(A) 8/25 - 0837 not worth wasting a lot of time on this

second point is the formulation by means of relative theory.

objects: $Q^t \subset T' \otimes Q$ graded T-subalg.

$\Omega_T(Q^t)$, ΩQ^t

$R_T(Q^t)$, RQ^t

$X_T(R_T(Q^t))$, $X(RQ)^t$

$I_T(Q^t)$, IQ^t

$F^p_{I_T(Q^t)} X_T(R_T(Q^t))$, $(F^pX)^t$

logic: we have defined filtrations on Q, ΩQ, RQ, $X(RQ)$, $F^p_{IQ} X(RQ)$

I am having trouble finding assertions seem to be forced into a long list of trivialities. What I am ultimately after are the formulas

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r, "\sim"] & X(RQ)^t \\
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[u, phantom, "\cup"] & (F^pX)^t \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

Thus we control the filtration

(B) Canonical identification with

$$\begin{tikzcd}
\Omega_T(Q^t) \arrow[r, "\sim"] & \Omega Q^t \\
\cup & \cup \\
F^p(\Omega_T(Q^t)) \arrow[r, "\sim"] & F^p(\Omega Q^t)
\end{tikzcd}$$

trace map

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] & L_q^t \otimes X(RB) \\
\cup & \cup \\
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r] & L_q^t \otimes F^p_{IB} X(RB)
\end{tikzcd}$$

~~identical to~~ identified with

$$\begin{tikzcd}
\Omega_T(Q^t) \arrow[r] & L_q^t \otimes \Omega B \\
\cup & \cup \\
F^p(\Omega_T(Q^t)) \arrow[r] & L_q^t \otimes F^p \Omega B
\end{tikzcd}$$

8/26 - 0532 Have decided that what I need to do after introducing

$$\begin{tikzcd}
X_S(R_S A) \arrow[r, phantom, "\sim"] & \Omega_S A \otimes_S \\
\cup & \cup \\
F^p_{I_S A} X_S(R_S A) \arrow[r, phantom, "\sim"] & F^p(\Omega_S A \otimes_S)
\end{tikzcd}$$

is to obtain

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r, "\sim"] & X(RQ)^t \\
\cup & \cup \\
F^p_{I_T Q} \arrow[r, "\sim"] & (F^p X)^t
\end{tikzcd}
\quad \text{and} \quad
\begin{tikzcd}
X(RQ)^t \arrow[r] & L_q^t \otimes X(RB) \\
\cup & \cup \\
(F^p X)^t \arrow[r] & L_q^t \otimes F^p_{IB}
\end{tikzcd}$$

(B) Now the stuff to work on is the D-theory. I ran into a problem before with ~~defining $F^p X_{\geq k}$~~ two possible L_D, h_D. First one takes the grading on Q, gets induced grading on RQ, $X(RQ)$, gets D, L_D, h_D. 2nd by one extends D on Q to Q^t, then gets D on $R_T(Q^t)$, L_D on $X_T(R_T(Q^t))$. Need consistency. But this is clear from

$$\begin{array}{ccc} Q^t & \subset & T'\otimes Q \\ D & \longleftrightarrow & 1\otimes D \\ R_T(Q^t) & \longrightarrow & T'\otimes RQ \\ X_T(R_T(Q^t)) & \longrightarrow & T'\otimes X(RQ) \\ L_D & & 1\otimes L_D \end{array}$$

$$\begin{array}{rcl} \phi : R_T(Q^t) & \longrightarrow & \Omega^2_T(R_T(Q^t)) \\ \| & & \| \\ RQ^t & & \Omega^2_T(RQ^t) \\ & & \| \\ & & \Omega^2(RQ)^t \end{array}$$

Go over points.

grading on Q

induces gradings on RQ, $X(RQ)$

hence $Q_{\geq k} = \bigoplus_{n\geq k} Q_n$ same relation holds for $RQ_{\geq k}$, $X(RQ)_{\geq k}$.

Ⓓ degree ops D on RQ, h_D on $X(RQ)$.

canonical ϕ and h_D.

next can do the lemma.
& its proof.

$Q^t \subset T' \otimes Q$

We know

$$\Omega_T(Q^t) \hookrightarrow T' \otimes \Omega Q$$

Apply basic identification $X_S(R_S A) = \Omega_S A \otimes_S$
in the case of Q^t rel T, $T' \otimes Q$ rel T'
get

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, phantom, "\wr\wr"] & T' \otimes X(RQ) \arrow[d, phantom, "\wr\wr"] \\
\Omega_T(Q^t) \arrow[r] & T' \otimes \Omega Q
\end{tikzcd}$$

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$$

$$F^p_{I_T}(Q^t) \qquad\qquad F^p_{I_{T'}}(T' \otimes Q)$$

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

$$F^p_{I_T}(Q^t) \longrightarrow F^p_{I_{T'}}(T' \otimes Q) = F^p_{T' \otimes IB} = T' \otimes F^p_{IB}$$

Have to add F^p ~~to~~

$$\Omega_T(Q^t) \longrightarrow T' \otimes \Omega Q$$

$$F^p(\Omega_T(Q^t)) \longrightarrow T' \otimes F^p \Omega Q$$

(E) OKAY what's missing

I have $\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega Q$

map of mixed complexes so that

$$F^p\Omega_T(Q^t) \longrightarrow F^p\Omega_{T'}(T' \otimes Q) = T' \otimes F^p\Omega Q$$

have to say $\Omega_T(Q^t) \simeq \Omega Q^t$ for the filt.

$$\Omega Q^t = \bigoplus t^k (\Omega Q)_{\geq k}$$

$$\therefore \quad F^p\Omega_T(Q^t) \simeq F^p(\Omega Q^t) = \bigoplus t^k F^p\Omega Q_{\geq k}$$

Conclude that we have canon isom.

$$\Omega_T(Q^t) \xrightarrow{\sim} \Omega Q^t$$

$$F^p\Omega_T(Q^t) \xrightarrow{\sim} \bigoplus t^k F^p(\Omega Q_{\geq k})$$

---

trace map Q

$$Q^t \longrightarrow L^t \otimes B$$

trace map of mixed cxs.

$$\Omega Q^t = \Omega_T Q^t \longrightarrow \Omega_{L^t}(L^t \otimes B) = L^t \otimes \Omega B$$

I am confused 8/26 - 1447 ~~what next~~.

(F) 8/27 - 0652

Start again

my construction

given $\theta, \theta' : A \longrightarrow L \otimes B$ cong mod $J \otimes B$

get $p + tq : A \longrightarrow S \otimes B$ hm. resp 1

induces $u : RA \longrightarrow S \otimes RB$

$IA \longrightarrow K \otimes RB + S \otimes IB$

get

$$\begin{tikzcd} X(RA) \arrow[r, "u_*"] & X(S \otimes RB) \arrow[r, "\alpha"] & S_\flat \otimes X(RB) \arrow[r, "\mu_m"] & J^{2m+1}_\# \otimes X(RB) \\ F^p_{IA} & F^p_{K \otimes RB + S \otimes IB} & \sum_{i \geq 0} q(K^i) \otimes F^{p-2i}_{IB} & J^{2m+1}_\# \otimes F^{p-2m}_{IB} \end{tikzcd}$$

get $\boxed{\mathcal{X}_A \longrightarrow J^{2m+1}_\# \otimes \mathcal{X}_B[2m]}$

get class $\boxed{ch^{2m}(\theta, \theta') \in HC^{2m}(\mathcal{X}_A, J^{2m+1}_\# \otimes \mathcal{X}_B)}$

version of Nistor construction

$Q = QA$ filtration $(Q_{\geq k})$, γ

ΩQ ind. filtration $\Omega Q_{\geq k}$, γ, $\gamma_- \Omega Q_{\geq 2j+1} = \gamma_- \Omega Q_{\geq 2j}$

