Clay Research Conference

The inaugural Clay Research Conference was held at Harvard University on May 14 and 15 at the Science Center. The lectures covered a wide range of fields: topology, algebraic geometry, number theory, real and complex dynamics, and geometric group theory. The Clay Research Awards (see sidebar and following article) were presented on the afternoon of May 14. Awardees were:

Alex Eskin (University of Chicago) for his work on rational billiards and geometric group theory, in particular, his crucial contribution to joint work with David Fisher and Kevin Whyte in establishing the quasi-isometric rigidity of sol.

Christopher Hacon (University of Utah) and James McKernan (UC Santa Barbara) for their work in advancing our understanding of the birational geometry of algebraic varieties in dimension greater than three, in particular, for their inductive proof of the existence of flips.

Michael Harris (Université de Paris VII) and Richard Taylor (Harvard University) for their work on local and global Galois representations, partly in collaboration with Laurent Clozel and Nicholas Shepherd-Barron, culminating in the solution of the Sato-Tate conjecture for elliptic curves with non-integral j-invariants.

Landon Clay, joined by James Carlson, congratulating Christopher Hacon (right front) and James McKernan (right rear) after they received the Clay Research Award.

Conference speakers were Peter Ozsváth, *Holomorphic disks and knot invariants*; William Thurston, *What is the future for 3-dimensional geometry and topology?*; Shigefumi Mori, *Recent progress in higher dimensional algebraic geometry I*; Alessio Corti, *Recent progress in higher dimensional algebraic geometry II*; Mark Kisin, *Modularity of 2-dimensional Galois representations*; Richard Taylor, *The Sato-Tate conjecture*; Curtis McMullen, *Algebraic dynamics on surfaces*; Alex Eskin, *Dynamics of rational billiards*; and David Fisher, *Coarse differentiation and quasi-isometries of solvable groups*.

Clay Research Awards

Previous recipients of the award, in reverse chronological order are:

2005	Manjul Bhargava (Princeton University) Nils Dencker (Lund University, Sweden)
2004	Ben Green (Cambridge University) Gérard Laumon (Université de Paris-Sud, Orsay) Bao-Châu Ngô (Université de Paris-Sud, Orsay)
2003	Richard Hamilton (Columbia University) Terence Tao (University of California, Los Angeles)
2002	Oded Schramm (Theory Group, Microsoft Research) Manindra Agrawal (Indian Institute of Technology, Kanpur)
2001	Edward Witten (Institute for Advanced Study) Stanislav Smirnov (Royal Institute of Technology, Stockholm)
2000	Alain Connes (College de France, IHES, Vanderbilt University) Laurent Lafforgue (Institut des Hautes Études Scientifiques)
1999	Andrew Wiles (Princeton University)

The Clay Mathematics Institute presents the Clay Research Award annually to recognize major breakthroughs in mathematical research. Awardees receive the bronze sculpture "Figureight Knot Complement vii/CMI" by Helaman Ferguson and are named Clay Research Scholars for a period of one year. As such they receive substantial, flexible research support. Awardees have used their research support to organize a conference or workshop, to bring in one or more collaborators, to travel to work with a collaborator, and for other endeavors.

Clay Research Conference

The recipients of the 2007 Clay Research Awards with Landon and Lavinia Clay. From left to right: Richard Taylor, Michael Harris, Lavinia Clay, Landon Clay, Alex Eskin, Christopher Hacon, and James McKernan.

Alessio Corti was unable to attend the conference because of security difficulties in London. Shigefumi Mori, on very short notice, graciously agreed to give Corti's lecture. David Fisher presented the work of Eskin, Fisher, and Whyte on the quasi-isometric rigidity of sol, a major problem in geometric group theory. Alex Eskin and Curtis McMullen spoke about problems in dynamics: rational billiards and the dynamics on complex algebraic surfaces, respectively. These are just two of the many active areas in the field of dynamics, which has important points of contact with number theory, representation theory, and other areas. Peter Ozsváth and William Thurston spoke about the past, present, and future of topology: Ozsváth on new knot invariants that have resolved many long-standing problems in the field; Thurston on the geometrization conjecture and what remains to be understood after the groundbreaking work of Perelman.

