So what are you trying to say, understand? e
You might want to divide E into two, to have an L^2 eigenvector inside

---

Cuntz's talk has lots of interesting constructions.

Γ group, F finite subset of Γ

$$\Sigma_F = \left\{ M \subset \Gamma \;\middle|\; \begin{matrix} s^{-1}t \in F \\ \forall s,t \in M \end{matrix} \right\} = \{ M \subset \Gamma \mid M^{-1}M \subset F \}$$

so F contains e.

$$s \in M \Rightarrow s^{-1}s = e \in F$$

$$s,t \in M \Rightarrow s^{-1}s,\ s^{-1}t,\ t^{-1}s,\ t^{-1}t \in F$$

Σ_F is a simplicial complex.

$A \xrightarrow{\Phi} A$ CP map of finite rank ~~is somehow~~ seems to be equivalent to factorization through a finite diml C^* alg.: $A \xrightarrow{\psi} F \xrightarrow{\varphi} A$ unital and CP.

Example. $A = C(X)$ X compact. Take a partition of unity $\sum_{i=1}^n \varphi_i = 1$ and points $(x_i)_1^n$ such that say $\varphi_i(x_i) = 1$. Then $F = \mathbb{C} \times \overset{n}{\cdots} \times \mathbb{C}$, n times, $A \xrightarrow{\psi} F$ sends a to $(a(x_i))_1^n$, and $\varphi = (\varphi_i)$, that is the ith idempotent e_i goes to $\varphi_i \in A$. Then

$$(\varphi\psi)(a) = \text{the function } \sum_{i=1}^n \varphi_i \, a(x_i)$$

Joachim mentioned $A = C_r(\Gamma)$, which is nuclear $\Longleftrightarrow$ Γ is amenable. In trying to reconstruct the factorization $A \xrightarrow{\psi} F \xrightarrow{\varphi} A$ he mentioned coproduct $C_r(\Gamma) \longrightarrow C_r(\Gamma) \otimes C_r(\Gamma)$ followed by $1 \otimes$ (some linear functional). Also he mentioned Schur product & convolution product.

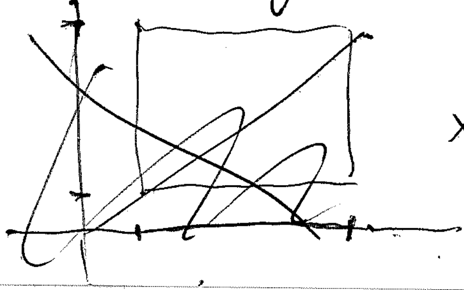
Γ amenable $\Longleftrightarrow$ $\exists$ net (h_i) of pos type functions $h_i \in C_c(\Gamma)$ such that $\lim h_i = 1$ uniformly on compact subsets of Γ. (French lady)

$C^*_r(\Gamma)$ exact $\Longrightarrow$ ($C^*_r(\Gamma) \hookrightarrow B(H)$ is nuclear. $\qquad H = \ell^2(G)$

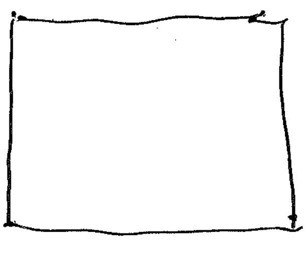
exact $\Longleftrightarrow$ $\exists$ net $\phi_i : C^*_r(\Gamma) \to B(H)$, CP finite rank, $\forall a \quad \phi_i(a) \longrightarrow a$.

You should be able to take $\Gamma = \mathbb{Z}$ and and show $C_r(\Gamma)$ (which should be $C(S^1)$) is nuclear, and thus see an example of Γ amenable.