$\iota_k \in HC^0(\Omega Q_{\geq k+1}, \Omega Q_{\geq k})$

$\exists$ $s_k \in HC^2(\Omega Q_{\geq k}, \Omega Q_{\geq k+1})$ inverse for ι_k up to S

s_k unique mod Ker S

can suppose s_k commutes with γ

$\exists$ $s'_{2j-1} \in HC^2(\gamma_- \Omega Q_{\geq 2j-1}, \gamma_- \Omega Q_{\geq j+1})$

inverse $\overset{\text{up to } S}{}$ to the class of the inclusion

(7) universal biv Chern charac. order 2m

$$ch^{2m}(\iota, \iota^\gamma) \equiv S'_{2m-1} \cdot S'_{2m-2} \cdots S'_1 \cdot ch^0(\iota, \iota^\gamma)$$

$$\in HC^{2m}(\Omega A, \gamma_- \Omega Q_{\geq 2m+1})$$

$ch^0(\iota, \iota^\gamma)$ class of

$$\Omega A \xrightarrow{\iota_*} \Omega Q \xrightarrow{\gamma_-} \gamma_- \Omega Q = \gamma_- \Omega Q_{\geq 1}$$

$ch^{2m}(\iota, \iota^\gamma)$ is

[redacted] trace map.

θ, θ' induce filt alg. hom.

$$Q \longrightarrow L \otimes B$$

$$Q_{\geq k} \longrightarrow J^k \otimes B \qquad \forall k$$

get

$$\Omega Q \longrightarrow L \otimes \Omega B \qquad \text{filt. DG alg hom.}$$

$$\Omega Q_{\geq k} \longrightarrow J^k \otimes \Omega B$$

get maps of mixed complexes

$$\Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$$

get $\ell_k(\theta, \theta') \in HC^0(\Omega Q_{\geq k}, J^k_\# \otimes \Omega B)$.

Def

$$\boxed{\begin{aligned} ch^{2m}(\theta, \theta') &= \ell_k(\theta, \theta') \cdot ch^{2m}(\iota, \iota^\gamma) \\ &\in HC^{2m}(\Omega A, J^{2m+1}_\# \otimes \Omega B). \end{aligned}}$$

Now the problem is to identify these two classes $ch^{2m}(\theta, \theta')$ (mod Ker S)

need an X version of Nistor construction. Outline the steps.

(H) from a minimum viewpoint we need

bifiltration $F^pX_{\geq k}$ of $X(RQ)$

L_D, h_D on $X(RQ)$

X - trace maps $X_{\geq k} \longrightarrow J^k_{\#} \otimes X(RB)$

with various properties:

basic homotopy equivalence $X(RQ) \sim \Omega Q$

. restricts to a htpy equiv $F^pX_{\geq k} \sim F^p\Omega Q_{\geq k}$

such that

$$\begin{tikzcd}
F^pX_{\geq k} \arrow[r] \arrow[d, "\sim"'] & J^k_{\#} \otimes F^p_{IB} \arrow[d, "\sim"'] \\
F^p\Omega Q_{\geq k} \arrow[r] & J^k_{\#} \otimes F^p\Omega B
\end{tikzcd}$$

Commutes.

need trace map

$$\begin{tikzcd}
X_{\geq k} \arrow[r] & J^k_{\#} \otimes X(RB) \\
F^pX_{\geq k} \arrow[u, phantom, "\cup" description] \arrow[r] & J^k_{\#} \otimes F^p_{IB} \arrow[u, phantom, "\cup" description]
\end{tikzcd}$$

$$\begin{tikzcd}
 & & F^pX_{\geq 0} \arrow[d, phantom, "\shortparallel" description] & & & \\
\cancel{X(RA)} & F^p_{IA} \arrow[r] \arrow[d, "\sim"'] & F^p_{IQ} \arrow[r, "\gamma_-"] \arrow[d, "\sim"'] & \gamma_- F^pX_{\geq 0} \arrow[r, phantom, "="] & \gamma_- F^pX_{\geq 1} \arrow[dd, "\sim"'] \\
 & F^p\Omega A \arrow[r] & F^p\Omega Q \arrow[r, "\gamma_-"] & \gamma_- F^p\Omega Q \arrow[d, phantom, "\shortparallel" description] & \\
 & & & F^p\gamma_-\Omega Q \arrow[r, phantom, "="] & F^p\gamma_-\Omega Q_{\geq 1}
\end{tikzcd}$$

Confused again.

(I) confused again.

X trace map:

$$\begin{tikzcd}
X_{\geq k} \arrow[r] & J^k_{\#} \otimes X(RB) \\
F^p X_{\geq k} \arrow[r] \arrow[u, phantom, "\cup"] & J^k_{\#} \otimes F^p_{IB} \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

agrees with Nistor traces map since

$$\begin{tikzcd}
F^p X_{\geq k} \arrow[r] \arrow[d, "\sim"'] & J^k \otimes F^p_{IB} \arrow[d, dash, "\sim"'] \\
F^p \Omega Q_{\geq k} \arrow[r] & J^k_{\#} \otimes F^p \Omega B
\end{tikzcd}$$

commutes

Identifies then our $l_k(\theta, \theta') \in HC^0(\mathcal{X}_{\geq k}, J^k_{\#} \otimes \mathcal{X}_B)$

with Nistor's $l_k(\theta, \theta') \in HC^0(\Omega Q_{\geq k}, J^k_{\#} \otimes \Omega B)$

Similarly for $Ch^0(\iota, \iota^{\gamma})$.

$$\begin{tikzcd}
F^p_{IA} \arrow[r, "\iota_*"] \arrow[d, dash, "\sim"'] & F^p_{IQ} = F^p X_{\geq 0} \arrow[r, "\gamma_-"] \arrow[d, dash, "\sim"'] & \gamma_- F^p X_{\geq 0} = \gamma_- F^p X_{\geq 1} \arrow[d, dash, "\sim"'] \\
F^p \Omega A \arrow[r] & F^p \Omega Q \arrow[r, "\gamma_-"] & F^p \gamma_- \Omega Q = F^p \gamma_- \Omega Q_{\geq 1}
\end{tikzcd}$$

identifies our

$ch^0(\iota, \iota^{\gamma}) \in HC^0(\mathcal{X}_A, \gamma_- \mathcal{X}_{\geq 1}$

and Nistor

$ch^0(\iota, \iota^{\gamma}) \in HC^0(\Omega A, \gamma_- \Omega Q_{\geq 1})$

Next point behavior of L_D, h_D

$L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$

$\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$

$h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$

(J) $\mathcal{X}_{\geq k} = \left(X_{\geq k} / F^p X_{\geq k} \right)_p$

class $\iota_k \in HC^0 (\mathcal{X}_{\geq k+1}, X_{\geq k})$ inclusion

$k \geq 1$

$$\begin{tikzcd}
1 - \frac{1}{k} L_D : & X_{\geq k} \arrow[r] & X_{\geq k+1} \\
& F^p X_{\geq k} \arrow[r] \arrow[u, phantom, "\cup"] & F^{p-2} X_{\geq k+1} \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

comp with superconformal structure

get $s_k : HC^2 (\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+1})$

$$1 - \left(1 - \frac{1}{k} L_D\right) = \left[\partial, \frac{1}{k} h_D\right] : X_{\geq k} \longrightarrow X_{\geq k}$$

$$\frac{1}{k} h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$$

$$1 - \iota_k s_k \sim 0 \quad : \mathcal{X}_{\geq k} \longrightarrow \mathcal{X}_{\geq k}[2]$$

$\gamma_- \mathcal{X}_{\geq k+1} = \gamma_- \mathcal{X}_{\geq k}$ for k even why.

