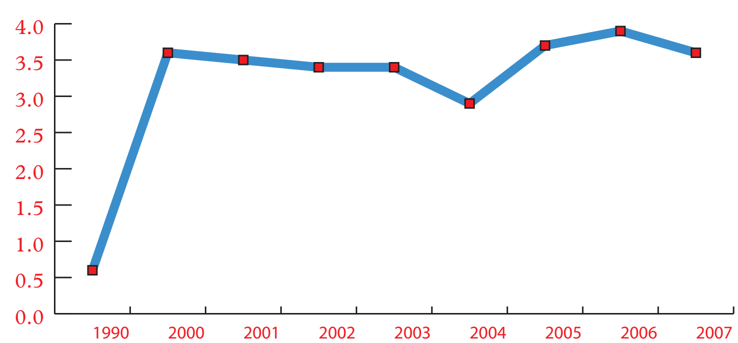
Kirsten Eisentraeger (Penn State) discussing "Hilbert's Tenth Problem for Function Fields of Varieties over C" at CMI's Rational Curves Workshop.

Program Allocation

Estimated number of persons supported by CMI in selected scientific programs for calendar year 2007:

Research Fellows, Research Awardees, Senior Scholars, Research Scholars, Book Fellows, Liftoff Fellows	45
Summer School participants and faculty	110
Student Programs, participants and faculty	140
CMI Workshops	95
Participants attending joint programs and the Independent University of Moscow	> 1000

IRS Qualifying Charitable Expenses of CMI Since Inception



Total through September 30, 2007: Over $28 million

Interview with Research Fellow Mircea Mustata

James Carlson and Mircea Mustata at the Tata Institute of Fundamental Research in Mumbai where CMI's 2007 "Clay Lectures on Mathematics" took place.

What first drew you to mathematics? What are some of your earliest memories of mathematics?

I am afraid that I don't have any math related memories from my early childhood. I began to show an interest in mathematics in elementary school, around the sixth grade. At that point more challenging problems started to come up, and one started doing rigorous proofs in plane Euclidean geometry. I was reasonably good at it, and there were interesting problems around, so I enjoyed doing it.

Could you talk about your mathematical education? What experiences and people were especially influential?

Maybe the turning point was when I started going to math olympiads, and to the circles organized around these contests. The main upshot was being around other kids who were enthusiastic about math. I then realized that I enjoyed spending my spare time doing math, and that this was not something completely out of ordinary. On the downside, I was never really good at these competitions, and at some point this got a bit frustrating. With hindsight, I think I shouldn't have spent this much time just with the olympiads, though this is what kept me being interested in math all through high school.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

I don't think that I had a real mentor while growing-up, though there have always been people around who influenced me. A key role was a tutor I had in the last grade in high school. That year I failed in one of the early stages of the math olympiad, and to cheer me up he gave me some books to read: some general topology, some real and complex analysis. This was my first encounter with serious mathematics, and it was an exciting experience. It certainly made the transition from high school to college much smoother.

Interview with Research Fellow Mircea Mustata

It would have been good to have a mentor during the first college years, but unfortunately I didn't have one. It would have helped in getting a better view of mathematics as a whole, and it might have motivated me to look at certain directions that I didn't know they existed until much later.

One thing that I think is good in Romania is that kids encounter mathematical reasoning at an earlier stage (through axiomatic Euclidean geometry, for example). Another feature of the Romanian education is that study during college is very focussed: with a couple of minor exceptions, all courses I have taken were in math...

You were educated in Romania. Could you comment on the differences between mathematical education in Romania and in the US?

One thing that I think is good in Romania is that kids encounter mathematical reasoning at an earlier stage (through axiomatic Euclidean geometry, for example). Another feature of the Romanian education is that study during college is very focussed: with a couple of minor exceptions, all courses I have taken were in math (with the embarrassing outcome that I did not take any college level course in physics). With so many math courses, one could get a strong background. The downside is that one insists maybe a bit too much on learning a lot of material, and on reading many books, and not really on using this information. What I like in the US is that students can get a taste of research at a very early stage. Myself, I started working on a problem only after I graduated from University. I was at the time a master's student in Bucharest.

What attracted you to the particular problems you have studied?

In my case, chance played a bit role. For example, I learned about the problem that influenced most of my work from my advisor, David Eisenbud, and from Edward Frenkel. At the time, by looking at several examples, they came up with a bold and unexpected conjecture. They very generously shared it with me, and this got me started in a completely new direction from what I had been doing.

In general, there are various reasons why I end up working on a particular problem. Most of the time, it is because it relates to other things I have thought about before. Of course, it helps if I believe that it is a problem other people would care about. In general, I find that problems that seem to have connections with other fields are particularly appealing. At least, they motivate me to try to learn some new things (though I have to say that in many cases it turned out that the new things were too hard and I got stuck).

Can you describe your research in accessible terms? Does it have applications to other areas?

Most of my research deals with singularities of algebraic varieties. These varieties are geometric objects defined by polynomial equations. It turns out that in many cases of interest, these objects are not "smooth-shaped"—they have singularities. In fact, what makes the local structure interesting are precisely these singularities. In my research, I deal with invariants that measure the singularities. Part of the motivation for what I do comes from questions that come up in the classification theory of algebraic varieties. Besides this, I don't know whether this topic has applications to other areas, but I believe that it has strong connections with various fields. In fact, the invariants that I have mentioned have all different origins (in valuation theory, commutative algebra or the theory of differential operators), but they turn out to be all related in ways that are still to be understood.

I always have a hard time saying which problems I will work on in six months (which makes writing NSF proposals a bit tricky). On the other hand, I have some long-term projects that I think about on and off over the years.

What research problems and areas are you likely to explore in the future?

I always have a hard time saying which problems I will work on in six months (which makes writing NSF proposals a bit tricky). On the other hand, I have some long-term projects that I think about on and off over the years. One problem that's been on my mind for a while has to do with the connections between some invariants of singularities that are defined in characteristic zero using valuations (or equivalently, using resolutions of singularities), and other invariants in positive characteristic that came out of an area in commutative algebra called tight closure theory. There is evidence that the connections between these invariants are quite subtle, and they have to do with the arithmetic properties of the varieties involved. At this point it is not clear whether there is a good framework for understanding these connections, but at least, this motivates me to learn a bit of number theory.

Could you comment on collaboration versus solo work as a research style? Are certain kinds of problems better suited to collaboration?

I believe that collaboration has more to do with personality than with the particular problems. Most of my work was done in collaboration, and I think that in general, this social aspect is a very rewarding one in our work (this is probably one of the things I got from my advisor). In my case, however, it is not so much a matter of choice: I realized that I get most of the ideas by talking to people --even when I don't fully understand what they are saying. On the other hand, I enjoy also the part of the work that's done in private, and I always need to take the time to think about a problem on my own.

What do you find most rewarding or productive?

Maybe the most rewarding experience is when you get an intuition how certain disparate pieces might fit together. I am not talking about figuring out how to prove a precise result, usually by the time the details are cleared the enthusiasm cools down. But rather about the moment when you realize that certain things might be connected in a way you hadn't expected. Of course, this does not happen very often, and sometimes this intuition is wrong, but it is always exhilarating, and it pays off for all the moments when I hit a dead end.

How has the Clay Fellowship made a difference for you?

Probably the most important thing was that for three years I had the freedom to choose where I want to be. I think that in general it made my postdoc years less stressful than they might have been: for example, I had no teaching duties (I ended up teaching two courses during this time, but this was my own choice, and actually got me much more enthusiastic about teaching). And of course, the stipend being pretty generous, I could appreciate the improvement in my life after being a graduate student.

What advice would you give to young people starting out in math (i.e., high school students and young researchers)?

Maybe the most rewarding experience is when you get an intuition how certain disparate pieces might fit together. I am not talking about figuring out how to prove a precise result, usually by the time the details are cleared the enthusiasm cools down. But rather about the moment when you realize that certain things might be connected in a way you hadn't expected.

I think that it is good to get involved in research early on, finding an easy, but interesting problem to work on. On the other hand, once you figure out a field you want to work in, it might be good to allow time to get also a broader view of that field, in addition to the technical mastery required for working on a specific problem. It is true that this might be hard to put in practice nowadays because of the way the programs are structured.

What advice would you give lay persons who would like to know more about mathematics — what it is, what its role in our society has been and so on? What should they read? How should they proceed?

A direct way of figuring things out would be by talking to mathematicians (though convincing them to discuss math with a non-mathematician might be considered a personal achievement). The good news is that there are by now several popularization books, either about famous mathematicians or about famous mathematical problems. I believe they can convey what "doing mathematics" means, why certain problems are important, and sometimes they can even put forward and explain interesting mathematical concepts.

How do you think mathematics benefits culture and society?

Interview with Research Fellow Mircea Mustata

Profile

Like other sciences, mathematics fulfils a need for figuring out the world around us. The special place of mathematics is due to the fact that it deals with an abstract realm. It is a common misconception that because of this fact, mathematics is out of touch with reality. It is indeed true that mathematical constructions do not need to be validated by "reality" (and I personally find this aspect very appealing). However, one should keep in mind that as physics taught us in the last hundred years, even very abstract models can help us to understand our own world.

Please tell us about things you enjoy when not doing mathematics.?

Whenever I have time, I enjoy hiking, reading fiction or watching movies, though my favorite pastime lately has been playing with my daughter. Another thing I enjoy a lot, which is one of the perks of a mathematician's life, is traveling. I always dreamt about traveling when I was a kid, but until my final years in grad school, I didn't realize that this was the way to go.

Mircea Mustata, a native of Rumania, finished the Ph.D program at University of California, Berkeley under the direction of David Eisenbud. Immediately afterwards he began his position as a Clay Research Fellow. He held this position from July 2001, to August 2004. During his time he visited Université de Nice, the Isaac Newton Institute (Cambridge), and Harvard University. In Septermber 2004, Mustata became Associate Professor of Mathematics at the University of Michigan. His reasearch is supported by the NSF and a Packard Fellowship.