Ex: $A = C(S^1) \cong C_r(\mathbb{Z})$. You want to understand why A is nuclear, and why $\mathbb{Z}$ is amenable, these are equivalent. Joachim explained why $C(X)$ is nuclear, you have a picture now of a CP map $C(X) \longrightarrow C(X)$



$X \times Y$:



$\int d\mu_x(y)\, f(y)$

A CP map $C(Y) \to C(X)$ seems to be a continuous family $x \longmapsto d\mu_x(y)$ of (≥ 0) measures on Y parametrized by $x \in X$. Joachim's construction: Here $Y = X$ and you want to approximate the identity $C(X) \xrightarrow{1} C(X)$, so $d\mu_x(x') = \delta_x(x')$ delta measure at x. You are supposed to get finite rank operator. Think of X as $[0,1]$. ~~There are two ingredients, namely~~ Think of the Thom form for $X \hookrightarrow X \times X$. Since all this takes place using functions and measures you want something like a heat kernel. ~~Things like this are not Vague~~ Two ingredients: Partition of unity for fine covering, and a probability measure in each open set of the covering. Use uniform cont. for cont. fns.

Next look at $A = C_r(\mathbb{Z})$ which is a special case of $C_r(\Gamma)$. Here $C_r(\Gamma)$ nuclear is supposed to equiv. to Γ amenable, so it seems there is link between a partition of unity on the dual group and net of pos type functions on the group approaching 1. ?

What about $C_r(\mathbb{Z})$? What does a CP map $C_r(\mathbb{Z}) \to C_r(\mathbb{Z})$ look like? Again it should corresp. to a family of probability measures on ~~the dual~~ S^1 param. by S^1

This looks interesting. Maybe you should approach this from the amenable end. $\exists$ invariant mean. One thing seems clear: you want ~~C~~ positive functionals $A \longrightarrow \mathbb{C}$, ~~which should amount to~~ finitely many, corresponding to the prob. measures chosen before. Why not subdivide S^1 evenly.

Notice that using δ measures ?

Γ group $\quad F$ finite subset of Γ

simplicial complex $\left\{ M \subset \Gamma \,\middle|\, \begin{matrix} M \neq \phi \\ M^{-1}M \subset F \end{matrix} \right\} = \Sigma_F$

$M \neq \phi \implies M^{-1}M$ contains 1

Also $(M^{-1}M)^{-1} = M^{-1}M$

so can replace F by $\bigcup\limits_{M \in \Sigma_F} M^{-1}M$ and suppose

$1 \in F$, $F = F^{-1}$. Note Γ left acts on Σ_F

$M \longmapsto gM$ as $(gM)^{-1}(gM) = M^{-1}\cancel{g^{-1}g}M$

What's happening? Consider $\Gamma \times \Gamma$ left action of Γ

Then $\Gamma \times \Gamma \longrightarrow \Gamma$ so the inverse

$(s, t) \longmapsto s^{-1}t$

image of F is a kind of <u>tube</u>, so Σ_F is a simplicial complex describing a nbd of the diagonal!

C^* alg A nuclear if it can be approximated by fd C^* algs:

$$A \xrightarrow{\psi_\alpha} F_\alpha \xrightarrow{\varphi_\alpha} A \qquad \varphi_\alpha, \psi_\alpha \text{ CP maps unital}$$

$$\dim(F_\alpha) < \infty \qquad \varphi_\alpha \psi_\alpha \longrightarrow id$$

~~Assume~~ F_α is a product of matrix algs over $\mathbb{C}$

$\{e_s\}_{s\in V}$ max family of pairwise orthogonal proj in F_α

diagonal subalg $D = \mathbb{C}^V$, $h_s = \varphi_\alpha(e_s)$ $s \in V$

?

