Interview with Research Fellow Mircea Mustata

James Carlson and Mircea Mustata at the Tata Institute of Fundamental Research in Mumbai where CMI's 2007 "Clay Lectures on Mathematics" took place.

What first drew you to mathematics? What are some of your earliest memories of mathematics?

I am afraid that I don't have any math related memories from my early childhood. I began to show an interest in mathematics in elementary school, around the sixth grade. At that point more challenging problems started to come up, and one started doing rigorous proofs in plane Euclidean geometry. I was reasonably good at it, and there were interesting problems around, so I enjoyed doing it.

Could you talk about your mathematical education? What experiences and people were especially influential?

Maybe the turning point was when I started going to math olympiads, and to the circles organized around these contests. The main upshot was being around other kids who were enthusiastic about math. I then realized that I enjoyed spending my spare time doing math, and that this was not something completely out of ordinary. On the downside, I was never really good at these competitions, and at some point this got a bit frustrating. With hindsight, I think I shouldn't have spent this much time just with the olympiads, though this is what kept me being interested in math all through high school.

Did you have a mentor? Who helped you develop your interest in mathematics, and how?

I don't think that I had a real mentor while growing-up, though there have always been people around who influenced me. A key role was a tutor I had in the last grade in high school. That year I failed in one of the early stages of the math olympiad, and to cheer me up he gave me some books to read: some general topology, some real and complex analysis. This was my first encounter with serious mathematics, and it was an exciting experience. It certainly made the transition from high school to college much smoother.

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Profile

It would have been good to have a mentor during the first college years, but unfortunately I didn't have one. It would have helped in getting a better view of mathematics as a whole, and it might have motivated me to look at certain directions that I didn't know they existed until much later.

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You were educated in Romania. Could you comment on the differences between mathematical education in Romania and in the US?

One thing that I think is good in Romania is that kids encounter mathematical reasoning at an earlier stage (through axiomatic Euclidean geometry, for example). Another feature of the Romanian education is that study during college is very focussed: with a couple of minor exceptions, all courses I have taken were in math (with the embarrassing outcome that I did not take any college level course in physics). With so many math courses, one could get a strong background. The downside is that one insists maybe a bit too much on learning a lot of material, and on reading many books, and not really on using this information. What I like in the US is that students can get a taste of research at a very early stage. Myself, I started working on a problem only after I graduated from University. I was at the time a master's student in Bucharest.

What attracted you to the particular problems you have studied?

In my case, chance played a bit role. For example, I learned about the problem that influenced most of my work from my advisor, David Eisenbud, and from Edward Frenkel. At the time, by looking at several examples, they came up with a bold and unexpected conjecture. They very generously shared it with me, and this got me started in a completely new direction from what I had been doing.

In general, there are various reasons why I end up working on a particular problem. Most of the time, it is because it relates to other things I have thought about before. Of course, it helps if I believe that it is a problem other people would care about. In general, I find that problems that seem to have connections with other fields are particularly appealing. At least, they motivate me to try to learn some new things (though I have to say that in many cases it turned out that the new things were too hard and I got stuck).

Can you describe your research in accessible terms? Does it have applications to other areas?

Most of my research deals with singularities of algebraic varieties. These varieties are geometric objects defined by polynomial equations. It turns out that in many cases of interest, these objects are not "smooth-shaped"—they have singularities. In fact, what makes the local structure interesting are precisely these singularities. In my research, I deal with invariants that measure the singularities. Part of the motivation for what I do comes from questions that come up in the classification theory of algebraic varieties. Besides this, I don't know whether this topic has applications to other areas, but I believe that it has strong connections with various fields. In fact, the invariants that I have mentioned have all different origins (in valuation theory, commutative algebra or the theory of differential operators), but they turn out to be all related in ways that are still to be understood.

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What research problems and areas are you likely to explore in the future?

I always have a hard time saying which problems I will work on in six months (which makes writing NSF proposals a bit tricky). On the other hand, I have some long-term projects that I think about on and off over the years. One problem that's been on my mind for a while has to do with the connections between some invariants of singularities that are defined in characteristic zero using valuations (or equivalently, using resolutions of singularities), and other invariants in positive characteristic that came out of an area in commutative algebra called tight closure theory. There is evidence that the connections between these invariants are quite subtle, and they have to do with the arithmetic properties of the varieties involved. At this point it is not clear whether there is a good framework for understanding these connections, but at least, this motivates me to learn a bit of number theory.