His main research interest is in algebraic geometry, in particular in various invariants of singularities of algebraic varieties, such as minimal log discrepancies, log canonical thresholds, multiplier ideals, Bernstein-Sato polynomials or F-thresholds. Various points of view and techniques come in the picture when studying these invariants: resolutions of singularities, jet schemes, D-modules or positive characteristic methods. Other interests include birational geometry, asymptotic base loci and invariants of divisors, and toric varieties.

Mircea Mustata at TIFR, Mumbai in December of 2007.

Recent Research Articles

"Invariants of singularities of pairs," with Lawrence Ein, International Congress of Mathematicians, Vol. II, 583–602, Eur. Math. Soc., Zurich, 2006.

"Asymptotic invariants of base loci," with Lawrence Ein, Robert Lazarsfeld, Michael Nakamaye, and Mihnea Popa, Ann. Inst. Fourier (Grenoble) 56 (2006), 1701–1734.

"Bernstein-Sato polynomials of arbitrary varieties," with Nero Budur and Morihiko Saito, Compos. Math. 142 (2006), 779–797.

"Ehrhart polynomials and stringy Betti numbers," with Sam Payne, Math. Ann. 333 (2005), 787–795.

"F-thresholds and Bernstein-Sato polynomials,"with Shunsuke Takagi and Kei-ichi Watanabe, European Congress of Mathematics, 341–364, Eur. Math. Soc., Zurich, 2005.

"Inversion of adjunction for local complete intersection varieties," with Lawrence Ein, Amer. J. Math. 126 (2004), 1355–1365.

Julia Robinson and Hilbert's Tenth Problem: Conference and Film

From top to bottom: Julia Robinson, courtesy of Constance Reid; Yuri Matiyasevich, photo by George Csicsery; David Hilbert, courtesy AK Peters, Ltd.

On March 15 and 16, 2007, CMI held a small conference at its Cambridge office on Hilbert's Tenth Problem. Participants included Martin Davis, Hilary Putnam, Yuri Matiyasevich, and Constance Reid, sister of Julia Robinson. The conference was coupled with a screening at the Museum of Science in Boston of a pre-release version of George Csicsery's film *Julia Robinson and Hilbert's Tenth Problem.*

The Problem

At the 1900 International Congress of Mathematicians in Paris, David Hilbert presented a list of twenty- three problems that he felt were important for the progress of mathematics. Tenth on the list was a question about Diophantine equations. These are polynomial equations like

$$x^2 + y^2 = z^2 \qquad \text{or} \qquad 3x^3 + 4y^3 + 5z^3 = 0$$

that have integer coefficients and for which we seek integer solutions. The first equation, which comes from the Pythagorean theorem, was known to the Babylonians and the Greeks. It has infinitely many solutions, of which the smallest is 3, 4, 5. The second, which defines an "elliptic curve," is the kind of object that played a crucial role in Wiles' proof of Fermat's last theorem and which is also important in modern cryptography: elliptic curves help keep your credit card data safe. In 1957, Selmer showed that the second important equation has no integer solutions.

Hilbert, in posing his Tenth Problem, asked whether it was possible "to devise a process according to which it can be determined in a finite number of operations whether the equation is solvable in rational integers." What is sought is a general method applicable to all Diophantine equations, not just to specific equations like those above, or even specific classes of equations such as

$$ax^2 + by^2 + cz^2 + dz^2 = 0. \qquad (*)$$

Today we would ask whether "the solubility of Diophantine equations is decidable." That is, we ask whether there is an algorithm or computer program which, given the equation, runs for a finite amount of time and then prints out the answer "yes, it is soluble" or "no, it is not soluble."

The Solution

The story of the solution of Hilbert's Tenth Problem is one of great intellectual challenge, adventure, and accomplishment. Hilbert himself worked on it, and probably thought that it could be solved in the affirmative. He knew, of course that the solvability of equations of the form (∗) could be determined by an algorithm of his colleague Minkowski. Nevertheless, the first real progress came in the 1930s with the work of Gödel on the undecidability of arithmetic. This work, which gave a negative solution of Hilbert's First Problem, presaged the solution of the Tenth Problem. Later in that decade came the work of Alan Turing and a group of logicians: Church, Kleene, Post, and Rosser. A key discovery was the existence of sets of numbers that could be listed by a Turing machine but for which no Turing machine could answer the problem "is x an element of the set" for all x.

The discovery of listable but non-computable sets set the stage for the contributions of Martin Davis and Hilary Putnam, and later Julia Robinson. By the late 1940s Davis had made substantial progress, and he formulated a bold conjecture relating listable sets with those defined by Diophantine equations. Julia Robinson, working independently, had been studying a seemingly simple question of the great

Julia Robinson and Hilbert's Tenth Problem: Conference and Film

Polish logician Alfred Tarski: can the set of powers of two be defined via Diophantine equations? Robinson was not able to solve the problem, but in a 1952 paper, she reduced it to the question of whether there was a set of pairs of numbers (a,b) that (i) grows exponentially (but not too fast) and (ii) is definable by Diophantine equations. Dubbed "JR" by Martin Davis, this hypothesis was to play a decisive role. Indeed, in a 1961 paper, Davis, Hilary Putnam, and Robinson reduced the solution of Hilbert's Tenth Problem to the problem of proving JR.

The last, crucial step was taken by Yuri Matiyasevich shortly after New Year's Day, 1970. As a sophomore at Leningrad State University, Matiyasevich had taken up Hilbert's Tenth Problem, but after several years of frustration, set it aside, vowing never to look at it again. Nevertheless, when asked to review a new paper by Julia Robinson, he saw almost immediately a way of proving JR, and within a few days had done so.

With Matiyasevich's work, Hilbert's Tenth Problem was at last solved. Despite the difficulties of communication during the Cold War, the good news quickly traveled from the USSR to the USA. Robinson and Matiyasevich exchanged letters, and thus began a long, fruitful, and generous collaboration among Davis, Matiyasevich, Putnam, and Robinson. For more about both the history and the mathematics, see "Decidability in Number Theory," by Bjorn Poonen, *The Notices of the AMS*, November 2008.

The Film

The threads of this story form the warp and weave of the film *Julia Robinson and Hilbert's Tenth Problem*, produced and directed by George Csicsery with major support from CMI and Will Hearst III. All the main players – Davis, Matiyasevich, Putnam, and Robinson — appear, as does Julia Robinson's sister, Constance Reid, author of the well-known biography of David Hilbert.

CMI organized a screening of a preliminary version of the film at the Museum of Science in Boston on March 15, in conjunction with a two-day conference, held March 15 and 16, on Hilbert's Tenth Problem. Following the film was a panel discussion moderated by Jim Carlson; panelists were George Csicsery, Kirsten Eisentrager, Martin Davis, Yuri Matiyasevich, Hilary Putnam, and Constance Reid.


Panelists George Csicsery and Constance Reid

Julia Robinson and Hilbert's Tenth Problem has now been released and is available on DVD from AK Peters (www.akpeters.com). It was shown to an enthusiastic audience at the winter meeting of the American Mathematical Society in San Diego on January 6, 2008.

The Conference

Held at CMI's offices at One Bow Street in Cambridge, the conference brought together all the living participants in the solution of the problem: Yuri Matiyasevich, Martin Davis, and Hilary Putnam. The talks given were as follows:

Constance Reid, *Genesis of the Hilbert Problems*
George Csicsery, *Film clip on life and work of Julia Robinson*
Bjorn Poonen, *Why number theory is hard?*
Yuri Matiyasevich, *My collaboration with Julia Robinson*
Martin Davis, *My collaboration with Hilary Putnam*
Yuri Matiyasevich, *Hilbert's Tenth Problem: What was done and what is to be done*
Bjorn Poonen, *Thoughts about the analogue for rational numbers*
Alexandra Shlapentokh, *Diophantine generation, horizontal and vertical problems, and the weak vertical method*
Yuri Matiyasevich, *Computation paradigms in the light of Hilbert's Tenth Problem*
Gunther Cornelisson, *Hard number-theoretical problems and elliptic curves*
Kirsten Eisentrager, *Hilbert's Tenth Problem for algebraic function fields*

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continued from page 7

The technical issues thus raised can all be successfully dealt with; indeed, working in the larger category of varieties with terminal singularities is crucial to the success of the minimal model program.

Another cost associated with flips is the difficulty of proving that they exist in sufficient generality. One can construct motivating examples (see below), but even these are somewhat complicated. Important special cases were proved by Tsunoda, Shokurov, Mori, and Kawamata. Finally, Mori proved the general existence theorem for flips in dimension three [8], [7, p. 268]. The Clay Research Award was given to Hacon and McKernan for their proof of the existence of flips in dimension n assuming termination of flips in dimension $n-1$. See [3]. This result suggests that one can prove the existence of minimal models inductively, a program carried out in [1].

Let us now discuss minimality and flips in a more substantitve way. To say that X is minimal is to say that its canonical bundle is *numerically effective*, or *nef*. The canonical bundle is the line bundle K whose local sections are holomorphic n-forms A line bundle L is nef if the integral

$$L \cdot C = \int_C \omega$$

is positive for all algebraic curves C on X, where ω is a differential form representing the first Chern class of L. This integral represents the intersection number, which, as noted, may be a rational number.

What is the relation of the new definition of minimality to that of the Castelnuovo-Enriques theory? By the adjunction formula, the value of the above integral, i.e., the intersection number $K \cdot C$, is nonnegative. Thus a variety of dimension two whose canonical class is nef has no (-1) curves. Consequently, surfaces that are minimal are classically minimal.