Go to Γ case: $B = A \rtimes \Gamma = \overline{\bigoplus_{s\in\Gamma} B_s}$ Γ-graded

$B_s^* = B_{s^{-1}}$, $B_s B_t = B_{st}$

Interested in $K_0(B)$, ~~but~~ instead look at

$K_0^{fin}(B) = \{ [(p, \bar{p})] \mid p, \bar{p} \text{ finite } \Gamma \text{ support} \}$

Then $p = \sum_{s\in\Gamma} p_s$ $\qquad p = p^* \Leftrightarrow p_s^* = p_{s^{-1}}$

$\qquad p^2 = p \Leftrightarrow p_t = \sum_s p_s p_{s^{-1}t}$

Form universal C^*-alg.

$$P_F = C^* \left\{ p_s, s \in \Gamma \;\middle|\; \begin{array}{ll} p_s^* = p_{s^{-1}} & p_t = \sum_s p_s p_{s^{-1}t} \\ p_s = 0 & s \notin F \end{array} \right\}$$

$$\mathrm{Hom}_{\Gamma}(P_F, B) \simeq \left\{ \begin{array}{l} e \text{ proj in } B \\ e_s = 0 \quad s \notin F \end{array} \right\}$$

Γ-graded $\qquad \varphi \longmapsto \varphi(p_s)$

j

Joachim has a C^*-algebra construction ~~[illegible]~~ associated to a group Γ a graded C^* algebra construction you should understand for $\Gamma = \mathbb{Z}$.

Motivation $B = A \rtimes \mathbb{Z} = A[u, u^{-1}]$

with $u a u^{-1} =$ the action of $\mathbb{Z}$ generator:

Concern: proj p in B with finite Γ support

$$p = \sum_{s \in \Gamma} p_s \qquad p_s \in B_s = A_s$$

$$P_F = C^*\left\{ p_s, s \in \Gamma \;\middle|\; \begin{array}{l} p_s^* = p_{-s} \quad p_t = \sum p_s p_{s^{-1}t} \\ p_s = 0, s \notin F \end{array} \right\}$$

Can you describe this for $\Gamma = \mathbb{Z}$

$\mathcal{E}_{\Sigma_F}$

$\Gamma = \mathbb{Z}$ $\qquad B = \overline{\sum B_n t^n}$ $\qquad$ You want a proj in B, finite support $\qquad p = \sum_n p_n$

$$p^* = p \qquad p^2 = p \quad \Big\} \quad p_n^* = p_{-n} \qquad p_n = \sum p_m p_{n-m+n}$$

$$p_n = 0 \qquad |n| > N$$

proj

k

G finite, ~~[illegible]~~ A G-algebra, look for proj in $A \rtimes G$ which is a G graded alg. Actually Joachim looks at an arb G-graded alg $B = \bigoplus_{s \in G} B_s$. Thus he considers projections in any G-graded *-alg and forms a universal such *-algebra having generators p_s, $s \in G$ relations $p_s^* = p_{-s}$ $p_t = \sum_{s \in G} p_s p_{s^{-1}t}$. This universal alg P_G is G-graded. Apparently G acts on P_G ~~[illegible]~~ preserving the grading, so that

$$P_G \xleftarrow{\sim} \mathcal{E}_{\Sigma_G} \rtimes G$$

Σ_G simplicial complex of all $\neq \phi$ subsets of G.

$\mathcal{K} = \{\text{compact ops}\}$ nuclear.

G finite group. ~~[illegible]~~ $B = \bigoplus_{s \in G} B_s$ a $*$ alg with G-grading, p proj in B, can write $p = \sum p_s$ where $p_s^* = p_{-s}$ $p^2 = p$ etc.

~~two things I'd like to understand.~~

Grading wrt a group G, more generally monoid. S a set, coalgebra $\mathbb{C}[S]$, $\Delta s = s \otimes s$.