$\gamma = (-1)^k$ on $X_{\geq k+1} / X_{\geq k}$

$$\begin{tikzcd}
& 0 \arrow[d] & 0 \arrow[d] & & \\
0 \arrow[r] & F^p X_{\geq k+1} \arrow[r] \arrow[d] & F^p X_{\geq k} \arrow[r] \arrow[d] & {} \arrow[r] & 0 \\
0 \arrow[r] & X_{\geq k+1} \arrow[r] \arrow[d] & X_{\geq k} \arrow[r] \arrow[d] & {} \arrow[r] & 0 \\
& \mathcal{X}^p_{\geq k+1} \arrow[r] \arrow[d] & \mathcal{X}^p_{\geq k} \arrow[d] & & \\
& 0 & 0 & &
\end{tikzcd}$$

$$\begin{tikzcd}
0 \arrow[r] & \boxed{K} \arrow[r] & \mathcal{X}^p_{\geq k+1} \arrow[r] & \mathcal{X}^p_{\geq k} \arrow[r] & \boxed{C} \arrow[r] & 0
\end{tikzcd}$$

$\boxed{K}$ subspace of $F^p X_{\geq k} / F^p X_{\geq k+1}$

$\boxed{C}$ quotient of $X_{\geq k} / X_{\geq k+1}$

(K)

$$\begin{tikzcd}
 & 0 & 0 & & \\
0 \arrow[r] & F^pX_{\geq k+1} \arrow[r] \arrow[d] & X_{\geq k+1} \arrow[r] \arrow[d] & \mathcal{X}^p_{\geq k+1} \arrow[r] \arrow[d] & 0 \\
0 \arrow[r] & F^pX_{\geq k} \arrow[r] \arrow[d] & X_{\geq k} \arrow[r] & \mathcal{X}^pX_{\geq k} \arrow[r] & 0 \\
 & {} & & &
\end{tikzcd}$$

Argument $\gamma_- X_{\geq 2m+1} = \gamma_- X_{\geq 2m}$

$\gamma_- F^pX_{\geq 2m+1} = \gamma_- F^pX_{\geq 2m}$

$\therefore \boxed{\gamma_- \mathcal{X}^p_{\geq 2m+1} = \gamma_- \mathcal{X}^p_{\geq 2m}}$

Other point is that basic heq $X(RQ) \sim \Omega Q$ induces a heq of towers $\mathcal{X}_{\geq k} \sim \theta(\Omega Q_{\geq k})$

Thus $s'_k \in$

How do I overcome the **obstacle**? 1245

Start again

~~X(RQ) introduce~~

my construction

version of Nistor const.

X-version of Nistor construction

objects $X(RQ)$, γ

bifilts. $F^pX_{\geq k}$

L_D, h_D

need to know that the basic heq $X(RQ) \sim \Omega Q$ induces

(L) need to know that the basic htpy equiv

$$X(RQ) \sim \Omega Q$$

induces $F^p X_{\geq k} \sim F^p \Omega Q_{\geq k}$

follows that $\mathcal{X}_{\geq k} \sim \theta(\Omega Q_{\geq k})$

In addition need trace maps

$$\Omega Q_{\geq k} \longrightarrow J_\#^k \otimes \Omega B$$ map of mix cxs.

$$\begin{tikzcd}
X_{\geq k} \arrow[r] & J_\#^k \otimes X(RB) \\
F^p X_{\geq k} \arrow[r] \arrow[u, phantom, "\cup"] & J_\#^k \otimes F^p_{IB} \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

and comm of

$$\begin{tikzcd}
F^p X_{\geq k} \arrow[r] \arrow[d, dash, "\sim"'] & J_\#^k \otimes F^p_{IB} \arrow[d, dash, "\sim"] \\
F^p \Omega Q_{\geq k} \arrow[r] & J_\#^k \otimes F^p \Omega
\end{tikzcd}$$

this gives

$$\begin{tikzcd}
\mathcal{X}_{\geq k} \arrow[r] \arrow[d, dash, "\sim"] & J_\#^k \otimes \mathcal{X}_B \arrow[d, dash, "\sim"] \\
\theta(\Omega Q_{\geq k}) \arrow[r] & \cancel{J_\#^k \otimes} \, \theta(J_\#^k \otimes \Omega B)
\end{tikzcd}$$

(M) Also need γ, L_D, h_D on X

$\exists$

$$\gamma - (-1)^k : F^pX_{\geq k} \longrightarrow F^pX_{\geq k+1}$$

$$L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$$

$$h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$$

$$\gamma_- F^pX_{\geq 2j+1} \Rightarrow \gamma_- F^pX_{\geq 2j} \qquad \forall j$$

$$1 - \frac{1}{k}L_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$$

induces $\mathcal{X}_{\geq k} \longrightarrow \mathcal{X}_{\geq k+1}[2]$

$$\delta_k \in HC^2(\mathcal{X}_{\geq k}, \mathcal{X}_{\geq k+1})$$

$$\delta'_{2j-1} \in HC^2(\gamma_-\mathcal{X}_{\geq 2j-1}, \gamma_-\mathcal{X}_{\geq 2j+1})$$

get $X(RA) \xrightarrow{\iota_*} X(RQ) \xrightarrow{\gamma_-} \gamma_-X_{\geq 0} = \gamma_-X_{\geq 1}$

$F^p_{IA} \longrightarrow F^pX_{\geq 0} \longrightarrow \gamma_-F^pX_{\geq 0} = \gamma_-F^pX_{\geq 1}.$

get $\mathcal{X}_A \longrightarrow \mathcal{X}_Q \xrightarrow{\gamma_-} \gamma_-\mathcal{X}_{\geq 0} = \gamma_-\mathcal{X}_{\geq 1}.$

$$ch^0(\iota, \iota^\gamma) \in HC^0(\mathcal{X}_A, \gamma_-\mathcal{X}_{\geq 1}).$$

$$ch^{2m}(\iota, \iota^\gamma) = \delta'_{2m-1}\cdot\delta'_{2m-3}\cdots\delta'_1\cdot ch^0(\iota,\iota^\gamma)$$

$$\in HC^{2m+}(\mathcal{X}_A, \gamma_-\mathcal{X}_{\geq 2m+1})$$

(N) so the problem is to define $F^pX_{\geq k}$ and verify all these properties.

Can define $F^pX_{\geq k}$ so that it corresponds under the basic identification $X(RQ) = \Omega Q$ to $F^p(\Omega Q_{\geq k})$. Since $F^p(\Omega Q_{\geq k})$ closed under b, d, etc. $F^pX_{\geq k}$ is a subcomplex of $X = X(RQ)$ and the basic heq $X(RQ) = \Omega Q$ (which consists of the maps cP, $c^{-1}P$, and the SDR's) induces heqs

$$F^pX_{\geq k} \sim F^p(\Omega Q_{\geq k}).$$

Conclusion $\mathcal{X}_{\geq k} \sim \Theta(\Omega Q_{\geq k})$ clear

Now trace map

$$\begin{tikzcd}
\Omega Q_{\geq k} \arrow[r] \arrow[d, dash, "\sim"'] & J^k_\# \otimes \Omega B \arrow[d, dash, "\sim"] \\
X_{\geq k} \arrow[r] & J^k_\# \otimes X(RB)
\end{tikzcd}$$

$\Uparrow$

defined via the basic ident.