Following [7, Example 12.1], we give an example, first observed by Atiyah, of a flop. We then modify the example to give a flip. Consider the affine variety C_0 given by the quadratic equation $xy - uv = 0$. Let C be its closure in $\mathbf{CP}^4$; it is the cone over a quadric $\mathbf{CP}^1 \times \mathbf{CP}^1$. Blow up the vertex of the cone to obtain a variety C_{12}. The resulting exceptional set $F = F_1 \times F_2$ is isomorphic to $\mathbf{CP}^1 \times \mathbf{CP}^1$. It is possible to blow down all the fibers $\{ x \} \times F_2$ of F to obtain a variety C_1 with a subvariety $E_1 \cong F_1 \cong \mathbf{CP}^1$. Likewise, we can blow down the fibers $F_1 \times \{ y \}$ to obtain a variety C_2 with a subvariety $E_2 \cong F_2 \cong \mathbf{CP}^1$. Thus $C_1 - E_1 \cong C_{12} - F \cong C_2 - E_2$. The birational map $C_2 \dashrightarrow C_1$ is the flop obtained by the surgery "cut out E_2 and glue in E_1." If $q_i : C_{12} \to C_i$ is the canonical projection, then the birational map $C_2 \dashrightarrow C_1$ is just the composition $q_1 q_2^{-1}$. In this case, the piece cut out and the piece glued in are both projective lines. Flops do not affect the intersection number with the canonical divisor.

To understand this flop better, consider the family of planes P_λ on C given by $x = \lambda u$, $v = \lambda y$. There is a corresponding family of projective planes in C and in C_{12}, and a corresponding family of hypersurfaces $\tilde{P}_\lambda$ in C_{12}. Each $\tilde{P}_\lambda$ is a projective plane with a point blown up. The blowup of the point is one of the fibers $\{ x \} \times F_2$.

Consider now the surfaces $q_1(\tilde{P}_\lambda) \subset C_1$. Obtained by blowing down $\{ x \} \times F_2$, these surfaces are disjoint for distinct λ, isomorphic to $\mathbf{CP}^2$, and each one meets E_1 in a single point. Thus $q_1(\tilde{P}_\lambda) \cdot E_1 = 1$. A more subtle computation yields $q_2(\tilde{P}_\lambda) \cdot E_2 = -1$. Thus the flop changes the intersection number of $q_i(P_\lambda)$ with the surgery loci E_i. In the example, the canonical bundle of C is defined and trivial at the vertex of the cone and so also trivial on the exceptional set F. Therefore $K \cdot E_i = 0$: the flop does not affect the positivity of the canonical bundle.

For the second example, we follow [7, Example 12.5]. Factor C_0 by the $\mathbf{Z}_2$ action given by the map $(x, y, u, v) \longrightarrow (x, -y, u, -v)$. The $\mathbf{Z}_2$ actions make sense on C_1, C_2, and C_{12}. However, there is a symmetry that is broken. In addition to the family of planes P_λ there is a family of planes Q_λ defined by $x = \lambda v$, $u = \lambda y$. They are interchanged by the map permuting u and v. This global symmetry is the reason why $C_1 \cong C_2$. However, the permutation of coordinates does not commute with the $\mathbf{Z}_2$ action. As a result, one finds that $C_1/\mathbf{Z}_2$ and $C_2/\mathbf{Z}_2$ are not isomorphic. Indeed, the intersection of the canonical class with $E_2/\mathbf{Z}_2$ is negative whereas its intersection with E_1/Z_2 is positive. The natural birational map $C_2/\mathbf{Z}_2 \dashrightarrow C_1/\mathbf{Z}_2$ can be described as "cut out $E_2/\mathbf{Z}_2$ and replace it by $E_1/\mathbf{Z}_1$." The quotients $E_i/\mathbf{Z}_2$ are projective lines. Thus, as in the first example, the surgery is obtained by removing a projective line and glueing it back in a different way. But in this case, the geometry of the variety subjected to surgery changes and positivity of the canonical bundle increases.

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For part (b) of the Mori-Reid conjecture, the differential geometric form of positivity for minimal models, consider first the case of algebraic curves. Via the uniformization theorem, every Riemann X has as universal cover which is a space of constant curvature $+1$, 0, or -1: the sphere in the case of genus zero, the plane in the case of genus one, and the upper half plane in the case of higher genus. For genus $g \geq 0$, the non-uniruled case, the curvature 0 or -1 condition is a strong differential-geometric form of nefness. In higher dimension, suppose that X is a smooth minimal variety of general type. By the base-point-free theorem, its canonical bundle is semi-ample, and so the first Chern class of the canonical bundle is represented by a semi-positive $(1,1)$ form. It follows from the solution of the Calabi Conjecture, independently given by Aubin and Yau in their work on the Monge-Ampère equation, that X has a metric whose Ricci curvature is negative (≤ 0). Alternatively, the Ricci form of the metric is positive-semidefinite. As noted, one generally has to admit terminal singularities in the minimal model. The best results for (b) in the general case are due to Eyssidieux, Guedj, and Zeriahi, generalizing the work of Aubin and Yau on Monge-Ampre equations.

Acknowledgments. The author relied heavily on [2], [4], *and* [7] *for this article, and would like to thank Herb Clemens and János Kollár for their help in its preparation.*

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2. The Sato-Tate Conjecture

Since the time of the Greeks, the study of Diophantine equations has driven some of the most important developments of mathematics. Of particular significance are the cubic equations in two variables, which we may put in the form $(*)$ $y^2 = x^3 + ax + b$. The set of complex solutions of $(*)$ plus the point at infinity is a torus; in general the solution set, including the point at infinity, is called an "elliptic curve," written E. The name is something of a historical accident having do with the problem of computing the lengths of arcs on ellipses. Its solution leads to integrals of the form $\int dx/y$, where $y = \sqrt{x^3 + ax + b}$. Of course the differential form dx/y plays a leading role in the modern theory of elliptic curves, e.g., by determining its Hodge structure.

It was Gauss, in his *Disquisitiones Arithmetica*, who formally introduced the idea of studying Diophantine equations by examining their reduction modulo a prime p. The notion certainly predates Gauss, however, and goes back at least as far as Fermat. The central problem is to count the number of points N on E modulo p. By N we mean the number of solutions of the equation $(*)$ modulo p, plus one for the point at infinity. A classical theorem of Hasse tells us how many solutions to expect:

$$|N - (p+1)| \leq 2\sqrt{p}.$$

Consider, for example, the elliptic curve defined by $y^2 = x^3 - x - 1$. Modulo 3 there is just one solution, namely the point at infinity. Modulo 5 there are eight. Further experiment reveals the behavior of N as a function of p to be quite random, suggesting a statistical interpretation: the expected value of N is $p + 1$ and the standard deviation is proportional to $\sqrt{p}$. To make more precise statement, consider the quantity

$$\delta = \frac{N - (p+1)}{2\sqrt{p}}.$$

According to Hasse's theorem, this normalized measure of the deviation of the number of solutions modulo p from its expected value is a number in the range $[-1,1]$. A deeper question, then, is the nature of the probability law governing the distribution of the numbers δ.

Around 1960, Mikio Sato and John Tate independently conjectured that the probability law for elliptic curves without complex multiplication ("extra symmetry") is given by the function $f(\delta) = (2/\pi)\sqrt{1-\delta^2}$. Sato was led to the conjecture by experimental evidence. Although the documentary record is sparse, there is still extant a letter from John Tate to Jean-Pierre Serre dated August 5, 1963, about Tate's thoughts on the conjecture. In this letter Tate adds, "Mumford tells me that Sato has found $f(\theta) = (2/\pi)sin^2(\theta)$ experimentally on one curve with thousands of p." The angle θ is $\cos^{-1}\delta$; the formulations in terms of δ and θ are equivalent. See [11] for computations now accessible to anyone.

Tate was led to the conjecture on theoretical grounds having to do with the connection between alebraic cycles and the zeroes and poles of L-functions. Starting from the Hasse-Weil function $L(s,E)$ of the elliptic curve, J.-P. Serre defined [6] a natural sequence of functions $L(s,E,sym^n)$ associated to the irreducible representations of $SU(2)$. When $n=1$, the $L(s,E,sym^n) = L(s,E)$. These functions were variants of those considered by Tate [7]. Serre showed, as Tate had predicted, that the Sato-Tate conjecture would follow from the assertion that $L(s,E,sym^n)$ has an extension to an analytic function in the half-plane $\Re(s) \geq n+1/2$ and is non-vanishing there. The work of Wiles [10] and of Taylor and Wiles [9] on Fermat's last theorem, and finally the work of Breuil, Conrad, Diamond, and Taylor, [1] established the crucial fact that the function $L(s,E)$ extends to an analytic function in the half-plane $\Re(s) \geq 3/2$ and is non-vanishing there.

One important ingredient in the proof is an extension of Wiles' technique for identifying L-functions of elliptic curves with L-functions of modular forms. This was begun in the paper [2] by Clozel, Harris, and Taylor and completed in later paper by Taylor [8]. Another is an extension of an idea of Taylor for proving meromorphic continuation of L-functions associated to two-dimensional Galois representations by applying to moduli spaces an approximation theorem of Moret-Baily. Execution of this plan relies on the existence of a suitable moduli space. Harris, Shepherd-Barron, and Taylor [4] found one suitable for studying n-dimensional representations for any even n. It is a twisted form of the moduli space for Calabi-Yau manifolds originally studied by Dwork in certain cases and now an important part of string theory.

The kind of probability distribution proved for elliptic curves is conjecturally far more general. See Barry Mazur's article in Nature [5].

Acknowledgments. The author relied heavily on the account in [3] *in preparing this article.*

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3. Virtual Quasi-Isometric Rigidity of Sol

In his 1983 ICM address, Mikhael Gromov proposed a program for studying finitely generated groups as geometric objects [6]. The story begins with the *Cayley graph*, a metric space $C_S(\Gamma)$ associated to a group Γ and a set of generators S. The set of vertices is Γ itself. Two vertices are connected by an edge if right multiplication by some generator maps one to the other. There is a natural action of the group on this graph given by left

Clay Research Awards

translation. The Cayley graph has a natural metric where each edge is isometric to a unit interval, and the distance between points is given by the length of a shortest path joining them.