$$\begin{array}{ccc} M & \xrightarrow{\mu} & \mathbb{C}[S]\otimes M \\ & & \Delta\otimes 1 \downarrow \ \downarrow 1\otimes\mu \\ & & \mathbb{C}[S]\otimes\mathbb{C}[S]\otimes M \end{array}$$

comodule $\mu = \sum_s s \otimes \mu_s$

$$(\Delta\otimes 1)\mu = \sum_s s\otimes s\otimes \mu_s \qquad (1\otimes\mu)\mu = \sum_s s \otimes \sum_t t\otimes \mu_t\mu_s$$

$$\mu_t\mu_s = \begin{cases} 0 & t \neq s \\ \mu_s & t = s. \end{cases}$$

counit $\mathbb{C}[S] \xrightarrow{\eta} \mathbb{C}$ $\quad \eta(s) = 1 \quad \forall s.$

$$\begin{array}{ccc} M & \xrightarrow{\mu} & \mathbb{C}[S]\otimes M \\ & {\scriptstyle 1}\searrow & \downarrow \eta\otimes 1 \\ & & M \end{array} \qquad \sum_s \mu_s = 1$$

~~comodule business~~

So a grading of M with respect to G is a comodule structure $M \xrightarrow{\mu} \mathbb{C}[G]\otimes M$. Suppose G monoid, you should a tensor product operation $M \longrightarrow \mathbb{C}[G]\otimes M$, $N \longrightarrow \mathbb{C}[G]\otimes N$

$$M\otimes N \longrightarrow \mathbb{C}[G]\otimes M \otimes \mathbb{C}[G]\otimes N$$

$$\parallel$$

$$\mathbb{C}[G]\otimes\mathbb{C}[G]\otimes M\otimes N \longrightarrow \mathbb{C}[G]\otimes M\otimes N$$

$$\bigoplus_s M_s \otimes \bigoplus_t N_t \longrightarrow \bigoplus_{st} s\otimes M_s \otimes t\otimes N_t \longmapsto \bigoplus_{st} st \otimes M_s \otimes N_t$$

m

$$\therefore\ (M \otimes N)_s = \bigoplus_t M_{st^{-1}} \otimes N_t$$

So you work in the category of G graded * algebras. denote this $\hat{G}$

$$\mathrm{Hom}_{\hat{G}\text{-alg}}\left(P_G,\ A = \bigoplus_{s \in G} A_s\right) = \left\{ p \in A \;\middle|\; \begin{matrix} p^* = p \\ p^2 = p \end{matrix} \right\}$$

$$p = \sum_{s \in G} p_s \qquad p_s^* = p_{s^{-1}} \qquad p_t = \sum_s p_s p_{s^{-1}t}$$

given such a p, is ~~$(p'_s)_s = (p_{gs})_s$~~ a similar thing

$$\cancel{(p')^* = (p_{gs})^* = p_{s^{-1}g^{-1}}}$$

$$p'_t = p_{gt} = \sum_s p_s p_{s^{-1}gt} = \sum_{gs} p_{gs} p_{(gs)^{-1}gt}$$

$$= \sum_s \underbrace{p_{gs}}_{p'_s} p_{s^{-1}t}$$

$$p'_{s^{-1}t}$$

$$\cancel{p'_t = p_{gt} = \sum_s p_s p_{s^{-1}gt}}$$

$$\cancel{\sum_s p'_s p'_{s^{-1}t} = \sum_s p_{gs} p_{gs^{-1}t} = \sum_s p_{g(g^{-1}s)^{-1}t}}$$

$$p'_t = p_{tg} = \sum_s p_s p_{s^{-1}tg}$$

$$\sum_s p'_s p'_{s^{-1}t} = \sum p_{sg} p_{s^{-1}}$$

P_Γ = C^* alg gen. by p_s, $s \in \Gamma$ $\Big|$ $p_s^* = p_{s^{-1}}$, $p_t = \sum_s p_s p_{s^{-1}t}$

$\mathcal{E}_{\Sigma_\Gamma}$ = C^* alg gen h_s, $s \in \Gamma$ $\Big|$ $h_s \geq 0$, $\sum h_s = 1$

$$P_\Gamma \longrightarrow \mathcal{E}_{\Sigma_\Gamma} \rtimes \Gamma$$

$$p_s \longmapsto h_1^{1/2} h_s^{1/2} s$$

$A \xrightarrow{\varphi} B$ φ CP when $\forall a_1, \dots, a_n$ $(\varphi(a_i^* a_j)) \in M_n(B)$ is ≥ 0.