Now without going into the details of $\Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$ it is not possible to see it commutes with d, b, although basically this is clear. In

(0) Now where are we?

All I have talked about is defining $F^pX_{\geq k}$ to corresp to $F^p(\Omega Q_{\geq k})$ under the basic ident. $X(BQ) = \Omega Q$.

Point: $F^p\Omega Q_{\geq k}$ closed under $d, b, \dots$ so $F^pX_{\geq k}$ is subcomplex of X, and basic heq restricts to a heq $F^pX_{\geq k} \sim F^p(\Omega Q_{\geq k})$, $\forall p, k$.

Similarly I define X-trace map via basic ident.

$$\begin{tikzcd}
F^pX_{\geq k} \arrow[r, "def"] \arrow[d, equal] & J^k_\# \otimes F^p_{IB} \arrow[d, equal] \\
F^p\Omega Q_{\geq k} \arrow[r, "\Uparrow \ \Omega\text{-trace map}"'] & J^k_\# \otimes F^p\Omega B
\end{tikzcd}$$

Then follows that

$$\begin{tikzcd}
F^pX_{\geq k} \arrow[r] \arrow[d, dash, "\sim"'] & J^k_\# \otimes F^p_{IB} \arrow[d, dash, "\sim"] \\
F^p\Omega Q_{\geq k} \arrow[r] & J^k_\# \otimes F^p\Omega B
\end{tikzcd}$$

Commutes.

All this works I am sure, but I would like things to work out better.

So the idea is to first establish

$$\Omega_T(Q^t) \xrightarrow{\sim} \Omega Q^t \subset T' \otimes \Omega Q = \Omega_{T'}(T' \otimes Q)$$

$$F^p(\Omega_T(Q^t)) \longrightarrow \bigoplus t^k F^p(\Omega Q_{\geq k})$$

(P) So how does this work? ~~I think I would like to~~ The point is to use the ~~[illegible]~~ basic identification ~~$F^p\Omega_T(Q^t)\otimes_T$~~ in relative form:

unnec.

$$\begin{tikzcd}
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r, phantom, "="] \arrow[d] & F^p(\Omega_T(Q^t)\otimes_T) \arrow[d] \\
F^p_{I_{T'}(T'\otimes Q)} X_{T'}(R_{T'}(T'\otimes Q)) \arrow[r, phantom, "="] \arrow[d, phantom, "\parallel"] & F^p(\Omega_{T'}(T'\otimes Q)\otimes_{T'}) \arrow[d, phantom, "\parallel"] \\
F^p_{T'\otimes IQ} X_{T'}(T'\otimes RQ) \arrow[d, phantom, "\parallel"] & F^p(T'\otimes \Omega Q) \arrow[d, phantom, "\parallel"] \\
T'\otimes F^p_{IQ} X(RQ) & T'\otimes F^p\Omega Q
\end{tikzcd}$$

you conclude

canonical ident.

$$\begin{tikzcd}
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r, phantom, "\boxed{=}"] \arrow[d, "\cong"'] & F^p\Omega_T(Q^t) \arrow[d, "\boxed{\cong}"] \\
(F^pX)^t \arrow[r, phantom, "\boxed{=}"] & F^p(\Omega Q^t)
\end{tikzcd}$$

due to lemma

definition of $F^pX_{\geq k}$

So maybe try listing assertions and the order they occur.

You have $Q^t \subset T'\otimes Q$

Tsubaly.

(Q)

$$\begin{tikzcd}
\Omega_T(Q^t) \arrow[r] & \Omega_{T'}(T'\otimes Q)
\end{tikzcd}$$

DG hom

univ.

$$\begin{tikzcd}
\Omega_T(Q^t)\otimes_T \arrow[r] \arrow[d, phantom, "\parallel"] & \Omega_{T'}(T'\otimes Q)\otimes_{T'} \arrow[d, phantom, "\parallel"] \\
X_T(R_T(Q^t)) \arrow[r] & X_{T'}(R_{T'}(T'\otimes Q))
\end{tikzcd}$$

map comp with d, b, K, etc.

$$\begin{tikzcd}
X(R_T(Q^t)) \arrow[r] \arrow[d, phantom, "\parallel"] & T'\otimes X(RQ) \arrow[d, phantom, "\parallel"] \\
\Omega_T(Q^t) \arrow[r] & T'\otimes \Omega Q
\end{tikzcd}$$

bottom arrow injective with image ΩQ^t

Thus if define $X_{\geq k}$ to corresp to $\Omega Q_{\geq k}$

Then we have

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[d, phantom, "\parallel"] & X(RQ)^t \arrow[d, phantom, "\parallel"] \\
\Omega_T(Q^t) \arrow[r, "\sim"] & \Omega Q^t
\end{tikzcd}$$

Sim. have

$$\begin{tikzcd}
F^p_{I_T}(Q^t) \arrow[r, hook] \arrow[d, phantom, "\parallel"] & T'\otimes F^p_{IQ} \arrow[d, phantom, "\parallel"] \\
F^p\Omega_T(Q^t) \arrow[r, hook] & T'\otimes F^p\Omega Q
\end{tikzcd}$$

Image of bottom is $\oplus\, t^k F^p(\Omega Q_{\geq k}) = F^p(\Omega Q^t)$

So if $F^p X_{\geq k} \longleftrightarrow F^p(\Omega Q_{\geq k})$ by def then

conclude

$$\begin{tikzcd}
F^p \cancel{I_T(Q^t)} X_T\cdots \arrow[r, "\sim"] \arrow[d, phantom, "\parallel"] & (F^p X)^t \arrow[d, phantom, "\parallel"] \\
F^p\,\Omega_T(Q^t) \arrow[r, "\sim"] & F^p(\Omega Q^t)
\end{tikzcd}$$

(R) Summarize:

What is your goal?

definition of the bifiltration $F^pX_{\geq k}$ of $X(RQ)$ with the required properties.

~~Two~~ There are two approaches.

1) Take the basic identification $X(RQ) = \Omega Q$, define $F^pX_{\geq k}$ to corresp to $F^p(\Omega Q_{\geq k})$. Since $F^p(\Omega Q_{\geq k})$ closed under d, b, K conclude that $F^pX_{\geq k}$ closed under ∂ and that basic heq $X \sim \Omega$ induces heq $F^pX_{\geq k} \sim F^p(\Omega Q_{\geq k})$. At this point you know that $\mathcal{X}_{\geq k} \sim \Theta(\Omega Q_{\geq k})$

2) Form $Q^t \subset T' \otimes Q$ use relative theory

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & T' \otimes X(RQ) \arrow[d, equal] \\
\Omega_T(Q^t) \arrow[r] & T' \otimes \Omega Q
\end{tikzcd}$$

Lemma shows bottom arrow injective.

identify image bottom arrow ~~with~~ ΩQ^t.

