CMI Summer Schools

2007	Homogeneous flows, moduli spaces, and arithmetic	De Giorgi Center, Pisa
2006	Arithmetic Geometry	Mathematisches Institut, Georg-August-Universität, Göttingen
2005	Ricci Flow, 3-manifolds, and Geometry	MSRI, Berkeley
2004	Floer Homology, Gauge Theory, and Low-dimensional Topology	Rényi Institute, Budapest
2003	Harmonic Analysis, Trace Formula, and Shimura Varieties	Fields Institute, Toronto
2002	Geometry and String Theory	Newton Institute, Cambridge UK
2001	Minimal surfaces	MSRI, Berkeley
2000	Mirror Symmetry	Pine Manor College, Boston

The Clay Mathematics Institute has conducted a program of research summer schools since 2000. Designed for graduate students and PhDs within five years of their degree, the aim of the summer schools is to furnish a new generation of mathematicians with the knowledge and tools needed to work successfully in an active research area. Three introductory courses, each three weeks in duration, make up the core of a typical summer school. These are followed by one week of more advanced minicourses and individual talks. Size is limited to roughly 100 participants in order to promote interaction and contact. Venues change from year to year, and have ranged from Cambridge, Massachusetts to Pisa, Italy. The lectures from each school are published in the CMI–AMS proceedings series, usually within two years' time.

Summer Schools
www.claymath.org/programs/summer_school

Summer School Proceedings
www.claymath.org/publications

2006 Arithmetic Geometry Summer School in Göttingen

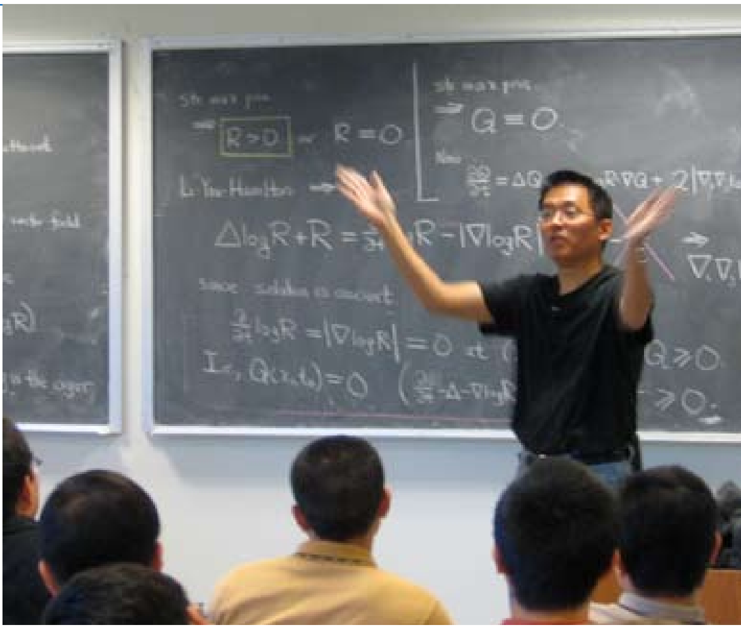
The 2006 summer school program will introduce participants to modern techniques and outstanding conjectures at the interface of number theory and algebraic geometry.

The main focus is the study of rational points on algebraic varieties over non-algebraically closed fields. Do such points exist? If not, can this fact be proven efficiently and algorithmically? When rational points do exist, are they finite in number, and can they be found effectively? When there are infinitely many rational points, how are they distributed?

For curves, a cohesive theory addressing these questions has emerged in the last few decades. Highlights include Faltings' finiteness theorem and Wiles' proof of Fermat's Last Theorem. Key techniques are drawn from the theory of elliptic curves, including modular curves and their parametrizations, Heegner points, and heights.

The arithmetic of higher-dimensional varieties is equally rich, offering a complex interplay of techniques including Shimura varieties, the minimal model program, moduli spaces of curves and maps, deformation theory, Galois cohomology, harmonic analysis, and automorphic functions. Many foundational questions about the structure of rational points remain open, however, and research tends to focus on properties of specific classes of varieties.

This summer school will offer three core courses (on curves, surfaces, and higher-dimensional varieties), supplemented by seminars on computational and algorithmic aspects of arithmetic geometry, and by mini-courses on more advanced topics.



Ben Chow's course on Ricci flow at the 2005 school

Summer School on Ricci Flow, 3-Manifolds, and Geometry

The 2005 school was held June 20–July 15 at the Mathematical Sciences Research Institute (MSRI) in Berkeley, California. Its aim was to give an in-depth introduction to the theory and applications of Ricci flow, beginning with Richard Hamilton's seminal 1982 paper and ending with a sketch of Perelman's claimed solution of the Poincaré conjecture.