Is it possible that a CP map induses a map in K-theory? Example: Let $p = p^* = p^2$ in A, so that $a \longmapsto pap$ is CP. from A to pAp. What happens ~~on~~ the K-theory level. Change p to e. Then have ideal AeA, Morita context $\begin{pmatrix} AeA & Ae \\ eA & eAe \end{pmatrix}$, so you have what.

$$\begin{tikzcd}
K_0(eAe) \arrow[d, equal] & & \\
K_0(AeA) \arrow[r] & K_0 A \arrow[r] & K_0(A/AeA) \arrow[d] \\
& \arrow[ul] & K_1(AeA) \arrow[l] \arrow[d, equal] \\
& & K_1(eAe)
\end{tikzcd}$$

OKAY it's clear that you have $K_*(eAe) \xrightarrow{\sim} K_*(AeA) \longrightarrow K_*(A)$ whereas

The CP map $a \mapsto eae$ goes the other way.

Back to Cuntz's construction for Γ finite.

$$P_\Gamma = C^*\left\{ p_s \;\; s \in \Gamma \;\middle|\; \begin{array}{l} p_s^* = p_{s^{-1}} \\ p_t = \sum_s p_s p_{s^{-1}t} \quad \forall t \end{array} \right\}$$

$$\mathcal{E}_{\Sigma_\Gamma} = C^*\left\{ h_s, s \in \Gamma \;\middle|\; h_s \geq 0 \,,\; \sum_s h_s = 1 \right\}$$

Γ acts on $\mathcal{E}_{\Sigma_\Gamma}$ (more gen on $\mathcal{E}_{\Sigma_F}$, F finite $\subset \Gamma$)

$\mathcal{E}_{\Sigma_\Gamma}$ is the noncommutative simplex with set of vertices Γ and Γ permutes the vertices. You have a choice of left or right action, which should be clear in the general case

$$\Sigma_F = \{ M \text{ finite} \neq \emptyset \subset \Gamma \mid M^{-1}M \subset F \}$$

then left action $M \longmapsto sM$ is required.

Since Γ acts on $\mathcal{E}_{\Sigma_F}$ can form $\mathcal{E}_{\Sigma_F} \rtimes \Gamma$

Cuntz wrote map

$$\begin{array}{ccc} P_F & \longrightarrow & \mathcal{E}_{\Sigma_F} \rtimes \Gamma \\ p_s & & h_1^{1/2} h_s^{1/2} s \end{array}$$

Why well-defined? $\left(h_1^{1/2} h_s^{1/2} s\right)^* = s^{-1} h_s^{\frac{1}{2}} h_1^{\frac{1}{2}} \overset{?}{=} h_1^{\frac{1}{2}} h_{s^{-1}}^{1/2} s^{-1}$

$$h_1^{1/2} h_t^{1/2} t \overset{?}{=} \sum_s \left(h_1^{1/2} h_s^{1/2} s \; h_1^{1/2} h_{s^{-1}t}^{1/2} \right) s^{-1}t \overset{?}{=} \sum_s h_1^{1/2} h_s^{1/2} h_s^{1/2} h_t^{1/2} t$$

ges

$$P_\Gamma = C^*\{ p_s, s\in\Gamma \mid p_s^* = p_{s^{-1}},\ p_t = \sum_{s\in\Gamma} p_s p_{s^{-1}t}\ \forall t\}$$

P_Γ should be Γ-graded $\deg(p_s) = s$.

$$\mathcal{E}_{\Sigma_\Gamma} = C^*\{ h_s, s\in\Gamma \mid h_s \geq 0, \sum_{s\in\Gamma} h_s = 1\}$$