The Cayley graph depends on the choice of generating set and so its geometry is not intrinsic to the group Γ. However, it turns out that there is a natural equivalence relation, quasi-isometry, relating graphs defined by different sets of generators. We say that two metric spaces X and Y are *quasi-isometric* if there is a map $f : X \to Y$ that does not distort distances too much. To make a precise statement, let d_X and d_Y denote the metrics. Suppose that there are constants $K \geq 1$ and $C \geq 0$ such that for every x_1, $x_2 \in X$

$$\frac{1}{K} d_X(x_1, x_2) - C \leq d_Y(f(x_1, f(x_2)$$

and

$$d_Y(f(x_1, f(x_2) \leq K d_X(x_1, x_2) + C$$

and such that the C-neighborhood of $f(X)$ is all of Y. Such a map is a (K, C) quasi-isometry. Two spaces are said to be quasi-isometric if there is a (K, C) quasi-isometry between them for some K and C.

The central question that Gromov raised was the classification of groups up to quasi-isometry, i.e., the enumeration and characterization of the quasi-isometry classes of finitely generated groups.

A modest beginning is to note that all finite groups are quasi-isometric, that is, quasi-isometric to a point. Thus Gromov's theory is a theory of infinite groups. For nontrivial examples, consider a group Γ that acts properly discontinuously by isometries on a nice space X, such as a connected Riemannian manifold. Suppose further that the quotient X/Γ is compact. (We say that Γ is "co-compact.") Then Γ and X are quasi-isometric. For example, the lattice $\mathbf{Z}^d$ in $\mathbf{R}^d$ is quasi-isometric to $\mathbf{R}^d$ — which is not quasi-isometric to a point. More generally, consider a lattice Γ in a Lie group G: a discrete subgroup such that the quotient G/Γ has finite volume relative to a left invariant Haar measure on G. If Γ is co-compact, then it is quasi-isometric to G, when G is equipped with any left invariant distance function. Moreover, if K is a compact subgroup of G and $X = G/K$ is the quotient, then Γ is quasi-isometric to X. Any cocompact lattice in $SO(1, n)$ or $PSO(1, n)$, for example, is quasi-isometric to real hyperbolic n-space. Likewise, any two such lattices are quasi-isometric to each other. In particular the fundamental groups Γ and Γ' of compact Riemann surfaces S and S' are quasi-isometric, so long as both surfaces have genus at least two.

Since any finitely generated group isomorphic to a cocompact lattice in a Lie group G is quasi-isometric to G, it is natural to ask whether the converse is true: whether any group quasi-isometric to G is a cocompact lattice. However, this fails for trivial reasons because passing to finite index subgroups or finite extensions does not change the quasi-isometry class of a group. If we say that two groups are *weakly commensurable* if they are the same, modulo applying a finite sequence of these two operations, then the most one can hope to show is that a finitely generated group quasi-isometric to G is weakly commensurable to a cocompact lattice. Proving this statement is the *quasi-isometric rigidity problem* for G. For G a semisimple Lie group, quasi-isometric rigidity holds: a deep theorem that is the result of the work of many people, including Sullivan, Tukia, Gromov, Pansu [9], Casson-Jungreis, Gabai, Schwartz [10], Kleiner-Leeb [7], Eskin [2], and Eskin-Farb [4].

A landmark in the development of Gromov's program was his polynomial growth theorem [5]. To state it, let $N(r)$ be the number of group elements within distance r of the identity element relative to the word metric. For the group $\mathbf{Z}^d$, the function $N(r)$ is bounded by a constant times r^d. A group for which $N(r) \leq Cr^d$ is said to have polynomial growth. The least integer d for which the preceding estimate holds is independent of the generating set, so this notion depends on the group alone. It is not hard to prove that a nilpotent group, as an iterated extension of abelian groups, has polynomial growth. For example, the group of matrices

$$\begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix}$$

with integer coefficients, has polynomial growth of degree 4, one greater than the the dimension of the corresponding nilpotent Lie group (the Heisenberg group) that contains it. Gromov's theorem gave a converse: that any group of polynomial growth is virtually nilpotent, that is, is weakly commensurable with a cocompact lattice in a nilpotent Lie group. This theorem can then be applied to prove quasi-isometric rigidity for many nilpotent groups. The classification of general nilpotent groups up to quasi-isometry is still wide a open problem.

The tractability of semisimple Lie groups comes from their curvature properties: they act on non-positively curved spaces, and for such manifolds powerful tools have been forged. Nilpotent groups are tractable for a different reason. Their asymptotic structure is well approximated by simple scale invariant models which are also nilpotent Lie groups. On the other hand, solvable Lie groups can fail to have either of these simplifying characteristics. The easiest such example is the group Sol, given by 3×3 matrices

$$\begin{pmatrix} e^{z/2} & x & 0 \\ 0 & 1 & 0 \\ 0 & y & e^{-z/2} \end{pmatrix}.$$

This group, with the invariant metric

$$ds^2 = e^{-z}dx^2 + e^z dy^2 + dz^2$$

gives one of the seven geometries in Thurston's geometrization program. It also represented a key obstacle in Gromov's program. Indeed, the question of whether a group quasi-isometric to a latice in Sol is virtually a lattice in Sol became known as the Farb-Mosher conjecture.

The Farb-Mosher conjecture was proved in the affirmative by recent work of Alex Eskin, David Fisher, and Kevin Whyte [3]. One of the main tools was the notion of *coarse differentiation*, which has found application to other areas, e.g., the geometry of Banach spaces [1] and combinatorics [8], was introduced by Eskin to the problem around 2005. Coarse differentiation may be viewed as a coarse variant of the theorem of Rademacher, which states that a Lipschitz function $R^n \longrightarrow R$ is differentiable almost everywhere; instead of stating that at almost every point a Lipschitz function has linear behavior on small scales, coarse differentation says that in a quantitative sense, Lipschitz functions $Sol \to R$ have linear behavior at large scales, at many points. Proving such a result requires one to develop a quantitative version of the Rademacher theorem.

Acknowledgments. The author thanks Bruce Kleiner and Domingo Toledo for their help in the preparation of this article.

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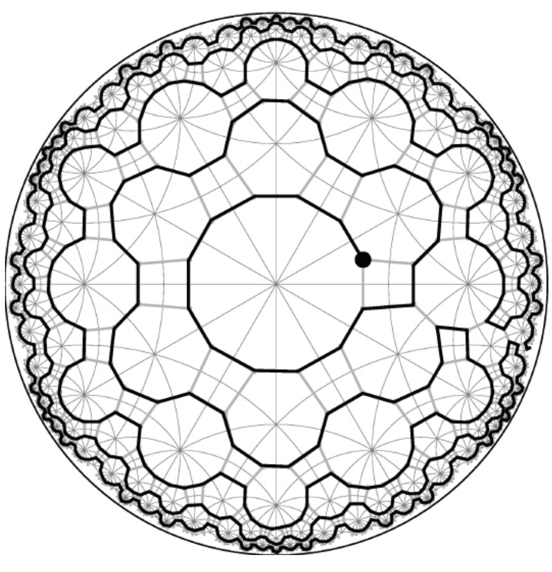


The group [6,4] is the triangle group generated by reflections in a right triangle with angles π/6 and π/4. The figure above shows a Hamiltonian path in the Cayley graph of this group relative to the standard set of generators. The heavy line segments, both solid and dashed, represent the Cayley graph; the light lines show the triangle tessellation. The Hamiltonian path consists of the solid line segments. Figure and text credit: Douglas Dunham, "Creating Repeating Hyperbolic Patterns—Old and New," Notices of the AMS, Volume 50, Number 4, April 2003, p. 453.

Homogeneous Flows, Moduli Spaces and Arithmetic, Pisa, Italy

Summer school

Aerial view of Pisa. © Peter Adams.

Clay Mathematics Institute 2007 Summer School on Homogeneous Flows, Moduli Spaces and Arithmetic

The Centro di Ricerca Matematica Ennio De Giorgi in Pisa, Italy provided a wonderful setting for the 2007 Clay Mathematics Institute Summer School. The school was designed to serve as a comprehensive introduction to the theory of flows on homogeneous spaces, moduli spaces and their many applications. These flows give concrete examples of dynamical systems with highly interesting behavior and a rich and powerful theory. They are also a source of many interesting problems and conjectures. Furthermore, understanding the dynamics of such concrete system lends to numerous applications in number theory and geometry regarding equidistributions, diophantine approximations, rational billiards and automorphic forms. The program was built around three foundation courses:

(1) *Unipotent flows and applications* by Alex Eskin and Dmitry Kleinbock;

(2) *Diagonalizable actions and arithmetic applications* by Manfred Einsiedler and Elon Lindenstrauss;

(3) *Interval exchange maps and translation surfaces* by Jean-Christophe Yoccoz.

These were supplemented by various Short Courses and Advanced Mini Courses and Lectures:

- *Equidistribution and L-Functions* by Gergely Harcos;
- *Reveiw of Vatsal's work on equidistribution and non-vanishing L-functions* by Nicolas Templier;

Organizers: David Ellwood (CMI), Manfred Einsiedler (Ohio State), Alex Eskin (Chicago), Dmitry Kleinbock (Brandeis), Elon Lindenstrauss, chair (Princeton), Gregory Margulis (Yale), Stefano Marmi (La Scuola Normale Superiore di Pisa), Peter Sarnak (Princeton), Jean-Christophe Yoccoz (Collège de France), Don Zagier (MIT)

Photo courtesy Centro di Ricerca Matematica Ennio De Giorgi.

