

Arithmetic Geometry at the Mathematisches Institut, Göttingen, Germany

**CLAY MATHEMATICS INSTITUTE
SUMMER SCHOOL 2006**
[JULY 17 - AUGUST 11]

LECTURERS
Dan Abramovich, Fedor Bogomolov, Antoine Chambert-Loir, Ching-I Chai, Henri Darmon, David Harari, Brendan Hassett, Andrew Kresch, Yuri Manin, Frans Oort, Joseph Stien, Yuri Tschinkel and others.

ORGANIZING COMMITTEE
Jim Carlson, Henri Darmon, David Elwood, Brendan Hassett and Yuri Tschinkel

ARITHMETIC GEOMETRY
at the Mathematisches Institut, Göttingen, Germany

Designed for graduate students and mathematicians within five years of their Ph.D., the program will introduce the participants to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

The main focus is rational points on algebraic varieties over non-algebraically closed fields. Do they exist? If not, can this be proven efficiently and algorithmically? When rational points do exist, are they finite in number and can they be found effectively? When there are infinitely many rational points, how are they distributed?

For curves, a cohesive theory addressing these questions has emerged in the last few decades. Highlights include Faltings' finiteness theorem and Wiles' proof of Fermat's Last Theorem. Key techniques are drawn from the theory of elliptic curves, including modular curves and parametrizations, Heegner points, and heights.

The arithmetic of higher-dimensional varieties is equally rich, offering a complex interplay of techniques including Shimura varieties, the minimal model program, moduli spaces of curves and maps, deformation theory, Galois cohomology, harmonic analysis, and automorphic functions. However, many foundational questions about the structure of rational points remain open, and research tends to focus on properties of specific classes of varieties.

This school will offer three core courses (on curves, surfaces, and higher-dimensional varieties), supplemented by seminars on computational and algorithmic aspects of arithmetic geometry, and by mini-courses on more advanced topics.

APPLICATION PROCEDURE

Application forms are available at www.claymath.org/summerschool

Interested participants should send the completed form along with a letter of recommendation from either their thesis/mathematical advisor or a senior mathematician. Completed application forms will be accepted by mail or fax. **The application deadline is February 28, 2006.**

Graduate and Postdoctoral Funding

Funding is available to graduate students and postdoctoral fellows (within 5 years of their PhD). Standard support amounts will include funds for local expenses and accommodation plus economy travel.

Additional Information

Please go to www.claymath.org/summerschool or write to summerschool@claymath.org

MAILING ADDRESS: CLAY MATHEMATICS INSTITUTE • Summer School • One Bow Street, Cambridge, MA 02138 USA • T. 617 995 2600 • F. 617 995 2660 • www.claymath.org

The 2006 CMI Summer School was held at the Mathematisches Institut of Georg-August-Universität, Göttingen, Germany. A leading scientific center since the time of Gauss, Göttingen became a Mecca for mathematics in the 20th century, with Hilbert, Klein and Minkowski, the first three chairholders at the Institute. The Mathematics Institute continues to be a leading international center for mathematical research, and the 107 participants¹ at the school enjoyed the excellent facilities and hospitality of the Institute from July 17 through August 11.

The main focus of the school was on rational points on algebraic varieties over non-algebraically closed fields. Do they exist? If not, can this be proven efficiently and algorithmically? When rational points do exist, are they finite in number and can they be found effectively? When there are infinitely many rational points, how are they distributed?

The school was organized around three core courses on *Curves*, *Surfaces*, and *Higher-dimensional*

1. In addition to the 107 participants funded by Clay, about 50 participants attended using their own funding.

Varieties, supplemented by seminars on *Computational and Algorithmic aspects of Arithmetic Geometry*, and by mini-courses on more advanced topics. For *Curves*, a cohesive theory addressing these questions has emerged in the last few decades. Highlights include Faltings' finiteness theorem and Wiles' proof of Fermat's Last theorem. Key techniques are drawn from the theory of elliptic curves, including modular curves and parametrizations, Heegner points and heights. Henri Darmon gave five lectures on *Key Finiteness Theorems* (Mordell-Weil theorem, Faltings' theorem, Modular curves and Mazur's theorem, Fermat curves and Wiles' theorem), followed by a more specialized set of lectures, focusing on elliptic curves and their rational points with special emphasis on the Heegner point construction arising from modularity and the theory of complex multiplication.

Brendan Hassett, Andrew Kresch and David Harari gave courses on *the Arithmetic of Surfaces*. Hassett lectured on the geometry of rational surfaces, with a view toward arithmetic applications. Kresch lectured on the theory of descent and the Brauer-



Brendan Hassett's course.

Manin obstruction to the Hasse principle and weak approximation, and Harari presented concrete applications of the theory of descent. The arithmetic of higher-dimensional varieties is equally rich, offering a complex interplay of techniques including Shimura varieties, the minimal model program, moduli spaces of curves and maps, deformation theory, Galois cohomology, harmonic analysis, and automorphic functions. Yuri Tschinkel gave eight lectures on the distribution of rational points with respect to heights, focusing on varieties closely related to linear algebraic groups, e.g., equivariant compactifications of groups and homogeneous spaces. Topics covered included the circle method and hypersurfaces, toric varieties, height zeta functions of toric varieties, flag varieties, compactifications of additive groups, spherical varieties and conjectures on rational and integral points. Boris Moroz lectured on the classical application of the circle method to the Waring problem, and then explained how Deligne's estimates on exponential sums lead to Heath-Brown's theorem on cubic forms in ten variables. Jason Starr gave three lectures, one on *the Tsen-Lang Theorem*, one on *Arithmetic over Function Fields of Curves* and one on *Arithmetic over Function Fields of Surfaces*. Dan Abramovich lectured on *Birational Geometry for Number Theorists* (Kodaira dimension and the birational classification of varieties, the minimal model program, the conjectures of Lang and Vojta, Campana's program, and applications to specific number-theoretic problems). Finally, Antoine

Chambert-Loir lectured on the distribution of points of "small" height on arithmetic varieties. Topics covered included equidistribution on the projective line, Arakelov geometry and equidistribution, and Equidistribution on Berkovich spaces

The school also included a three week-workshop on *Computational Aspects of Arithmetic Algebraic Geometry*, as well as advanced mini-courses on *Moduli of Abelian Varieties and p -Divisible Groups* (Frans Oort and Ching-Li Chai), *Zink's Theory of Displays and Crystalline Dieudonné Theory* (William Messing), *Non-commutative Cartier Isomorphism and Hodge-to-de Rham Degeneration* (Dmitry Kaledin), *Classical and Iterated Shimura Symbols* (Yuri Manin), *Geometry over Finite Fields* (Fedor Bogomolov), *André-Oort Conjectures* (Emmanuel Ullmo), and *Varieties over Finite Fields* (Bjorn Poonen).



Downtown Göttingen, photo courtesy Ulrich Derenthal.