Γ acts on $\mathcal{E}_{\Sigma_\Gamma}$ $\quad t h_s = h_{ts}$

~~Cdp.~~

$$P_\Gamma \longrightarrow \mathcal{E}_{\Sigma_\Gamma} \rtimes \Gamma = \bigoplus_{s\in\Gamma} \mathcal{E}_{\Sigma_\Gamma} s$$

$$p_s \longmapsto h_1^{1/2} h_s^{1/2} s$$

$$(h_1^{1/2} h_s^{1/2} s)^* = s^{-1} h_s^{1/2} h_1^{1/2} = h_1^{1/2} h_{s^{-1}}^{1/2} s^{-1}$$

$$\sum_s h_1^{1/2} h_s^{1/2} s\, h_1^{1/2} h_{s^{-1}t}^{1/2} s^{-1}t = \sum_s h_1^{1/2} h_s^{1/2} h_s^{1/2} h_t^{1/2} t =$$

$$h_1^{1/2}\left(\sum h_s\right) h_t^{1/2} t = h_1^{1/2} h_t^{1/2} t$$

$$(h_1^a h_s^b s)^* = s^{-1} h_s^b h_1^a = h_1^b h_{s^{-1}}^a s^{-1}$$

~~Rp~~ Repeat this construction

Γ-graded algebra $\quad B = \bigoplus_{s\in\Gamma} B_s \quad (B_s)^* = B_{s^{-1}}$, $\quad B_s B_t \subset B_{s+t}$

~~[illegible]~~ consider $p \in B \quad p = \sum_s p_s$

$$p^* = p = p^2,\quad (p_s)^* = p_{s^{-1}},\quad p_t = \sum_s p_s p_{s^{-1}t}$$

yields P_Γ with these generators + relations

P_Γ has a Γ- ~~[illegible]~~ grading, canonical 9

$\deg(p_s) = s$.

$$\mathcal{E}_{\Sigma_\Gamma} = C^*\{h_s, s\in\Gamma \mid h_s \geq 0, \sum h_s = 1\}$$

non comm. simplex on Γ. Γ acts on it

$t h_s = h_{ts}$.

Let's go over Baum's question about $C_r(\mathbb{Z}\times\mathbb{Z}) = C(S^1\times S^1)$ and the Bott class.

$$K^{top}\Gamma \xrightarrow{\mu} K_*(C_r(\Gamma)) \qquad \text{index map}$$

$$\| $$

$$KK^\Gamma(\underline{E}\Gamma, \mathbb{C}) = \varinjlim_F KK^\Gamma(\mathcal{E}^{ab}_{\Sigma_F}, \mathbb{C})$$

Interested in $K^{fin}_*(C_r(\Gamma)) = {}''K_*(\mathbb{C}[\Gamma])$

$$p_s^* = (h_1^{1/2} h_s^{1/2} s)^* = s^{-1} h_s^{1/2} h_1^{1/2}$$

$$p_s^* p_s = s^{-1} h_s^{1/2} h_1^{1/2} h_1^{1/2} h_s^{1/2} s = h_1^{1/2} h_{s^{-1}} h_1^{1/2}$$

$$p_{s^{-1}} p_s = h_1^{1/2} h_{s^{-1}}^{1/2} s^{-1} h_1^{1/2} h_s^{1/2} s = h_1^{1/2} h_{s^{-1}}^{1/2} h_{s^{-1}}^{1/2} h_1^{1/2}$$

$$p_s p_s^* = h_1^{1/2} h_s h_1^{1/2}$$

essentially $p_s = (?) s^{\mathcal{E}\Sigma_\Gamma}$

Goal $P_\Gamma \longrightarrow \mathcal{E}_{\Sigma_\Gamma} \rtimes \Gamma$ $\qquad p_s \longmapsto h_1^{1/2} h_s^{1/2} s$

$$(p_s s^{-1})^* = s\, p_{s^{-1}} \qquad \sum p_s s^{-1} p_{s^{-1}t} t^{-1} s$$

$p_s \longmapsto h_1^{1/2} h_s^{1/2} s$ $\qquad \underbrace{P_\Gamma \longrightarrow \mathcal{E}_{\Sigma_\Gamma} \rtimes \Gamma}$ map of $\hat{\Gamma}$-alg. (Γ graded)

P_Γ universal for projections in a Γ-graded alg.