- *Homogeneous flows, buildings and tilings* by Shahar Mozes;
- *Fuchsian groups, geodesic flows on surfaces of constant negative curvature and symbolic coding of geodesics* by Svetlana Katok
- *Chaoticity of the Teichmuller flow* by Artur Avila;
- *Eigenfunctions of the laplacian: a semi-classical study* by Nalini Anantharaman;
- *Equidistribution on homogeneous spaces and the analytic theory of L-functions* by Akshay Venkatesh;
- *Counting and equidistribution on homogeneous spaces, via mixing and unipotent flows* by Hee Oh;
- *Informal introduction to unipotent flows* by Gregory Margulis;
- *Modular shadows* by Yuri Manin;
- *On the regularity of solutions of the cohomological equation for IET's and translation flows Modular Shadows* by Giovanni Forni;
- *The Distribution of free path lengths in the Periodic Lorentz Gas* by J. Marklof;
- *Uniform spectral gap bounds and arithmetic applications* by A. Gamburd;
- *Multi-valued Hamiltonians and Birkhoff sums over rotations and IET* by C. Ulcigrai;
- *Random hyperbolic surfaces and measured laminations* by M. Mirzakhani.

One way to orientate oneself within the formidable mathematical landscape explored in the school is to consider the familiar space $SL(2,\mathbb{R})/SL(2,\mathbb{Z})$: to a hyperbolic geometer, it is the unit tangent bundle of a hyperbolic surface; to a number theorist, it is the space of elliptic curves; to a low-dimensional topologist it is the moduli space of flat metrics (with an associated vector field) on genus 1 surfaces; to those who study Diophantine approximation, it is the space of unimodular lattices in $\mathbb{R}^2$; and to Lie theorists it is a motivating example of a finite volume homogeneous space G/Γ, that is, of a lattice Γ inside a Lie group G.

Two principal generalizations provided the setting for much of the material presented at the school:

Dynamics on the space of lattices: Let $X_n = SL(n,\mathbb{R})/SL(n,\mathbb{Z})$. This is the space of unimodular lattices in $\mathbb{R}^n$, as well as a homogeneous space G/Γ. On our motivating example X_2 there are two important dynamical systems arising from the left action of one-parameter subgroups: the *geodesic flow*, given by action of the diagonal subgroup

$$A = \left\{ g_t = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix} \right\}_{t\in\mathbb{R}};$$

Homogeneous Flows, Moduli Spaces and Arithmetic, Pisa, Italy

and the *horocycle flow*, given by action of the unipotent subgroup

$$U = \left\{ h_t = \begin{pmatrix} 1 & t \\ 0 & 1 \end{pmatrix} \right\}_{t \in \mathbb{R}}.$$

Two sets of lectures focused primarily on generalizations of each of these actions: Alex Eskin and Dmitry Kleinbock (with a two lecture prequel by Grigorii Margulis) delivered a lecture series on the action of unipotent subgroups H on homogeneous spaces G/Γ, exploring the results of Dani, Margulis, and others on non-divergence of orbits; Margulis's use of unipotent dynamics to prove the Oppenheim conjecture on the values of quadratic forms; and Ratner's classification of orbit closures and invariant measures. Here, even in the case $n = 2$, the situation is quite rigid: for example, all orbits of the group U are either periodic or dense.

In contrast, the action of A for $n = 2$ is remarkably chaotic: given any $1 \leq \alpha \leq 3$, one can produce an orbit whose closure has Hausdorff dimension α. However, for $n \geq 3$, there are conjectures of Margulis on the rigidity of the action of the diagonal subgroup. Two of the main contributors to this field, Manfred Einsiedler and Elon Lindenstrauss, gave lectures on the progress made toward these conjectures, focusing on the theory of entropy; the applications to the Littlewood conjecture on simultaneous diophantine approximation; and the theory of quantum unique ergodicity.

Building on these lectures, Nalini Anantharaman, Gergely Harcos, Hee Oh, and Akshay Venkatesh gave shorter series of more advanced lectures, indicating applications to quantum chaos, automorphic forms, and counting points on varieties. In addition, a special session on Diophantine approximation was organized by Dmitry Kleinbock, allowing many of the younger participants to present their recent results in the subject.

Dynamics on the moduli space of flat surfaces: Instead of considering the space X_n of lattices in higher dimensions, we can consider the moduli spaces $\mathcal{H}$ of flat metrics (with associated vector fields) on surfaces of higher *genus* $g \geq 2$. There is a natural $SL(2,\mathbb{R})$ action on this space, coming from the linear action on $\mathbb{R}^2$. The action of the subgroup A is called *Teichmüller geodesic flow*, and the orbit of a point $x \in \mathcal{H}$ under this flow yields information about the ergodic properties of the associated vector field.

Elon Lindenstrauss conducting a session in Pisa.

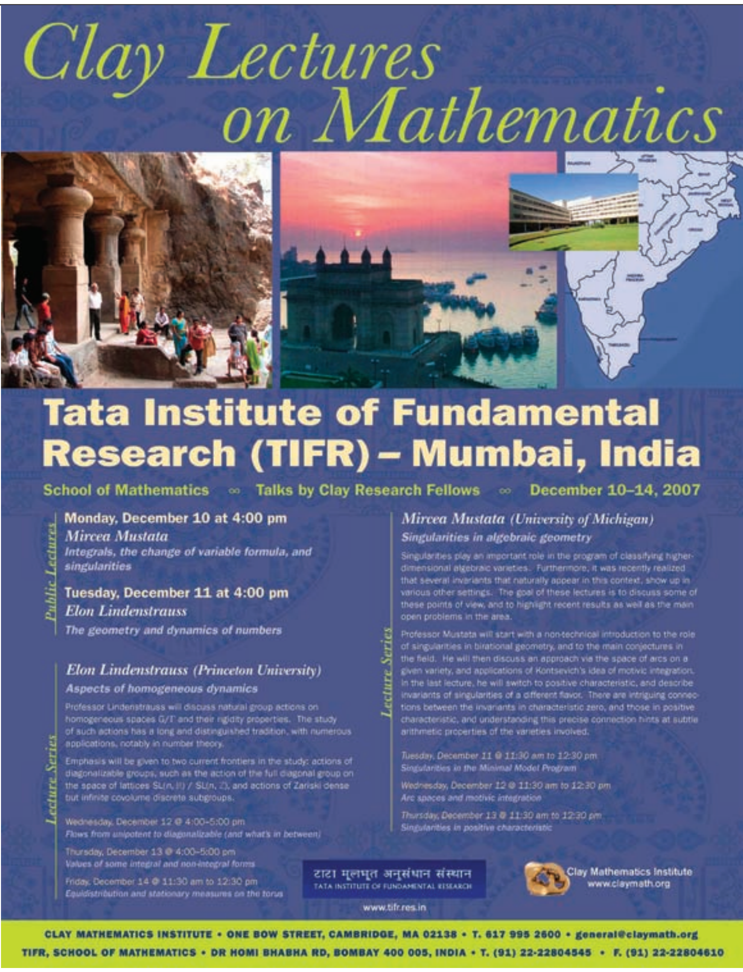
A seemingly unrelated family of dynamical systems are *interval exchange maps*: given a partition of the unit interval into n labeled subintervals, rearrange them according to a permutation $\pi \in S_n$. However, if we take a first return map for the flow associated to a vector field $x \in \mathcal{H}$ to a transverse interval, we obtain exactly one of these exchange maps.

In the last foundational course, Jean-Chrstophe Yoccoz explored this connection from the perspective of combinatorics and dynamics of interval exchanges. In particular, he showed how to use the ergodicity of Teichmüller flow and associated renormalization procedures on the space of interval exchanges to show the resolution (by Masur & Veech) of the Keane conjecture that almost every interval exchange map is uniquely ergodic.

Following Yoccoz's lectures, Artur Avila, Giovanni Forni, and Maryam Mirzakhani gave further talks on Teichmüller dynamics, exploring applications to the ergodic theory of polygonal billiards, and studying closely related spaces of foliations and laminations on surfaces.

All the lecturers made a special effort to ensure their presentations would be accessible to *all* the participants in the summer school, from beginning graduate students on upwards. Ninety-seven young mathematicians participated, from Europe, Asia, the Middle East, and the Americas. In addition to bringing many of the participants in these fields together to exchange ideas, the summer school has hopefully helped spark the interest of a new generation of mathematicians in these beautiful areas.

Clay Lectures on Mathematics at the Tata Institute of Fundamental Research



Clay Lectures on Mathematics, Mumbai

The 2007 Clay Lectures on Mathematics were hosted by the School of Mathematics, Tata Institute of Fundamental Research (TIFR) in India from December 10 through 14, 2007.

The lecturers were Mircea Mustaţă of the University of Michigan and Elon Lindenstrauss of Princeton University.

Mustaţă delivered a series of three lectures:

- Singularities in the Minimal Model Program
- Arc Spaces and Motivic Integration
- Singularities in Positive Characteristic

and one public lecture

- Integrals, the Change of Variable Formula, and Singularities.

Lindenstrauss delivered a parallel series of three lectures:

- Flows from Unipotent to Diagonalizable (and what is in between)
- Values of some Integral and Non-integral Forms
- Equidistribution and Stationary Measures on the Torus

and one public lecture

- The Geometry and Dynamics of Numbers.

The following is a summary of Mustaţă's and Lindenstrauss's lectures.