Define $X_{\geq k}$ so that the image of top arrow is X^t. Clearly $X_{\geq k}$ subcomplex of $X(RQ)$, and $X_{\geq k} = \Omega Q_{\geq k}$ under the basic identification. More generally have $\forall p$

$$\begin{tikzcd}
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & T' \otimes F^p_{IQ} X(RQ) \arrow[d, equal] \\
F^p \Omega_T(Q^t) \arrow[r] & T' \otimes F^p \Omega Q
\end{tikzcd}$$

(5) Image bottom arrow is $F^p(\Omega Q^t) = \bigoplus t^k F^p(\Omega Q_{\geq k})$. ~~Define Image~~ Define $F^p X_{\geq k}$ so that image of top arrow is $\bigoplus t^k F^p X_{\geq k} = (F^p X)^t$. Then $F^p X_{\geq k} = F^p(\Omega Q_{\geq k})$. under basic defn. & we have established

$$\begin{tikzcd}
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[d, equal] & (F^p X)^t \arrow[d, equal] \\
F^p \Omega_T(Q^t) \arrow[r, "\sim"] & F^p(\Omega Q^t)
\end{tikzcd}$$

Somehow say that horizontal arrows are canonical isos. compatible with all structure, vertical arrows are like a coordinate system.

Now try to do the trace map.

filt. alg hom
$$Q \longrightarrow L \otimes B$$
$$Q_{\geq k} \longrightarrow J^k \otimes B$$
assemble into $Q^t \longrightarrow L^t \otimes B$.

$$\begin{tikzcd}
~~get~~\; X(RQ)^t \arrow[r, phantom, "="] \arrow[d, equal] & X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{L^t}(R_{L^t}(L^t \otimes B)) \arrow[r, phantom, "="] \arrow[d, equal] & L^t_\natural \otimes X(RB) \arrow[d, equal] \\
\Omega Q^t \arrow[r, phantom, "="] & \Omega_T(Q^t) \arrow[r] & \Omega_{L^t}(L^t \otimes B) \arrow[r, phantom, "="] & L^t_\natural \otimes \Omega B
\end{tikzcd}$$

bottom arrow assembles the Ω trace maps
$$\Omega Q_{\geq k} \longrightarrow J^k_\# \otimes \Omega B$$
top arrow is the X version
$$X(RQ)_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$
More generally have

(I) $\forall p$

$$\begin{tikzcd}
(F^pX)^t \arrow[r] \arrow[d, equal] & L^t_{\flat} \otimes F^p_{IB} X(RB) \arrow[d, equal] \\
F^p(\Omega Q^t) \arrow[r] & L^t_{\flat} \otimes F^p \Omega Q
\end{tikzcd}$$

i.e.

$$\begin{tikzcd}
X(RQ)_{\geq k} \arrow[r] & J^k_{\#} \otimes X(RB) \\
F^pX_{\geq k} \arrow[u, phantom, "\cup"] \arrow[r] & J^k_{\#} \otimes F^p_{IB} \arrow[u, phantom, "\cup"]
\end{tikzcd} \qquad \forall p.$$

I don't yet have a minimal set of assertions. Alternatively instead of the $\begin{matrix} X \\ \cup \\ \Omega \end{matrix}$ diagram I could put

$$\begin{tikzcd}
X(RQ)^t \arrow[r, phantom, "="] & X_T(R_T(Q^t)) \arrow[r] & X_{T'}(T'\otimes RB) \arrow[r] & T'\otimes X(RB) \\
(F^pX)^t \arrow[u, phantom, "\cup"] \arrow[r, phantom, "="] & F^p_{I_T(Q^t)} \arrow[u, phantom, "\cup"] \arrow[r] & F^p_{T'\otimes IB} \arrow[u, phantom, "\cup"] \arrow[r] & T'\otimes F^p_{IB} \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

this might not be so good because it is hidden within

$$X_S(R_S(S\otimes B)) = S_{\flat} \otimes X(RB)$$

$$F^p_{I_SB} X_S(R_S(S\otimes B)) = F^p_{S\otimes IB}(X_S(S\otimes RB))$$

$$= S_{\flat} \otimes F^p_{IB} X(RB).$$

So far 1815 it seems that we understand the filtration part of the program. Next stage will be D game.

(u) 8/28 - 0611

Let's go over the first part of X version of Nistor construction. Aim to define bifilt $F^pX_{\geq k}$ of $X = X(RQ)$ with certain properties.

One property is that the basic heq $X(RQ) \sim \Omega Q$ restricts to a heq $F^pX_{\geq k} \sim F^p(\Omega Q_{\geq k})$ $\forall p, k$.

Another property is that the filtered alg homom. $Q \longrightarrow L \otimes B$, $Q_{\geq k} \longrightarrow J^k \otimes B$ induces trace maps $F^pX_{\geq k} \longrightarrow J^k_{\#} \otimes F^p_{IB}$ as k, p vary and ~~compatible~~ commuting compatible with the basic heqs $F^pX_{\geq k} \sim F^p(\Omega Q_{\geq k})$, $F^p_{IB} \sim F^p \Omega B$

To do this ~~consider~~ define $F^pX_{\geq k}$ to corresp. to $F^p(\Omega_{\geq k})$ under $\underbrace{\text{the basic identification } X(RQ) = \Omega Q}$. As $F^p(\Omega_{\geq k})$ closed under d, b, κ, etc. explicit formulas show $F^pX_{\geq k}$ subcomplex of $X(RQ)$ and that the basic heq restricts. Also this should work for the trace map.

Other approach $T' = \mathbb{C}[t, t^{-1}]$, $T = \mathbb{C}[t^{-1}] \subset T'$

graded T-subalg $Q^t = \bigoplus t^k Q_{\geq k} \subset T' \otimes Q$.

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T' \otimes Q) = T' \otimes \Omega Q$$

Lemma injective

image is $\Omega Q^t = \bigoplus t^k \Omega Q_{\geq k}$

has

given elt $t^{k_i}x_i \in Q^t$, $x_i \in Q_{\geq k_i}$ one has in $T' \otimes \Omega Q$

$$t^{k_0}x_0 \, d(t^{k_1}x_1) \cdots d(t^{k_n}x_n) = t^{k_0 + \cdots + k_n} x_0 \, dx_1 \cdots dx_n$$

Concludes canon. isom $\boxed{\Omega_T(Q^t) \xrightarrow{\sim} \Omega Q^t}$.

Also $\boxed{F^p\Omega_T(Q^t) \xrightarrow{\sim} F^p(\Omega Q^t) = \bigoplus t^k F^p(\Omega Q_{\geq k}).}$

Next

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{T'}(R_{T'}(T' \otimes Q)) \arrow[r, phantom, "="] \arrow[d, equal] & T' \otimes X(RQ) \arrow[d, equal] \\
\Omega_T(Q^t) \arrow[r] & \Omega_{T'}(T' \otimes Q) \arrow[r, phantom, "="] & T' \otimes \Omega Q
\end{tikzcd}$$

bottom arrow inj with image ΩQ^t

top arrow - inj with image $X(RQ)^t = \bigoplus t^k X(RQ)_{\geq k}$

(V)

canon. isom $\boxed{X_T(R_T(Q^t)) \xrightarrow{\sim} X(RQ)^t}$

coinciding with ( ) under basic ident.

similarly $\boxed{F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \xrightarrow{\sim} (F^pX)^t}$

