Clay Lectures at Cambridge University

On November 28, 2006, the Clay Institute launched the Clay Lectures in Mathematics, an annual series of talks given by CMI's past or current research fellows. The talks, extending over a period of four days, feature three research talks and one public lecture by each of two fellows.

The lecture series is aimed at young mathematicians, as well as experts from other fields, and aims to develop a theme related to the research fellows' interests.

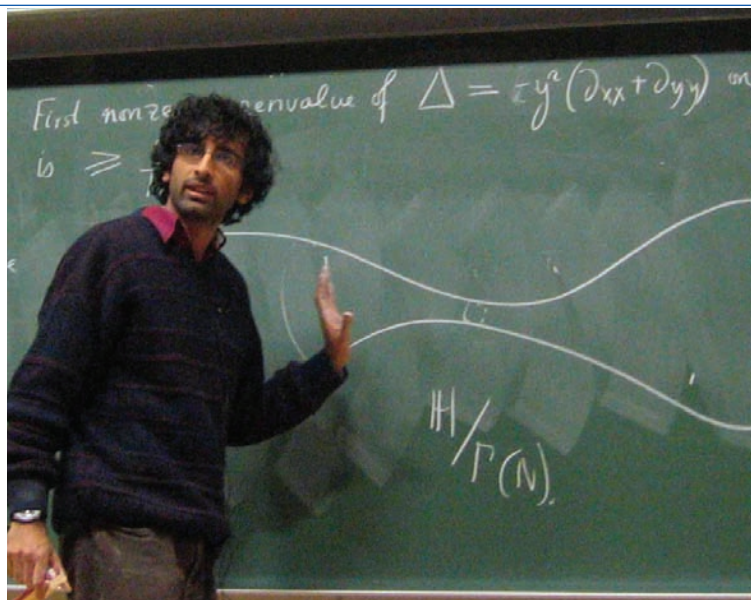
The Cambridge lectures were delivered November 28–December 1, by fellows Ben Green (2005–07) and Akshay Venkatesh (2004–06). Green is now at Cambridge University and Venkatesh is at the Courant Institute of Mathematics (New York University).

Venkatesh gave three lectures entitled *Abelian and Nonabelian Symmetry in Analytic Number Theory,* and a public lecture *Adding Square Numbers*. The operation of adding together square numbers (1, 4, 9, 16, 25, ...) gives rise to complex and beautiful patterns, that have motivated mathematicians from ancient times to the present.

Venkatesh's talks began with a discussion of harmonic analysis on the circle and one of its early triumphs in the 1918 paper of Hardy-Ramanujan, which gave an exact formula for the number of partitions of an integer (e.g., 4 = 3+1 = 2+1+1 = 2+2 = 1+1+1+1). Modular forms already made their appearance in the Hardy-Littlewood paper; behind them lurks the nonabelian group $SL_2(\mathbf{R})$.

Green gave three lectures entitled *Themes in Additive Combinatorics* and a public lecture, *Adding Prime Numbers*. While it has been noted that it is more natural to multiply primes than to add them, many famous open problems in number theory are concerned with adding primes. The study of these problems has led to some fascinating mathematics, including the question of the existence and abundance of arithmetic progressions in the primes. The latter question was resolved by the recent work of Green and Tao.

Green's lectures on additive combinatorics dealt with additive properties of sets of integers. If a set A is somewhat closed under addition, what is the structure of A? What do we need to know about A in order to be able to locate very regular structures, such as arithmetic progressions, inside A? How does the Fourier transform of A reflect the additive structure of A?



Akshay Venkatesh delivering one of the Clay Lectures at DPMMS.

Akshay Venkatesh (Courant Institute)

Lecture Series:

Abelian and Nonabelian Symmetry in Analytic Number Theory

- Some theorems of Hardy, Littlewood and Ramanujan. Partitions and sums of squares
- Some theorems of Linnik, Duke and Iwaniec
- A survey of modern developments

Public Lecture:

Adding Square Numbers

Ben Green (University of Cambridge / CMI)

Lecture Series:

Themes in Additive Combinatorics

The structure theory of set addition. Freiman's theorem

Gowers norms and nilsequences

The idempotent theorem: an application of additive combinatorics to harmonic analysis

Public Lecture:

Adding Prime Numbers

Ben Green delivering one of the Clay Lectures at DPMMS.

The public lectures bring recent research developments to the educated general public. For the Cambridge event, the Centre for Mathematical Sciences converted its central atrium into a massive lecture hall. A capacity crowd, with many of the Centre's 900 undergraduate mathematics majors, attended.

The 2007 lectures will be held at the Tata Institute in Mumbai, India, with the talks to be given by fellows Elon Lindenstrauss (2003–05) and Mircea Mustata (2001–04). Lindenstrauss and Mustata are at Princeton University and the University of Michigan, respectively.