1. Singularities in Algebraic Geometry: Mircea Mustaţă

The role of singularities in the program of classifying higher-dimensional algebraic varieties is well-known. The public lecture gave an introduction to some results relating invariants of singularities (like the log canonical threshold) with various integration theories. The other lectures covered various aspects of invariants of singularities, discussing some of the recent results, as well as the main open problems and conjectures in the field. The content of the talks was roughly the following:

Integrals connected to singularities (complex powers, p-adic zeta functions): Let K be a field with an absolute value $|\,.\,|$ and a measure. Consider the corresponding product measure on K^n. Roughly speaking, the goal is to relate the singularities of a polynomial $f \in K[x_1, \cdots, x_n]$ with the asymptotic behavior of $\mu(\{x \in K^n \mid |f(x)| < \epsilon\})$, when ϵ goes to zero. One way of encoding this asymptotic behavior is by studying certain integrals. The key is to use a log resolution of singularities for f and some version of the Change of Variable Formula. The cases $K = \mathbb{C}$, $K = \mathbb{Q}_p$ (or more general p-adic fields) and $K = \mathbb{C}((t))$ are the main examples. The first case that was discussed was the Archimedean case (that is, when $K = \mathbb{R}$ or $\mathbb{C}$), which goes back to a question of Gelfand, subsequently proved by Atiyah, Bernstein and Gelfand. Mustaţă then gave an overview of the p-adic side of the theory. There was a discussion of p-adic integration, the Igusa zeta function and the main result

Clay Lectures on Mathematics at the Tata Institute of Fundamental Research

of Igusa about the rationality of the zeta function. In this context, he emphasized the connection between the largest pole of the zeta function and the log canonical threshold.

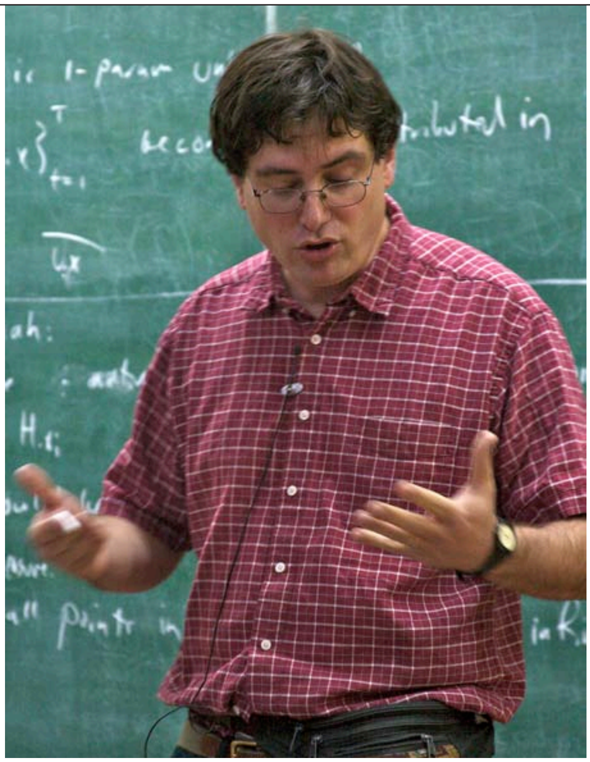
Invariants of singularities (the log canonical threshold) in birational geometry: In this lecture Mustaţă discussed the role that singularities play in higher dimensional birational geometry. The first part covered the role of vanishing theorems and the canonical divisor. This motivated the definition of various classes of singularities. After a brief overview of the basic setup in the Minimal Model Program, and of the recent progress in this field, Mustaţă explained a conjecture of Shokurov and its relevance in this setting.

Spaces of arcs and singularities: The talk covered some basic facts about spaces of arcs, with an emphasis on geometric aspects and applications to singularities. One result that was discussed was a theorem of Kontsevich relating the spaces of arcs of two smooth varieties X and Y, with a proper birational morphism between them. This result is the key ingredient in the theory of motivic integration. In the rest of the talk, Mustaţă explained how this result can be used to relate the log canonical threshold to the codimension of certain subsets in spaces of arcs. This result can be considered an analogue in the context of spaces of arcs of Igusa's results in the p-adic setting.

Invariants of singularities in positive characteristic: The invariants of singularities that appear in birational geometry (in characteristic zero) are defined via valuations, or equivalently, via resolution of singularities. In positive characteristic, one can define invariants in a very algebraic way, exploiting the Frobenius morphism. In this talk. Mustaţă discussed analogies, results, and conjectures relating the invariants such as the log canonical threshold with algebraic invariants in positive characteristic.

2. Aspects of Homogeneous Dynamics

The Geometry and Dynamics of Numbers: Lindenstrauss began his public lecture by explaining Minkowski's idea that the study of lattice points in $\mathbb{R}^n$ implies many deep results about number fields. Given the determinant, some lattices be-



Elon Lindenstrauss delivering one of his lectures at TIFR.

have more "unboundedly" than others in the sense that they contain very short vectors. The group $SL(n,\mathbb{R})$ acts on the space of lattices in $\mathbb{R}^n$ while preserving the determinant. The key observation here is that orbits of actions of large enough diagonal subgroups of $SL(n,\mathbb{R})$ have to either contain an "unbounded" lattice or be periodic. Two theorems from number theory, Diophantine approximation and the existence of solutions to Pell's equation, can be demonstrated as elegant examples of these two situations. After mentioning the three dimensional setting (Dirichlet Theorem), Lindenstrauss discussed Cassels and Swinnerton-Dyer conjecture, which was reformulated by Margulis in the following way: in the moduli space of n-dimensional lattices, $SL(n,\mathbb{Z})\backslash SL(n,\mathbb{R})$, any orbit of the full diagonal subgroup of $SL(n,\mathbb{R})$ is either unbounded or periodic. It was orbserved by Cassels and Swinnerton-Dyer(1955) and Margulis (1997) that this conjecture would imply the famous Littlewood conjecture: $\forall(\alpha,\beta)\in\mathbb{R}^2$, $\inf_{n\in\mathbb{N}} n\,\|n\alpha\|\;\;\|n\beta\| = 0$, where $\|x\| = \min_{z\in\mathbb{Z}}|x-z|$. By studying dynamics on $SL(n,\mathbb{Z})\backslash SL(n,\mathbb{R})$, Einsiedler, Katok, and Lindenstrauss recently proved that the exceptional set of Littlewood's conjecture is of Hausdorff dimesion 0. Lindenstrauss also talked about applications of geometry of numbers to the study of number fields.

Rigidity of unipotent and diagonal flows: The general setting is always that of a homogeneous space $\Gamma\backslash G$ where G is a linear algebraic group and Γ one of its discrete subgroups. From Ratner's work, it is well known that unipotent actions are rigid: if H is a subgroup of G generated by unipotent one-parameter subgroups, then any H-invariant probability measure on $\Gamma\backslash G$ is homogeneous and any orbit of a one-parameter unipotent subgroup is equidistributed in the closure of the H orbit. A result of Mozes and Shah claims that for a sequence $\{\mu_i\}$ of natural measures on periodic H-orbits, there would be a subsequence converging to a homogeneous measure unless all points in their orbits escape to infinity. There remain several challenges in the domain of unipotent dynamics: spaces $\Gamma\backslash G$ of infinite volume, polynomial trajectories and effective estimates. Lindenstruass discussed these challenges, and some of the recent progress that has been made.

The second part of the lecture focused on the rigidity of diagonal flows. Although the linearization technique does *not* work in this case, there is a weaker analogue called isolation, that essentially says that any point close enough to (but not inside) a periodic orbit has an unbounded orbit itself. However, isolation techniques are proven only for $G = SL(n, \mathbb{R})$. Let A be the full diagonal subgroup; when $G = SL(2, \mathbb{R})$ the orbits are known to behave in a nasty way. But for $G = SL(n, \mathbb{R}), n \geq 3$, it is conjectured that any ergodic A-invariant measure is homogeneous. Lindenstruass discussed high entropy techniques, as well as the recent work of Maucourant and its relation to this conjecture.

Values of integral and non-integral forms: The main focus of this lecture was on homogeneous polynomials F that can be written in the form $F = \prod_{i=1}^{n}(\sum_{i=1}^{n} h_{ij}x_j)$ where $H = (h_{ij})$ is a non-degenerate $n \times n$ real matrix. Lindenstrauss discussed the example of such forms arising as the norm of an element in a number field of degree n given an integral basis. The objective is to study the distribution of $F(\mathbb{Z}^n)$, which is preserved under the action of $GL(n, \mathbb{Z})$ given by $\gamma.F(x) = F(x\gamma)$. One observes that the dynamics of $GL(n, \mathbb{Z})$ on the space Y_n of equivalence classes of F up to scaling is closely related to that of A on

$$PGL(n, \mathbb{Z})\backslash PGL(n, \mathbb{R}) = SL(n, \mathbb{Z})\backslash SL(n, \mathbb{R}).$$

Conditions on the distribution of $F(\mathbb{Z}^n)$ can be translated into conditions on the A-orbit of $[H] \in PGL(n, \mathbb{Z})\backslash PGL(n, \mathbb{R})$. By such translation, the reformulation of Margulis mentioned in his public lecture is equivalent to the original Cassels-Swinnerton-Dyer conjecture. Lindenstrauss explained his work with Einsiedler and Katok on the Hausdorff dimension of the exceptional set as well as more recent work with Einsiedler, Michel, and Venkatesh that gives an upper bound $\#\{orbit\ GL(n, \mathbb{Z}).F \subset Y_n\ of\ determinant\ D :\ F(\mathbb{Z}^n\backslash\{0\}) \cap [-\delta, \delta] = \emptyset\} <<_{\epsilon,\delta} D^{\epsilon}, \forall \epsilon, \delta, D$.

The Torus: Equidistribution and Stationary Measures: Suppose S is the multiplicative semigroup generated by two multiplicatively independent postive integers a and b. Let S act naturally on $T = \mathbb{R}/\mathbb{Z}$. The Furstenberg theorem asserts that any closed S-invariant subset of T is either full or finite. Furstenburg also conjectured that any ergodic S-invariant probability measure μ is either Lebesgue or finitely supported. Though the conjecture is still open, Rudolph and Johnson showed if $h(\mu, a) > 0$ then μ has to be Lebesgue measure. Lindenstrauss discussed how an effective Rudolph-Johnson theorem can be used to prove an effective version of Furstenberg's theorem. He then discussed how higher-rank analogues of these problems on $(\mathbb{R}/\mathbb{Z})^d, d > 1$ diverge into two cases. The first type of problems deal with two commuting matrices $A, B \in SL(d, \mathbb{Z})$ acting on $(\mathbb{R}/\mathbb{Z})^d$, there are results by Berend, Kalinin-Katok-Spatzier and Einsiedler-Lindenstrauss. The other extreme is a pair of non-commuting matrices A and B generating a Zariski-dense subgroup S in $SL(d, \mathbb{Z})$.