$\mathcal{E}_{\Sigma_\Gamma}$ universal for partitions of 1 indexed by Γ

: $\quad h_s \geq 0, s \in \Gamma \qquad \sum_{s\in\Gamma} h_s = 1$

$\Gamma = \{1\} \qquad P_\Gamma = C^*\{p \mid p = p^*, p = p^2\}.$

$\mathcal{E}_{\Sigma_\Gamma} = \mathbb{C}^*\{h \mid h \geq 0, h = 1\}.$

Cuntz ~~said~~ said $\sum h_s = 1$ is to be interpreted formally, meaning $(\sum h_s - 1) h_t = 0$ $h_t(\sum h_s - 1) = 0, \forall t$. This means $\mathcal{E}_{\Sigma_\Gamma} = C^*\{h_1 \mid h_1 \geq 0, h_1^2 = h_1\}$

Then $p_1 = h_1^{1/2} h_1^{1/2} 1 = h_1 1$. True in general.

$(p_s = h_1^{1/2} h_s^{1/2} s) \qquad p_1 = h_1 1.$

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admissible $\quad 0 \to I \to A \to A/I \to 0$

Banach alg $A \ni \mathcal{H}^1(A, M) = 0 \quad \forall$ dual bimodules M
called amenable $\qquad$ n-amenable: $\mathcal{H}^n(A, X^*) = 0$ all X^*

weak bidim $\inf n$ $\mathcal{H}^n = 0$ all X^*

Try again $\quad \Gamma = \{1\} \qquad \mathcal{E}_{\Sigma_\Gamma} = C^*\{h_1 \mid h_1 \geq 0, h_1 = 1\}$

maybe should be $(h_1 - 1) h_1 = 0$

Repeat. Γ finite. $P_\Gamma = C^*\left\{ p_s, s\in\Gamma \,\middle|\, \begin{array}{l} p_s^* = p_{s^{-1}} \\ p_t = \sum_s p_s p_{s^{-1}t} \end{array} \right\}$ S

P_Γ is a Γ-graded algebra with degree $(p_s) = s$.

~~So you can form~~ $\cancel{P_\Gamma \rtimes \hat{\Gamma}}$ Universal prop.

$$\operatorname{Hom}_{\hat{\Gamma}\text{alg}}(P_\Gamma, B) = \text{Projections in } B = \bigoplus_{s\in\Gamma} B_s$$

Now because P_Γ is a $\hat{\Gamma}$-alg you can form $P_\Gamma \rtimes \hat{\Gamma}$

$P_\Gamma \rtimes \hat{\Gamma}$, which is supposed to be Mor. eq to

$$\mathcal{E}_{\Sigma_\Gamma} = C^*\left\{ h_s, s\in\Gamma \,\middle|\, h_s^* \geq 0 \quad \underbrace{\sum h_s^2 = 1}_{\text{fuzzy condition}} \right\}$$

noncommutative simplex
acted on by Γ $\qquad t h_s = h_{ts}$ Can form

$$\mathcal{E}_{\Sigma_\Gamma} \rtimes \Gamma = \bigoplus_{s\in\Gamma} \mathcal{E}_{\Sigma_\Gamma} s \quad , \text{map} \quad P_\Gamma \to \mathcal{E}_\Gamma \rtimes \Gamma$$

$$p_s \longmapsto h_1 h_s s \qquad (h_1 h_s s)^* = s^{-1} h_s h_1 = h_1 h_{s^{-1}} s^{-1}$$

$$\sum_s p_s p_{s^{-1}t} \longmapsto \sum_s h_1 h_s s\, h_1 h_{s^{-1}t} s^{-1} t = \sum_s h_1 h_s^2 h_t t = h_1 h_t t$$

Look at $\Gamma = \mathbb{Z}$. First look at ∞ group.