Ultimately I would like to write 4 diagrams

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r, "\sim"] & X(RQ)^t \arrow[r, phantom, "\subset"] & T'\otimes X(RQ) \\
F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[u, phantom, "\cup"] \arrow[r, "\sim"] & (F^pX)^t \arrow[u, phantom, "\cup"] \arrow[r, phantom, "\subset"] & T'\otimes F^p_{IQ} X(RQ) \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

$$\begin{tikzcd}
\Omega_T(Q^t) \arrow[r, "\sim"] & \Omega Q^t \arrow[r, phantom, "\subset"] & T'\otimes \Omega Q \\
F^p\Omega_T(Q^t) \arrow[u, phantom, "\cup"] \arrow[r, "\sim"] & F^p(\Omega Q^t) \arrow[u, phantom, "\cup"] \arrow[r, phantom, "\subset"] & T'\otimes F^p\Omega Q \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

$$\begin{tikzcd}
X(RQ)^t \arrow[r, phantom, "\simeq"] & X_T(R_T(Q^t)) \arrow[r] & X_{L^t}(R_{L^t}(L^t\otimes B)) \arrow[r, phantom, "="] & L^t_\flat \otimes X(RB) \\
(F^pX)^t \arrow[u, phantom, "\cup"] \arrow[r, phantom, "\simeq"] & F^p_{I_T(Q^t)} \arrow[u, phantom, "\cup"] \arrow[r] & F^p_{I_{L^t}(L^t\otimes B)} \arrow[u, phantom, "\cup"] \arrow[r, phantom, "="] & L^t_\flat \otimes F^p_{IB} \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

$$\begin{tikzcd}
\Omega Q^t \arrow[r, phantom, "\simeq"] & \Omega_T(Q^t) \arrow[r] & \Omega_{L^t}(L^t\otimes B)\otimes_{L^t} \arrow[r, phantom, "="] & L^t_\flat \otimes \Omega B \\
F^p(\Omega Q^t) \arrow[r, phantom, "\simeq"] & F^p\Omega_T(Q^t) \arrow[u, phantom, "\cup"] \arrow[r] & F^p\Omega_{L^t}(L^t\otimes B)\otimes_{L^t} \arrow[u, phantom, "\cup"] \arrow[r, phantom, "="] & L^t_\flat \otimes F^p\Omega B \arrow[u, phantom, "\cup"]
\end{tikzcd}$$

state they agree wrt the basic identification and are homotopy equivalent via the basic homotopy equivalence

(W) 1428 ~~Main points are~~

~~$Q^t \subset T' \otimes Q$~~

~~induces~~

~~$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$~~

~~$F^p_{I_T}(Q^t) \longrightarrow F^p_{I_{T'}}(T' \otimes Q) = T' \otimes F^p_{IQ}$~~

~~injective arrows. get $X_T(R_T(Q^t)) \xrightarrow{\sim} X(RQ)^t$~~

~~$F^p_{I_T}(Q^t) \xrightarrow{\sim} (F^pX)^t$~~

Forget this and consider the D side. OKAY.

1435. Apply ~~formula~~ result

$$X_T(R_T(Q^t)) \xrightarrow{\sim} X(RQ)^t \subset T' \otimes X(RQ)$$

~~[illegible] Q~~

grading ~~[illegible]~~ $Q = \bigoplus_n Q_n$ induces a grading on RQ as alg. and on $X(RQ)$ as supercomplex

grading on Q induces gradings on RQ, $X(RQ)$ comp. w. structure, because RQ depends only on the underlying vector space + identity elt.

~~[illegible]~~

Define L_D on $X_T(R_T(Q^t))$ $X_{T'}(R_{T'}(T' \otimes Q))$

Maybe better to define the [illegible] grading 1454

You want to explain the grading on $X(RQ)$.

$Q \longrightarrow$

grading on V equivalent

forget this stuff and concentrate on the essentials. The aim is to show

(X) $$L_D - k : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k+1}$$

$$\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}$$

$$h_D : F^p X_{\geq k} \longrightarrow F^{p-2} X_{\geq k}$$

Idea of proof is to define L_D, γ, h_D on $X_T(R_T(Q^t))$ consistent with their defintion on $X(RQ)$ and the arrow

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$$

How do I propose to proceed 1508

~~Take~~ use the grading to define

$$\begin{tikzcd} Q \arrow[r] \arrow[d, phantom, "D"] & T' \otimes Q \arrow[d, phantom, "t\partial_t"] \\ RQ \arrow[r] & R_{T'}(T' \otimes Q) = T' \otimes RQ \end{tikzcd}$$

lin resp 1

Again I am getting involved in the background.

In simple terms you extend D on Q to a derivation D on RQ and consider L_D on $X(RQ)$. canonical ϕ yields h_D.

This probably works in the relative context. Define D on Q^t [illegible] to be rest. of $1 \otimes D$ on $T' \otimes Q$. Why defined?

$$T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

$$t^a \otimes x \longmapsto t^a t^D x$$

$$Q^t = \bigoplus t^k Q_{\geq k}$$

Continue. Yes.

So it certainly works for [illegible]

(y) [illegible]

To define L_D, γ, h_D on $X(RQ)$

To define these on $X(RQ)^t$ and then prove that

$$L_D - t\partial_t : (F^pX)^t \longrightarrow t^{-1}(F^{p-2}X)^t$$

$$\gamma - [illegible]\gamma^t : (F^pX)^t \longrightarrow t^{-1}(F^pX)^t$$

$$h_D \quad : (F^pX)^t \longrightarrow (F^{p-2}X)^t$$

Idea of the proof is to use the canonical isom.

$$\begin{tikzcd} X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[d, phantom, "\cup" description] & X(RQ)^t \arrow[d, phantom, "\cup" description] \\ F^p_{I_T(Q^t)} X_T(R_T(Q^t)) \arrow[r, "\sim"] & (F^pX)^t \end{tikzcd}$$

Then argue that h_D is $h^\phi(1, D)$ for

$$\phi : R_T(Q^t) \longrightarrow \Omega^2_T(R_T(Q^t))$$

$$D : R_T(Q^t) \longrightarrow R_T(Q^t) \text{ derivation}$$

$\frac{1}{2}$ hr before dinner [illegible] 1750

Final stuff about D. Organize what to say.

Recall things worked out

$$\begin{tikzcd} \mathbb{C} \arrow[r, phantom, "\subset"] \arrow[d, phantom, "\cap" description] & S \arrow[d, phantom, "\cap" description] \\ T \arrow[r, phantom, "\subset"] & L^t \end{tikzcd} \qquad \begin{tikzcd} Q \arrow[r, "\xi"] \arrow[d, "t^D"'] & S \otimes B \arrow[d, phantom, "\cap" description] \\ Q^t \arrow[r] & L^t \otimes B \end{tikzcd}$$

$$\begin{tikzcd} X(RQ) \arrow[r, "\alpha V_*"] \arrow[d, "t^{L_D}"'] & S_q \otimes X(RB) \arrow[d, phantom, "\cap" description] \\ X(RQ)^t \arrow[r] & L^t_q \otimes X(RB) \end{tikzcd}$$

shows that the trace map is the T-module extension of $[illegible] : X(RQ) \longrightarrow S_q \otimes X(RB) \subset L^t_q \otimes X(RB)$

(2) So what did I use in the course of this argument. Used

$$X(RQ)^t = X_T(R_T(Q^t))$$

Also I used

$$\begin{array}{ccc} T\otimes X(RQ) & \xrightarrow{\sim} & X(RQ)^t \\ 1\otimes \xi & \longmapsto & t^{L_D}\xi \end{array}$$

SCRATCH.

$$\begin{tikzcd} Q \arrow[r, hook] \arrow[rr, bend left, "t^D"] & T\otimes Q \arrow[r, "\sim"] & Q^t \arrow[r, phantom, "\subset"] & T'\otimes Q \\ D & 1\otimes D & & 1\otimes D \\ D & t\partial_t\otimes 1 + 1\otimes D & & t\partial_t\otimes 1 \\ 0 & -t\partial_t\otimes 1 & & 1\otimes D - t\partial_t\otimes 1 \end{tikzcd}$$

$$\begin{tikzcd} X(RQ) \arrow[r, hook] \arrow[rrr, bend left, "t^{L_D}"] & T\otimes X(RQ) \arrow[r, "\sim"] & X(RQ)^t \arrow[r, phantom, "\subset"] & T'\otimes X(RQ) \\ L_D & 1\otimes L_D & & 1\otimes L_D \\ L_D & t\partial_t\otimes 1 + 1\otimes L_D & & t\partial_t\otimes 1 \\ 0 & -t\partial_t\otimes 1 & & 1\otimes L_D - t\partial_t\otimes 1 \end{tikzcd}$$

identifications confusing things.