Mustata delivering one of his lectures at TIFR.

George Mackey Library

George Mackey in his office at Harvard, c. 1970s. Photo courtesy of the Mackey family.

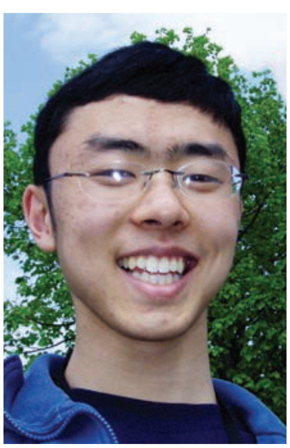
The Clay Mathematics Institute is honored to be the recipient of Professor George Mackey's extensive mathematical library — a gift to the Institute by his widow Alice Mackey. The library adds 1220 volumes to the 720 previously received from the family of Raoul Bott in 2005. The addition of these books is a great benefit to CMI, both in carrying out its specific functions and in creating a general atmosphere that is conducive to doing mathematics, including the presentation of workshops at One Bow Street.

George Mackey was born February 1, 1916, and died on March 15, 2006. He is survived by his widow, Alice Mackey, and his daughter, Ann Mackey. His mathematical work and legacy are beautifully described in a twenty-seven page article in the Notices of the American Mathematical Society (Volume 54, number 7, August 2007), to which thirteen authors, among them many of his twenty-three students, contributed.

George Mackey received a bachelors degree in physics from the Rice Institute (now Rice University) in 1938, where he studied chemistry, physics, and mathematics. As one of the top five William Lowell Putnam Contest winners, he earned a scholarship to Harvard for graduate work, where he received his doctorate in mathematics in 1942 under the direction of Marshall Stone. Following a one-year postdoctoral position at the Illinois Institute of Technology, Mackey returned to Harvard as an instructor. He was appointed the Landon T. Clay Professor of Mathematics and Theoretical Physics, and he remained at Harvard until he retired in 1985.

Mackey's main areas of interest were in representation theory, group actions, ergodic theory, functional analysis, and mathematical physics. He had a long and abiding interest in the mathematical foundations of quantum mechanics, an interest that led to his now classic work, *Mathematical Foundations of Quantum Mechanics* (1963). He was an influential and beloved figure, and a mentor to many.

2007 Olympiad Scholar Andrew Geng



Andrew Geng

On May 21, 2007, Andrew Geng of Westford, Massachusetts received the ninth Clay Mathematics Olympiad Scholar Award at the 36th USA Mathematical Olympiad Awards Ceremony in Washington, DC, the nation's premier high school level mathematics problem-solving competition. The USAMO is a two-day, nine-hour, six-question, essay-proof examination that is one of a series of national contests administered by the American Mathematics Competitions and sponsored by the Mathematical Association of America and several other organizations.

The Clay Mathematics Olympiad Scholar Award is given for the solution to an Olympiad problem judged most creative. It consists of a plaque and cash award to the recipient, and a cash award to the recipient's school. It is presented each year at the official awards dinner for the US American Mathematics Olympiad (USAMO) held in June in Washington, DC at the State Department Ballroom.

Andrew is currently enjoying classes at MIT where he has chosen to major in mathematics and devotes much of his free time to teaching in the Educational Studies Program.

Selected Articles by Research Fellows

MODHAMMED ABOUZAID

"On the Fukaya categories of higher genus surfaces," *Advances in Mathematics* (2007).

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"Compatibility of local and global Langlands correspondences," with Richard Taylor. *J. Amer. Math. Soc.*, 20-2 (2007), 467-493.

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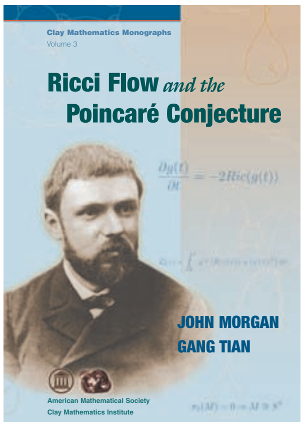
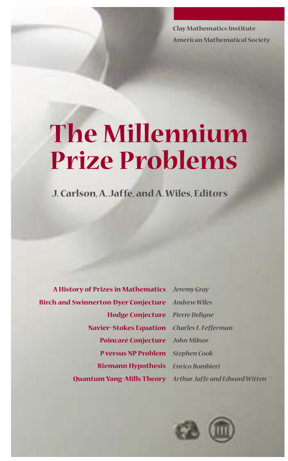
Books & Videos

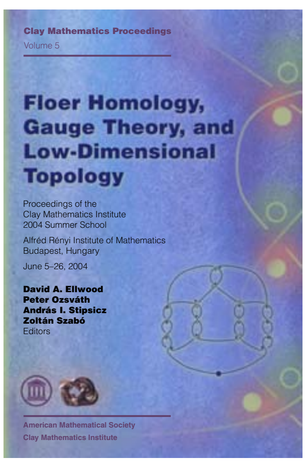
Analytic Number Theory; A Tribute to Gauss and Dirichlet; Editors: William Duke, Yuri Tschinkel. CMI/AMS, 2007, 265 pp. www.claymath.org/publications/Gauss_Dirichlet. This volume contains the proceedings of the Gauss–Dirichlet Conference held in Göttingen from June 20–24 in 2005, commemorating the 150th anniversary of the death of Gauss and the 200th anniversary of Dirichlet's birth. It begins with a definitive summary of the life and work of Dirichlet by J. Elstrodt and continues with thirteen papers by leading experts on research topics of current interest within number theory that were directly influenced by Gauss and Dirichlet.



Ricci Flow and the Poincaré Conjecture; Authors: John Morgan, Gang Tian. CMI/AMS, 2007, 521 pp., www.claymath.org/publications/ricciflow. This book presents a complete and detailed proof of the Poincaré Conjecture. This conjecture was formulated by Henri Poincaré in 1904 and has remained open until the recent work of Grigory Perelman. The arguments given in the book are a detailed version of those that appear in Perelman's three

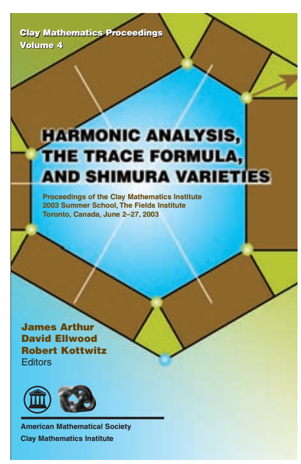


The Millennium Prize Problems; Editors: James Carlson, Arthur Jaffe, Andrew Wiles. CMI/AMS, 2006, 165 pp., www.claymath.org/publications/Millennium_Problems. This volume gives the official description of each of the seven problems as well as the rules governing the prizes. It also contains an essay by Jeremy Gray on the history of prize problems in mathematics.



Floer Homology, Gauge Theory, and Low-Dimensional Topology; Proceedings of the 2004 CMI Summer School at Rényi Institute of Mathematics, Budapest. Editors: David Ellwood, Peter Ozsváth, András Stipsicz, Zoltán Szábo. CMI/AMS, 2006, 297 pp., www.claymath.org/publications/Floer_Homology. This volume grew out of the summer school that took place in June of 2004 at the Alfréd Rényi Institute of Mathematics in Budapest, Hungary. It provides a state-of-the-art introduction to current research, covering material from Heegaard Floer homology, contact geometry, smooth four-manifold topology, and symplectic four-manifolds.

Lecture Notes on Motivic Cohomology; Authors: Carlo Mazza, Vladimir Voevodsky, Charles Weibel. CMI/AMS, 2006, 210 pp., www.claymath.org/publications/Motivic_Cohomology. This book provides an account of the triangulated theory of motives. Its purpose is to introduce the reader to Motivic Cohomology, to develop its main properties, and finally to relate it to other known invariants of algebraic varieties and rings such as Milnor K-theory, étale cohomology and Chow groups.



Surveys in Noncommutative Geometry; Editors: Nigel Higson, John Roe. CMI/AMS, 2006, 189 pp., www.claymath.org/publications/Noncommutative_Geometry. In June of 2000, a summer school on Noncom- mutative Geometry, organized jointly by the American Mathematical Society and the Clay Mathematics Institute, was held at Mount Holyoke College in Massachusetts. The meeting centered around several series of expository lectures that were intended to introduce key topics in noncommutative geometry to mathematicians unfamiliar with the subject. Those expository lectures have been edited and are reproduced in this volume.

Harmonic Analysis, the Trace Formula and Shimura Varieties; Proceedings of the 2003 CMI Summer School at Fields Institute, Toronto. Editors: James Arthur, David Ellwood, Robert Kottwitz. CMI/AMS, 2005, 689 pp., www.claymath.org/publications/Harmonic_Analysis. The subject of this volume is the trace formula and Shimura varieties. These areas have been especially difficult to learn because of a lack of expository material. This volume aims

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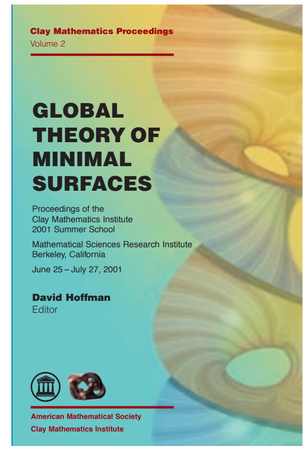
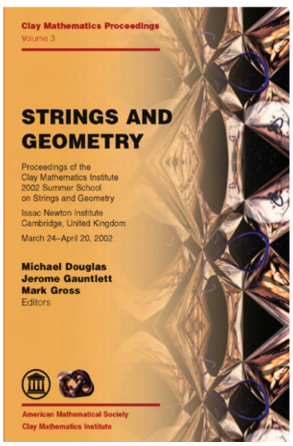
to rectify that problem. It is based on the courses given at the 2003 Clay Mathematics Institute Summer School. Many of the articles have been expanded into comprehensive introductions, either to the trace formula or the theory of Shimura varieties, or to some aspect of the interplay and application of the two areas.