Videos of the lectures are available at **www.claymath.org/publications/videos**

Abstracts of Talks

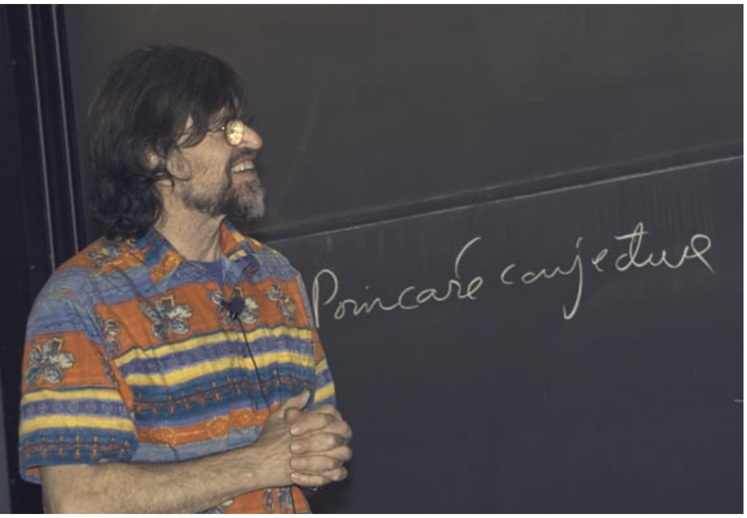
Peter Ozsváth (Columbia University)
Holomorphic disks and knot invariants

Heegaard Floer homology is an invariant for three- and four-manifolds defined using techniques from symplectic geometry. More specifically, a Heegaard diagram is used to set up an associated symplectic manifold equipped with a pair of Lagrangian submanifolds. The Heegaard Floer homology groups of the three-manifold are then defined as the homology groups of a chain complex whose differential counts pseudo-holomorphic disks in the symplectic manifold. These methods also lead to invariants for knots, links, and four-manifolds.

Ozsváth discussed applications of this theory, along with some recent calculational advances that have rendered the knot Floer homology groups purely combinatorial. Heegaard Floer homology was defined in joint work with Zoltan Szabo, in addition to further work with other collaborators, including Ciprian Manolescu, Sucharit Sarkar, and Dylan Thurston.

William Thurston (Cornell University)
What is the future for 3-dimensional geometry and topology?

Though the geometrization conjecture is now proved, there is still much to be understood in achieving a simple big picture of three-manifold topology, e.g., how is the set of hyperbolic manifolds, ordered by volume, organized; what are the deeper connections with number theory via the field of traces associated with the length of geodesic loops?



William Thurston delivering his talk at the conference.

Shigefumi Mori (RIMS, Kyoto)
Recent progress in higher dimensional algebraic geometry I

Higher dimensional algebraic geometry has recently undergone major developments related to the minimal model program. Mori reviewed the basic definitions of the minimal model program and surveyed some of the recent achievements and their applications.

Alessio Corti (Imperial College, London)
Recent progress in higher dimensional algebraic geometry II

Corti explained some of the key ideas in the recent work of Hacon and McKernan on the higher dimensional minimal model program for algebraic varieties.

Mark Kisin (University of Chicago)
Modularity of 2-dimensional Galois representations

Kisin discussed the recently proved conjecture of Serre on 2-dimensional mod p Galois representations, and its implications for modularity of 2-dimensional motives and p-adic Galois representations.

Richard Taylor (Harvard University)
The Sato-Tate conjecture

A fixed elliptic curve over the rational numbers is known to have approximately p points modulo p for any prime number p. In about 1960, Sato and Tate gave a conjectural distribution for the error term. Laurent Clozel, Michael Harris, Nicholas Shepherd-Barron and Taylor recently proved this conjecture in the case that the elliptic curve has somewhere multiplicative reduction.

