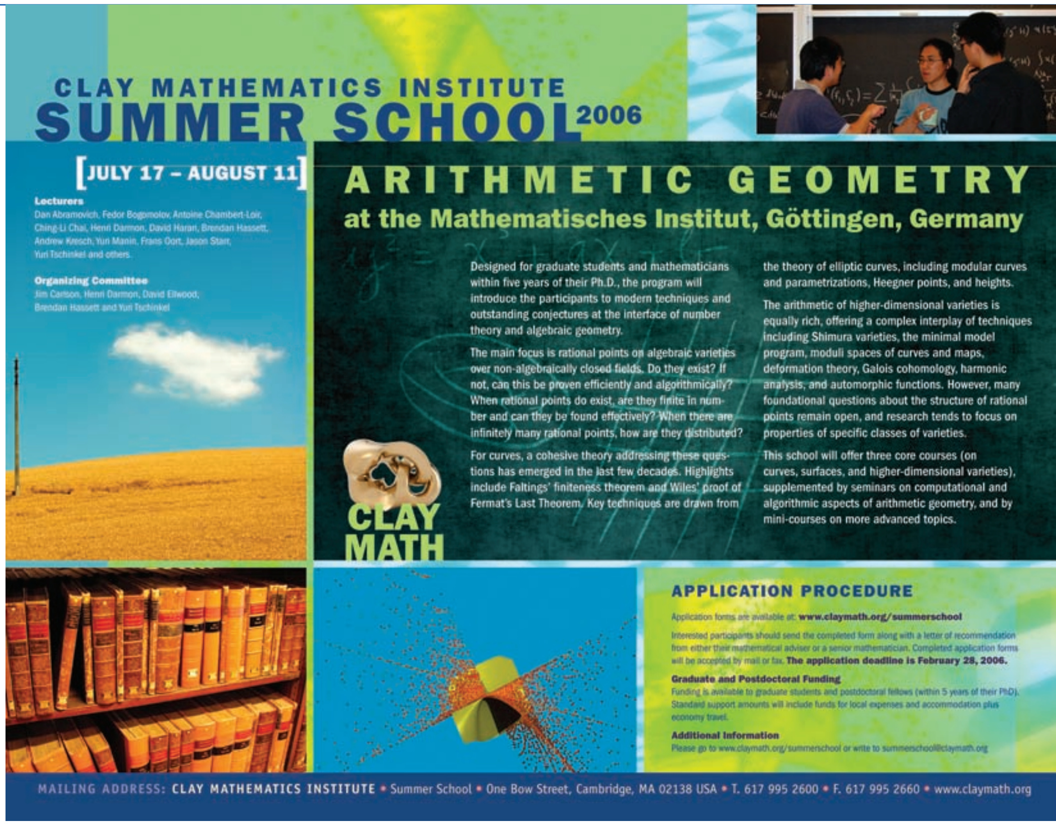
# Arithmetic Geometry at the Mathematisches Institut, Göttingen, Germany




**The 2006 CMI** Summer School was held at the Mathematisches Institut of Georg-August-Universität, Göttingen, Germany. A leading scientific center since the time of Gauss, Göttingen became a Mecca for mathematics in the 20th century, with Hilbert, Klein and Minkowski, the first three chair-holders at the Institute. The Mathematics Institute continues to be a leading international center for mathematical research, and the 107 participants[1] at the school enjoyed the excellent facilities and hospitality of the Institute from July 17 through August 11.

The main focus of the school was on rational points on algebraic varieties over non-algebraically closed fields. Do they exist? If not, can this be proven efficiently and algorithmically? When rational points do exist, are they finite in number and can they be found effectively? When there are infinitely many rational points, how are they distributed?

The school was organized around three core coures on *Curves, Surfaces, and Higher-dimensional Varieties*, supplemented by seminars on *Computational and Algorithmic aspects of Arithmetic Geometry*, and by mini-courses on more advanced topics. For *Curves*, a cohesive theory addressing these questions has emerged in the last few decades. Highlights include Faltings' finiteness theorem and Wiles' proof of Fermat's Last theorem. Key techniques are drawn from the theory of elliptic curves, including modular curves and parametrizations, Heegner points and heights. Henri Darmon gave five lectures on *Key Finiteness Theorems* (Mordell-Weil theorem, Faltings' theorem, Modular curves and Mazur's theorem, Fermat curves and Wiles' theorem), followed by a more specialized set of lectures, focusing on elliptic curves and their rational points with special emphasis on the Heegner point construction arising from modularity and the theory of complex multiplication.

Brendan Hassett, Andrew Kresch and David Harari gave courses on *the Arithmetic of Surfaces*. Hassett lectured on the geometry of rational surfaces, with a view toward arithmetic applications. Kresch lectured on the theory of descent and the Brauer-

1. In addition to the 107 participants funded by Clay, about 50 participants attended using their own funding.

*Brendan Hassett's course.*

Manin obstruction to the Hasse principle and weak approximation, and Harari presented concrete applications of the theory of descent. The arithmetic of higher-dimensional varieties is equally rich, offering a complex interplay of techniques including Shimura varieties, the minimal model program, moduli spaces of curves and maps, deformation theory, Galois cohomology, harmonic analysis, and automorphic functions. Yuri Tschinkel gave eight lectures on the distribution of rational points with respect to heights, focusing on varieties closely related to linear algebraic groups, e.g., equivariant compactifications of groups and homogeneous spaces. Topics covered included the circle method and hypersurfaces, toric varieties, height zeta functions of toric varieties, flag varieties, compactifications of additive groups, spherical varieties and conjectures on rational and integral points. Boris Moroz lectured on the classical application of the circle method to the Waring problem, and then explained how Deligne's estimates on exponential sums lead to Heath-Brown's theorem on cubic forms in ten variables. Jason Starr gave three lectures, one on *the Tsen-Lang Theorem*, one on *Arithmetic over Function Fields of Curves* and one on *Arithmetic over Function Fields of Surfaces*. Dan Abramovich lectured on *Birational Geometry for Number Theorists* (Kodaira dimension and the birational classification of varieties, the minimal model program, the conjectures of Lang and Vojta, Campana's program, and applications to specific number-theoretic problems). Finally, Antoine Chambert-Loir lectured on the distribution of points of "small" height on arithmetic varieties. Topics covered included equidistribution on the projective line, Arakelov geometry and equidistribution, and Equidistribution on Berkovich spaces

The school also included a three week-workshop on *Computational Aspects of Arithmetic Algebraic Geometry*, as well as advanced mini-courses on *Moduli of Abelian Varieties and p-Divisible Groups* (Frans Oort and Ching-Li Chai), *Zink's Theory of Displays and Crystalline Dieudonné Theory* (William Messing), *Non-commutative Cartier Isomorphism and Hodge-to-de Rham Degeneration* (Dmitry Kaledin), *Classical and Iterated Shimura Symbols* (Yuri Manin), *Geometry over Finite Fields* (Fedor Bogomolov), *André-Oort Conjectures* (Emmanuel Ullmo), and *Varieties over Finite Fields* (Bjorn Poonen).

*Downtown Göttingen, photo courtesy Ulrich Derenthal.*