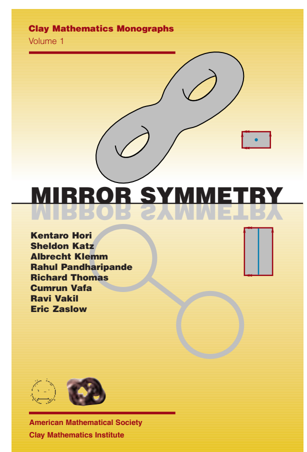
Global Theory of Minimal Surfaces; Proceedings of the 2001 CMI Summer School at MSRI. Editor: David Hoffman. CMI/AMS, 2005, 800 pp., www.claymath.org/publications/Minimal_Surfaces. This book is the product of the 2001 CMI Summer School held at MSRI. The subjects covered include minimal and constant-mean-curvature submanifolds, geometric measure theory and the double-bubble conjecture, Lagrangian geometry, numerical simulation of geometric phenomena, applications of mean curvature to general relativity and Riemannian geometry, the isoperimetric problem, the geometry of fully nonlinear elliptic equations, and applications to the topology of three-manifolds.





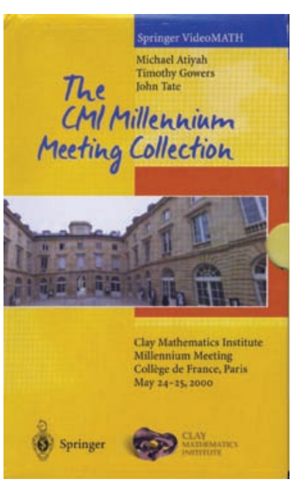
Strings and Geometry; Proceedings of the 2002 CMI Summer School held at the Isaac Newton Institute for Mathematical Sciences, UK. Editors: Michael Douglas, Jerome Gauntlett, Mark Gross. CMI/AMS publication, 376 pp., Paperback, ISBN 0-8218-3715-X. List: $69. AMS Members: $55. Order code: CMIP/3. To order, visit www.ams.org/bookstore.

Mirror Symmetry; Authors: Kentaro Hori, Sheldon Katz, Albrecht Klemm, Rahul Pandharipande, Richard Thomas, Ravi Vakil. Editors: Cumrun Vafa, Eric Zaslow. CMI/AMS publication, 929 pp., Hardcover. ISBN 0-8218-2955-6. List: $124. AMS Members: $99. Order code: CMIM/1. To order, visit www.ams.org/bookstore.



Strings 2001; Authors: Atish Dabholkar, Sunil Mukhi, Spenta R. Wadia. Tata Institute of Fundamental Research. Editor: American Mathematical Society (AMS), 2002, 489 pp., Paperback, ISBN 0-8218-2981-5, List $74. AMS Members: $59. Order code: CMIP/1. To order, visit www.ams.org/bookstore

Video Cassettes



The CMI Millennium Meeting Collection; Authors: Michael Atiyah, Timothy Gowers, John Tate, François Tisseyre. Editors: Tom Apostol, Jean-Pierre Bourguignon, Michele Emmer, Hans-Christian Hege, Konrad Polthier. Springer VideoMATH, © Clay Mathematics Institute, 2002. Box set consists of four video cassettes: The CMI Millennium Meeting, a film by François Tisseyre; The Importance of Mathematics, a lecture by Timothy Gowers; The Millennium Prize Problems, a lecture by Michael Atiyah; and The Millennium Prize Problems, a lecture by John Tate. VHS/NTSC or PAL. ISBN 3-540-92657-7. List: $119, EUR 104.95. To order, visit www.springer-ny.com (in the United States) or www.springer.de (in Europe).

These videos document the Paris meeting at the Collège de France where CMI announced the Millennium Prize Problems. The videos are for anyone who wants to learn more about these seven grand challenges in mathematics.

Videos of the 2000 Millennium event are available online and in VHS format from Springer-Verlag. To order the box set or individual tapes visit www.springer.com.

2008 Institute Calendar

JANUARY FEBRAURY MARCH	Cycles, Motives and Shimura Varieties at TIFR, Mumbai, India. January 3–12. Recent Progress on the Moduli Space of Curves at Banff International Research Station, Canada. March 16–21. CMI Workshop on K3s: modular forms, moduli, and string theory. March 20–23. Conference on Algebraic Cycles II at Ohio State University, Columbus, Ohio. March 24–29.
APRIL MAY	Additive Combinatorics, Number Theory, and Harmonic Analysis at the Fields Institute, Toronto, Canada. April 5–13. "The Music of the Primes," Clay Public Lecture by Marcus du Sautoy at MIT. May 8. Workshop on Global Riemannian Geometry National Autonomous University of Mexico, Cuernavaca (IMATE-UNAM Cuernavaca), Mexico. May 11–18. Clay Research Conference at MIT. May 12–13. HIRZ80 at Emmy Noether Research Institute for Mathematics at Bar Ilan University, Ramat Gan, Israel. May 18–23. Senior Scholars John Lott and Gang Tian at Institut Henri Poincaré. May 1–June 30.
JUNE JULY AUGUST	Conference on Motives, Quantum Field Theory and Pseudodifferential Operators, Boston University. June 4–14. A Celebration of Raoul Bott's Legacy in Mathematics at CRM, Montreal. June 9–20. Conference on Aspects of Moduli Theory at the De Giorgi Center, Scuola Normale Superior di Pisa June 16–28. Senior Scholar Rob Lazarsfeld at PCMI/IAS. July 6–26. CMI Summer School: Evolution Equations at the ETH, Eidgenossische Technische Hochschule, Swiss Federal Institute of Technology, Zurich, Switzerland. June 23–July 18. Algebraic Geometry, D-modules, Foliations and their Interactions, Buenos Aires, Argentina. July 14–24. 60 Miles: A conference in Honor of Miles Reid's 60th Birthday, LMS, London. July 18–19.
SEPTEMBER OCTOBER NOVEMBER DECEMBER	Senior Scholar Henri Gillet at the Fields Institute Program on Arithmetic Geometry, Hyperbolic Geometry and Related Topics. September 1–November 30. Senior Scholar Fedor Bogomolov at the Centro di Ricerca Matematica Program on Groups in Algebraic Geometry. September 1–November 30. Senior Scholar Richard Schoen at the Mittag-Leffler Institute Program on Geometry, Analysis and General Relativity. September 1–December 15. Senior Scholar Werner Mueller at the MSRI Program on Analysis of Singular Spaces. September 10–October 21. Senior Scholar Gunther Uhlmann at the MSRI Program on Analysis of Singular Spaces. September 16–December 15.

The line drawing above was created by Polish artist Marlena Bocian. It is a rendering of "Figureight Knot Complement vii/ CMI," by Helaman Ferguson. This elegant bronze sculpture was commissioned by CMI in 1999 and designed and cast by Ferguson to be given as an award. It has also established a distinct identity for the Clay Mathematics Institute by figuring, as the icon, in the Clay logo. Bocain's drawing will replace older photographic images of the topological sculpture that had been used in earlier versions of the Institute's logo.

Marlena Bocian, a European artist, earned her degree in art and business administration in Poland and she continues to pursue her studies here in the States between her own painting and other work. She exhibits frequently in galleries on both continents, showing recent works that emphasize science, math, and the women of these worlds.

Top photo: *Peter Ozsváth presenting his talk "Holomorphic disks and knot invariants" at the inaugural Clay Research Conference at Harvard on May 14, 2007.*

Middle photo: *Princeton University Professor Ingrid Daubechies next to Milton Academy mathematics instructor Keith Hilles-Pilant and his students (from left to right): Hae-Jin Koo (in front), TaeYoon Song (in back), then Elsie Kenyon, Clare Choi, Sanghoon Ouh, Hee Seung Yang, Tim Walsh, and finally Fabrice Jules of the Math Department of the Lycée Duby at Aix-en-Provence, France, who was visiting Milton at the time.*

Bottom photo: *Cliff Taubes lecturing at the CMI workshop on "Recent Developments in Symplectic Topology."*

Workshops at CMI

CMI holds a limited number of workshops at its One Bow Street office in Cambridge, Massachusetts. Although the format is flexible, these are typically of three to six days duration with eight to sixteen participants. Participants are fully funded. Workshops in topics undergoing rapid development can be organized with little lead time. Please contact Jim Carlson (carlson@claymath. org) or David Ellwood (ellwood@claymath.org) to discuss a proposal.

Workshops to date have been held on the following topics: Goodwillie Calculus (Haynes Miller and Jack Morava), Algebraic Statistics and Computational Biology (Bernd Sturmfels, Lior Pachter, and Seth Sullivant), Eigenvarieties (Barry Mazur and Richard Taylor), Moduli Spaces of Vector Bundles (Emma Previato and Montserrat Teixidor-i-Bigas), Hopf Algebras and Props (David Ellwood, Jean-Louis Loday, and Richard Stanley), Recent Developments in Symplectic Topology (Denis Auroux, Victor Guillemin, Tomasz Mrowka, and Katrin Wehrheim), Recent Developments in Higher-dimensional Algebraic Geometry (in conjunction with the 2007 Clay Research Conference), Solvability and Spectral Instability (Nils Dencker and Maciej Zworski), Computational Arithmetic Geometry (Jim Carlson, David Harvey, Kiran Kedlaya and William Stein), Rational Curves and Diophantine Problems over Function Fields (Brendan Hassett, Johan de Jong, Jason Starr and Yuri Tschinkel).

See www.claymath.org/workshops

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