Curtis McMullen (Harvard University)
Algebraic dynamics on surfaces

McMullen discussed the role of Hodge theory, Salem numbers, and Coxeter groups in the construction of new dynamical systems on compact complex surfaces.

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Alex Eskin, (University of Chicago)

Dynamics of rational billiards

Eskin called his talk "a short and extremely biased survey of recent developments in the study of rational billiards and Teichmüller dynamics."

David Fisher (Indiana University)

Coarse differentiation and quasi-isometries of solvable groups

In the early 1980s Gromov initiated a program to study finitely generated groups up to quasi-isometry. This program was motivated by rigidity properties of lattices in Lie groups. A lattice T in a group G is a discrete subgroup where the quotient G/T has finite volume. Gromov's own major theorem in this direction is a rigidity result for lattices in nilpotent Lie groups.

In the 1990s, a series of dramatic results led to the completion of the Gromov program for lattices in semisimple Lie groups. The next natural class of examples to consider are lattices in solvable Lie groups, and even results for the simplest examples were elusive for a considerable time. Fisher's joint work with Eskin and Whyte in which they proved the first results on quasi-isometric classification of lattices in solvable Lie groups was discussed. The results were proven by a method of coarse differentiation, which was outlined.

Some interesting results concerning groups quasi-isometric to homogeneous graphs that follow from the same methods will also be described.

Satellite Workshop at the Clay Mathematics Insitute May 16–17

A satellite workshop held at the Clay Mathematics Institute in the days following the Conference consisted of more detailed talks on recent progress in higher dimensional algebraic geometry. On this occasion, Christopher Hacon and James McKernan spoke on the existence of flips and MMP scaling to an audience of advanced graduate students in the field.

Clay Research Awards

The Clay Research Awards

Below is a brief account of the mathematics of the work for which each of the three Clay Research Awards were given. – jc

1. Minimal Models in Algebraic Geometry

Let X be projective algebraic variety over the complex numbers, that is, the set of common zeroes of a system of homogeneous polynomial equations. The meromorphic functions on X form a field, the *function field* of X. For the Riemann sphere (the projective line $\mathbf{CP}^1$) this field is $\mathbf{C}(t)$, the field of rational functions in one variable. For an elliptic curve $y^2 = x^3 + ax + b$, it is the field obtained by adjoining the algebraic function $y = \sqrt{x^3 + ax + b}$ to $\mathbf{C}(x)$. Two varieties are *birationally equivalent* if they have isomorphic function fields.

The *birational equivalence problem* is a fundamental one in algebraic geometry. Given two varieties X and Y, how do we recognize whether they are birationally equivalent? In the case of elliptic curves, there is an easy answer: the fields are isomorphic if and only if the quantity b^2/a^3 is the same in both cases. What can we say about other varieties? On what data does the birational equivalence class of a variety depend?

Consider first the case of complex dimension one. Every algebraic curve is birational to a smooth one, its normalization. Thus two curves are birational if and only if their smooth models are isomorphic. Consequently, the birational equivalence problem is the same as the moduli problem. Take, for example, the algebraic curves defined by the affine equations $x+y = 1$, $x^2+y^2 = 1$ and $x^2+y^2+x^3+y^3 = 0$.

The first two curves are smooth and isomorphic to the Riemann sphere, as one sees by stereographic projection. The last curve has one singular point, but its smooth model is the Riemann sphere, as we see by the parametrization

$$x = -(1 + t^2)/(1 + t^3),\ y = -t(1 + t^2)/(1 + t^3).$$

Thus all three varieties are birationally equivalent, with function field $\mathbf{C}(t)$.

Varieties of higher dimension are birationally equivalent to a smooth one by Hironaka's resolution of singularities theorem. Nonetheless, this powerful result does not answer the birational equivalence problem. To see why, consider a smooth algebraic