Ben Chow gave a three-week-long course on Ricci flow, beginning with the background needed to understand Hamilton's theorem on 3-manifolds with positive Ricci curvature. This theorem states that Ricci flow transports an initial metric on a closed Riemannian manifold with positive Ricci curvature toward one that has constant positive sectional curvature; convergence is exponentially fast. Notes for Ben's course, partly based on the books *The Ricci flow: An introduction,* by Chow-Knopf and *Hamilton's Ricci flow,* by Chow-Lu-Ni, were posted on the web in advance of the school, giving participants a chance to prepare themselves. See www.claymath.org/programs/summer_school/2005/program.php

Bruce Kleiner and John Lott gave a three-week course on Perelman's work: Perelman's non-collapsing theorem, Perelman's reduced volume, the entropy functional, Kappa-ancient solutions and classification in three dimensions, analysis of the large curvature part of Ricci flow solutions, and applications to geometrization.

John Morgan and Jeff Cheeger gave a two-week course. Morgan spoke on geometrization: the eight three-dimensional geometries, prime decomposition of 3-manifolds, incompressible tori, Thurston's geometrization conjecture on 3-manifolds, and graph manifolds. Cheeger spoke on compactness theorems in Riemannian geometry, manifolds of nonnegative curvature, and Alexandrov spaces. Their lectures were followed by a one-week course on minimal surfaces by Tobias Colding, David Hoffman, and Gabriele La Nave.

The fourth week of the school was devoted to advanced courses that introduced participants to important problems of current research. Lectures were given by Jeff Cheeger, Bennett Chow, Tobias Colding, Richard Hamilton, David Hoffman, Bruce Kleiner, Gabriele La Nave, John Lott, Peng Lu, John Morgan, Andre Neves, Lei Ni, and Gang Tian.

The organizing committee for the school consisted of Gang Tian, John Lott, John Morgan, Bennett Chow, Tobias Colding, Jim Carlson, David Ellwood, and Hugo Rossi.

Ricci Flow Summer School participants

CMI Summer Schools

Bruce Kleiner's course on Perelman's work at the 2005 Ricci Flow School

The Ricci flow equation is much like the heat equation, which was first intensively studied by Joseph Fourier in his "Théorie Analytique de la Chaleur" in 1822. While the heat equation governs the way temperature in a material body evolves over time, the Ricci flow equation governs the evolution of an object's geometry, that is, its metric. The heat equation is linear, and any roughness in the initial temperature distribution smooths out as time increases. If now my coffee is very hot and my martini is very cold, then (alas) both will be lukewarm in the not-too-distant future. The Ricci flow equation, however, is nonlinear. Thus, while smoothing of the geometry can occur – little bumps of curvature can spread and disappear – curvature can also concentrate in some regions and lead to the formation of singularities. For example, join two round spheres by a smooth neck. Ricci flow progressively narrows the neck, which then pinches off. Thus, the geometry of a manifold can undergo not only quantitative but also qualitative change.

There are very few natural equations that can describe the evolution of the geometry of a manifold. On one side must be the time derivative of the metric tensor, the quantity that defines geometry. On the other side must be a tensor of the same kind, constructed from the metric tensor. The simplest such tensor besides the metric tensor itself is the Ricci tensor. The equation

$$\frac{\partial g_{ij}}{\partial t} = -2R_{ij}$$

is the natural (and inspired) guess. This is the Ricci Flow Equation, formulated in Richard Hamilton's celebrated 1982 paper. In that paper Hamilton showed that a compact three-manifold of positive definite Ricci curvature evolves by Ricci flow (and rescaling) to a manifold of constant positive sectional curvature. If the manifold is also simply-connected, then it must be the three-sphere. Subsequently, Hamilton made a detailed study of the development of singularities under Ricci flow, and he introduced and studied the notion of Ricci flow with surgery to deal with singularity formation. Underlying this effort was the allure of a proof of the Poincaré conjecture and, more generally, of Thurston's geometrization conjecture.

The Poincaré conjecture characterizes the simplest of all closed three-manifolds – the three-sphere – as the only 3-manifold in which closed loops (think of little strings) can be shrunk to a point without breaking them or letting them move off the sphere. Thurston's geometrization conjecture asserts that all possible closed three-dimensional manifolds can be built in standard ways from basic building blocks which we understand; it includes the Poincaré conjecture as a special case.

Grisha Perelman, in a series of three papers posted on arXiv.org beginning in November of 2002, announced a solution of the Poincaré and the geometrization conjectures using Ricci flow. His papers contained a series of breakthroughs that allowed one to successfully deal with all types of singularity formation, with the problems encountered in Ricci flow with surgery, and with the formation of hyperbolic pieces as time tends to infinity. In particular, Pereleman's papers (listed below) gave a crucial argument to show that the set of surgery times is discrete.

Perelman's work set off a flurry of activity aimed at understanding and verifying his claims. Most of that work to date is available publically via the web page www.math.lsa.umich.edu/~lott/ricciflow/perelman.html, maintained by Bruce Kleiner and John Lott. Perelman's first two papers on Ricci flow were:

arXiv:math.DG/0303109 — November 11, 2002
The entropy formula for the Ricci flow and its geometric applications

arXiv:math.DG/0303109 — March 10, 2003
Ricci flow with surgery on three-manifolds