$$\begin{tikzcd} X(RQ) \arrow[r] \arrow[d, equal] & X_T(R_T(T\otimes Q)) \arrow[r, "\sim"] \arrow[d, equal] & X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{T'}(R_{T'}(T'\otimes Q)) \arrow[d, equal] \\ X(RQ) \arrow[r, phantom, "\subset"] & T\otimes X(RQ) \arrow[r, "\sim"] & X(RQ)^t \arrow[r, phantom, "\subset"] & T'\otimes X(RQ) \\ L_D & 1\otimes L_D & & 1\otimes L_D \\ L_D & t\partial_t\otimes 1 + 1\otimes L_D & & t\partial_t\otimes 1 \end{tikzcd}$$

(a) 8/29

simple points

$$Q^t = \oplus t^k Q_{\geq k} = \bigoplus_{n\geq k} t^k Q_n \subset T'\otimes Q$$

$$\left| \begin{aligned} D &= 1\otimes D \quad \text{on} \quad T'\otimes Q \\ t\partial_t &= t\partial_t \otimes 1 \end{aligned}\right.$$

$$\begin{aligned} D &= n \quad \text{on} \quad t^k Q_n \\ t\partial_t &= k \quad \text{on} \quad t^k Q_n \end{aligned} \qquad \therefore \quad Q^t \text{ closed under } D,\ t\partial_t$$

$$D - t\partial_t = n - k \quad \text{on} \quad t^k Q_n$$

$$\therefore \quad D - t\partial_t : Q^t \longrightarrow \bigoplus_{n>k} t^k Q_n = t^{-1} Q^t$$

$$\gamma - \gamma^t = (-1)^D - (-1)^{t\partial_t} = (-1)^n - (-1)^k$$

$$\gamma - \gamma^t : Q^t \longrightarrow t^{-1} Q^t.$$

Next to understand how D defined on $R_T(Q^t)$ and $X_T(R_T(Q^t))$.
You would like to say this is obvious because $D : Q^t \longrightarrow Q^t$ is T-linear resp 1, thus extends to a derivation of $R_T(Q^t)$.

I think there should be a simple method.

8/29 - 1525 - 1529

The problem is how to introduce D on really the consequences of the grading.
Start with $Q = \oplus Q_n$
From this I want to obtain D on RQ h_D on $X(RQ)$ and h_D also

(b) 1534 The idea is that D results from the grading although in principle ~~anything~~ any linear map $D: Q \to Q$ respecting the filtration should ~~give~~ extend to a Derivation on RQ.

I have this:

$$X_T(R_T(Q^t)) \xrightarrow{\sim} X(RQ)^t \subset T' \otimes X(RQ)$$

Let's carefully proceed.

Start again. I have Q, RQ, X(RQ) objects, and $Q = \oplus Q_n$ grading.

There are various claims.

X(RQ) inherits a grading $(X(RQ)_n)$ consistent with all structure

in particular RQ inherits a grading

[redacted] (RQ_n) compat with alg st. + ρ

get D on RQ derivation comp. with ρ,

D on Q. L_D grading of on X(RQ).

get $h_D \ni L_D = [\partial, h_D]$, $[L_D, h_D] = 0$

[redacted] also grading + filt. (inherited from $Q_{\geq k} = \oplus_{n \geq k} Q_n$) are compatible.

objects Q, RQ, X(RQ), [redacted]

Q_n, RQ_n, $X(RQ)_n$,

D on Q, D on RQ, [redacted]

L_D, h_D on X(RQ).

∃ lots of relations and I want to find the best way to summarize them.

(c) Ultimately I need to have L_D, h_D defined on $X_T(R_T(Q^t))$ agreeing with the way they are defined on $X(RQ)$.

The idea is this:

The grading on Q ~~equivalent to a lifting~~ gives

$$Q \longrightarrow T' \otimes Q$$

(lin resp 1
section of $t \mapsto 1$
specialization
image graded
(closed under $t\partial_t$

induces

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

(alg homom.
section of $t \mapsto 1$ specialization
im alg closed under $t\partial_t$.

induces $X(RQ) \longrightarrow X_{T'}(R_{T'}(T' \otimes RQ)) = T' \otimes X(RQ)$

compatible w. ∂
section of spec.

These seem to handle grading.

Next we've already done something with the filtration. Defined $Q^t \subset T' \otimes Q$ and proved

$$X_T(R_T(Q^t)) \xrightarrow{\sim} X(RQ)^t \subset T' \otimes X(RQ).$$

Now we know that things

Wait defined $Q^t \subset T' \otimes Q$

this induces $X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$

proved this is injective and checked that our defn of $(X(RQ)_{\geq k})$ agrees with $X(RQ)^t$ = image.

But now we have the grading on Q

This gives $Q \longrightarrow T' \otimes Q$, $RQ \longrightarrow T' \otimes RQ$

(d) grading on Q gives

$$Q \longrightarrow T' \otimes Q$$

(lin. resp 1, section of δ, image closed under $t\partial_t$

$$RQ \longrightarrow T' \otimes RQ$$

(alg hom. section of δ, image closed under $t\partial_t$

$$X(RQ) \longrightarrow T' \otimes X(RQ)$$

Next condition $Q_{\geq k} = \bigoplus_{n \geq k} Q_n$

$\Longrightarrow$ $T \otimes Q \xrightarrow{\sim} Q^t$ extn of $t^D : Q \to Q^t$

$\Longrightarrow$ $T \otimes RQ = R_T(T \otimes Q) \xrightarrow{\sim} R_T(Q^t) \xrightarrow{\sim} RQ^t$

$\Longrightarrow$ $T \otimes X(RQ) = X_T(R_T(T \otimes Q)) \xrightarrow{\sim} X_T(R_T(Q^t)) \xrightarrow{\sim} X(RQ)^t$

~~amount to [illegible]~~

yield. $RQ_{\geq k} = \bigoplus_{n \geq k} RQ_n$ and simil. for $X(RQ)_{\geq k}$.

Furthermore consider

$$T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

$$t^{-i} x \longmapsto t^{-i} t^D x = t^{-i+|x|} x$$

commutes with D

$$\begin{tikzcd}
T \otimes Q \arrow[r] & T' \otimes Q & \\
t^{-i} x \arrow[r, mapsto] \arrow[d, mapsto, "D"] & t^{-i} t^D x \arrow[rd, mapsto, "D"] & \\
t^{-i} D x \arrow[r, mapsto] & t^{-i} t^D D x = & D(t^{-i} t^D x)
\end{tikzcd}$$

$$\begin{tikzcd}
t^{-i} x \arrow[r, mapsto] \arrow[d, mapsto, "t\partial_t + D"] & t^{-i} t^D x \arrow[r, mapsto, "t\partial_t"] & (-i+D) t^{-i} t^D x \\
(-i+D) t^{-i} x \arrow[rru, mapsto] & &
\end{tikzcd}$$

(e) $T \otimes Q \hookrightarrow T' \otimes Q$

$t^{-i} \otimes x \longmapsto t^{-i} t^{D} x = t^{-i+|x|} x$

$D \longleftrightarrow D$

$t\partial_t + D \longleftrightarrow t\partial_t$

$-t\partial_t \longleftrightarrow D - t\partial_t$

what am I after w. all this calc?
The point is that because $T \otimes RQ = R_T(Q^t)$
the canonical ϕ needn't be mentioned.