Could you comment on collaboration versus solo work as a research style? Are certain kinds of problems better suited to collaboration?

I believe that collaboration has more to do with personality than with the particular problems. Most of my work was done in collaboration, and I think that in general, this social aspect is a very rewarding one in our work (this is probably one of the things I got from my advisor). In my case, however, it is not so much a matter of choice: I realized that I get most of the ideas by talking to people --even when I don't fully understand what they are saying. On the other hand, I enjoy also the part of the work that's done in private, and I always need to take the time to think about a problem on my own.

What do you find most rewarding or productive?

Maybe the most rewarding experience is when you get an intuition how certain disparate pieces might fit together. I am not talking about figuring out how to prove a precise result, usually by the time the details are cleared the enthusiasm cools down. But rather about the moment when you realize that certain things might be connected in a way you hadn't expected. Of course, this does not happen very often, and sometimes this intuition is wrong, but it is always exhilarating, and it pays off for all the moments when I hit a dead end.

How has the Clay Fellowship made a difference for you?

Probably the most important thing was that for three years I had the freedom to choose where I want to be. I think that in general it made my postdoc years less stressful than they might have been: for example, I had no teaching duties (I ended up teaching two courses during this time, but this was my own choice, and actually got me much more enthusiastic about teaching). And of course, the stipend being pretty generous, I could appreciate the improvement in my life after being a graduate student.

What advice would you give to young people starting out in math (i.e., high school students and young researchers)?

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I think that it is good to get involved in research early on, finding an easy, but interesting problem to work on. On the other hand, once you figure out a field you want to work in, it might be good to allow time to get also a broader view of that field, in addition to the technical mastery required for working on a specific problem. It is true that this might be hard to put in practice nowadays because of the way the programs are structured.

What advice would you give lay persons who would like to know more about mathematics — what it is, what its role in our society has been and so on? What should they read? How should they proceed?

A direct way of figuring things out would be by talking to mathematicians (though convincing them to discuss math with a non-mathematician might be considered a personal achievement). The good news is that there are by now several popularization books, either about famous mathematicians or about famous mathematical problems. I believe they can convey what "doing mathematics" means, why certain problems are important, and sometimes they can even put forward and explain interesting mathematical concepts.

How do you think mathematics benefits culture and society?

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Like other sciences, mathematics fulfils a need for figuring out the world around us. The special place of mathematics is due to the fact that it deals with an abstract realm. It is a common misconception that because of this fact, mathematics is out of touch with reality. It is indeed true that mathematical constructions do not need to be validated by "reality" (and I personally find this aspect very appealing). However, one should keep in mind that as physics taught us in the last hundred years, even very abstract models can help us to understand our own world.

Please tell us about things you enjoy when not doing mathematics.?

Whenever I have time, I enjoy hiking, reading fiction or watching movies, though my favorite pastime lately has been playing with my daughter. Another thing I enjoy a lot, which is one of the perks of a mathematician's life, is traveling. I always dreamt about traveling when I was a kid, but until my final years in grad school, I didn't realize that this was the way to go.

Mircea Mustata at TIFR, Mumbai in December of 2007.

Mircea Mustata, a native of Rumania, finished the Ph.D program at University of California, Berkeley under the direction of David Eisenbud. Immediately afterwards he began his position as a Clay Research Fellow. He held this position from July 2001, to August 2004. During his time he visited Université de Nice, the Isaac Newton Institute (Cambridge), and Harvard University. In Septermber 2004, Mustata became Associate Professor of Mathematics at the University of Michigan. His reasearch is supported by the NSF and a Packard Fellowship.

His main research interest is in algebraic geometry, in particular in various invariants of singularities of algebraic varieties, such as minimal log discrepancies, log canonical thresholds, multiplier ideals, Bernstein-Sato polynomials or F-thresholds. Various points of view and techniques come in the picture when studying these invariants: resolutions of singularities, jet schemes, D-modules or positive characteristic methods. Other interests include birational geometry, asymptotic base loci and invariants of divisors, and toric varieties.

Recent Research Articles

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