F finite $\subset \Gamma \rightsquigarrow \Sigma_F = \{\emptyset \neq M \subset \Gamma \mid M^{-1}M \subset F\}$.
(loc fin) simp. complex

$$\text{“}\Sigma_\Gamma\text{”} = \bigcup_F \Sigma_F \qquad \Gamma \text{ acts by left mult.}$$

$$\underbrace{\varinjlim_F KK_*^\Gamma(\mathcal{E}_{\Sigma_F}, A)}_{K_*^{\text{fin}}(A \rtimes \Gamma)} \longrightarrow K_*(A \rtimes_r \Gamma)$$

Other point is that $\mathcal{E}^{ab}_{\Sigma_F} \sim C(\underline{E}_\Gamma)$ t

You want to understand what Paul Baum said in the case of $\Gamma = \mathbb{Z} \times \mathbb{Z}$

$$\underbrace{\lim_F KK^\Gamma(\overset{\mathbb{R}^2}{\underline{E}_\Gamma}, \mathbb{C})}_{\parallel} \qquad 0$$

$$\lim_F KK^\Gamma(\mathcal{E}^{ab}_{\Sigma_F}, \mathbb{C}) \qquad\qquad K_*^{fin}(C_r(\Gamma))$$

$$\downarrow \qquad\qquad\qquad \uparrow$$

$$\lim_F KK^\Gamma(\mathcal{E}_{\Sigma_F}, \mathbb{C}) \qquad K_*^{fin}(\mathbb{C}[\Gamma])$$

$\wr\wr$ Baaj Skandalis

$$\lim_F KK^\Gamma(P_F \rtimes \hat{\Gamma}, \mathbb{C})$$

$\wr\wr$?

$$\lim_F KK^\Gamma(\mathcal{E}_\Sigma$$

BC $\quad KK_*^\Gamma(\underline{E}_\Gamma, A) \longrightarrow K_*(A \rtimes_r \Gamma)$

$$\lim_F KK_*^\Gamma(\mathcal{E}_{\Sigma_F}, A) \qquad\qquad K_*^{fin}(A \rtimes \Gamma) \uparrow$$

$K_*(A \rtimes_r \Gamma)$. $C_r(\mathbb{Z}) = C(S^1)$ want C pos.

map $C(S^1) \longrightarrow F \longrightarrow C(S^1)$ which approxs identity. ~~This first~~ F is a f.d. C^*-alg big effort to get straight

Cuntz constructions. $B_s B_t \subset B_{st}$

cat of Γ-graded *algebras : $B = \bigoplus_{s\in\Gamma} B_s$ $B_s^* = B_{s^{-1}}$

$$\mathrm{Hom}_{\Gamma\text{-gr-alg}}(P_\Gamma, B) = \{p \in B \mid p = p^* = p^2\}.$$

cat of Γ-algs. A.

construction $B = A \rtimes \Gamma = \bigoplus_{s\in\Gamma} As$

$a_1 s\, a_2 t = a_1 s(a_2)\, st$. This goes from Γ-algebras to Γ-gr-algs.

$$\mathrm{Hom}_{\Gamma\,\mathrm{gr\,alg}}(A \rtimes \Gamma, B) \overset{?}{=} \mathrm{Hom}_{\mathrm{algs}}(A, B_e)$$

Given A with Γ action you can form $A \rtimes \Gamma = A \otimes \mathbb{C}[\Gamma]$ with mult $(a \otimes s)(a' \otimes t) = a\,s(a') \otimes st$

assume algebras are unital and given

$$\begin{tikzcd} \varphi : A \tilde{\otimes} \mathbb{C}[\Gamma] \arrow[r] & \bigoplus_{s\in\Gamma} B_s \\ \mathbb{C}[\Gamma] \arrow[u] \arrow[ur] & \end{tikzcd}$$

so actually you get $\varphi(1 \otimes s) \in B_s$

so you get ~~a sep~~ a famil

various scratch work, outlines, connected with Morita invariance for K_1.