What I want is the relative version
of $h(F^p) \subset (F^{p-2})$

$$F^p_{I_T(Q^t)} \subset X_T(R_T(Q^t))$$

What to say? You have D_A derivation on $R_T(Q^t)$
T linear, you have $\phi : R_T(Q^t)$

Repeat the ideas

8/30 - 0645 Anyway let's continue

$X_T(R_T(Q^t))$ spanned by elements

- $\rho x_1 \cdots \rho x_m$ $\hbar(\rho x_1 \cdots \rho x_m \, d(\rho x_{m+1}))$

where the $x_i \in Q^t$ and ρ is T-linear : $Q^t \to R_T(Q^t)$

$\rho : Q^t \longrightarrow R_T(Q^t) = RQ^t$

We know that $D - t\partial_t : Q^t \longrightarrow t^{-1}Q^t$
same true for $D - t\partial_t$ on RQ^t since

$$(D-t\partial_t)(\rho x_1 \cdots \rho x_m) = \sum \cdots \underbrace{(D-t\partial_t)\rho x_i}_{= \rho((D-t\partial_t)x_i)} \cdots$$

(f) Also $I_T(Q^t) = IQ^t$

$$(D - t\partial_t)\left((IQ^t)^n\right) \subset t^{-1}(IQ^t)^{n-1}$$

$$\begin{aligned}(D - t\partial_t)\,(IQ^t)^{n+1} &\subset \sum_0^n (I^tQ)^i\,(D - t\partial_t)(IQ^t)\,(I^tQ)^{n-i}\\ &\subset \sum_0^n (I^tQ)^i\, t^{-1}RQ^t\,(I^tQ)^{n-1}\\ &\subset t^{-1}\,(I^tQ)^{n-1}.\end{aligned}$$

Better if $y_i \in IQ^t$ then

$$(D - t\partial_t)(y_1 \cdots y_m) = \sum_1^m y_1 \cdot y_{i-1}\,\underbrace{(D - t\partial_t)(y_i)}_{\in t^{-1}RQ^t} \cdots$$

$$\in$$

$\Delta = \frac{1}{t}(D - t\partial_t)$ OKAY for RQ^t

$$(L_D - t\partial_t)\,\natural(y_1 \cdots y_n\, dy_{n+1})$$

$$(\gamma - \gamma^t)(y_1 \cdots y_n)$$

$$= \sum \gamma_{y_1} \cdots \gamma_{y_{i-1}}(\gamma - \gamma^t)(y_i)\,\gamma^t_{y_{i+1}} \cdots \gamma^t_{y_n}$$

The goal is to find assertions.
& sketch the proofs
Start with
Start again. I am in the grading situation

$$Q \longrightarrow T' \otimes Q$$

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

$$X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$

(g) Alternative $t^{D}Q \longrightarrow Q^t$

$T \otimes Q \xrightarrow{\sim} Q^t$

$$T\otimes RQ = R_T(T\otimes Q) \xrightarrow{\sim} R_T(Q^t) = RQ^t$$

$$T\otimes X(RQ) = X_T(T\otimes RQ) \xrightarrow{\sim} X_T(R_T(Q^t)) = X(RQ)^t$$

D on Q^t, RQ^t, $(L_D$ on $X(RQ)^t)$

corresp to $1\otimes D$ $(1\otimes L_D$

ought to get conclusions straight first.

on $X(RQ)^t$ we have L_D, γ, h_D

related by $\gamma = (-1)^{L_D}$, $L_D = [\partial, h_D]$, $[L_D, h_D] = 0$.

All conclusions at the end refer to $X(RQ)$

$$\begin{tikzcd} Q \arrow[r, phantom, "\subset"] \arrow[rrr, bend left, "t^D"] & T\otimes Q \arrow[r, "\sim"] & Q^t \arrow[r, phantom, "\subset"] & T'\otimes Q \end{tikzcd}$$

gives rise to

$$\begin{tikzcd} X(RQ) \arrow[r] \arrow[d, equal] & X_T(R_T(T\otimes Q)) \arrow[r, "\sim"] \arrow[d, equal] & X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal] & X_{T'}(R_{T'}(T'\otimes Q)) \arrow[d, equal] \\ X(RQ) \arrow[r, phantom, "\subset"] \arrow[rrr, bend right, "t^{L_D}"'] & T\otimes X(RQ) \arrow[r, "\sim"] & X(RQ)^t \arrow[r, phantom, "\subset"] & T'\otimes X(RQ) \end{tikzcd}$$

L_D	$1\otimes L_D$	$1\otimes L_D$
L_D	$t\partial_t\otimes 1 + 1\otimes L_D$	$t\partial_t\otimes 1$
0	$-t\partial_t\otimes 1$	$1\otimes L_D - t\partial_t\otimes 1$

(h) So what next?
somewhere have to ~~say~~ argue that

$$h^{\bullet}_D : F^p_{I_T(Q^t)} \longrightarrow F^{p-2}_{I_T(Q^t)}$$

relative version of

$$h^\phi(u,\dot u) : F^p_I X(R) \longrightarrow F^{p-2}_J X(S)$$

provided $u(I) \subset J$.

where $u = id : R_T(Q^t) = (RQ)^t$ ⟲
$\dot u = D :$ ________ ⟲

situation: $u = id$ on R^t | T linear
$\dot u = D$ on R^t
$\phi : R^t \longrightarrow \Omega^2_T(R^t)$ canonical
ideal $I^{\bullet}_T(Q^t) = IQ^t$
kernel of $R^t \longrightarrow Q^t$

---

seems we have another proof that $[L_D\, h_D] = 0$, namely

$$\begin{tikzcd}
X(RQ) \arrow[r] \arrow[d, "h_D"] & T \otimes X(RQ) \arrow[r, "\sim"] \arrow[d, "1\otimes h_D"] & X_T(R_T Q^t)) \arrow[r, hook] \arrow[d, "h_D"] & T' \otimes X(RQ) \arrow[d, "1\otimes h_D"] \\
X(RQ) \arrow[r] & T \otimes X(RQ) \arrow[r, "\sim"] & X_T(R_T(Q^t)) \arrow[r, hook] & T' \otimes X(RQ)
\end{tikzcd}$$

more precisely you define h_D on $X_T(R_T(Q^t))$ as ~~$1\otimes h_D$ on $T\otimes$~~ T linear extn of h_D on $X(RQ)$. but you need to know this h_D on $X_T(R_T(Q^t))$ agrees with $1\otimes h_D$ on $T'\otimes X(RQ)$. Thus need

$$\begin{tikzcd}
X(RQ) \arrow[r, "t^{L_D}"] \arrow[d, "h_D"] & T' \otimes X(RQ) \arrow[d, "1\otimes h_D"] \\
X(RQ) \arrow[r, "t^{L_D}"] & {}
\end{tikzcd}$$

(i) $\natural(x\,dy)$ $x, y \in R^t$

$L_D - t\partial_t$

$$L_D \natural(x\,dy) = \natural(Dx\,dy + x\,d(Dy))$$

Same true for $t\partial_t$ because it is associated to the grading on

A derivation of A preserving the subalg S should act in $\Omega_S A$ and $X_S(A)$ compatible with structure.

1543 I have two hours to function.
I want to find the essential points.

First V v.s. then a grading on V is equivalent to a ~~section~~ map

$$V \longrightarrow T' \otimes V$$

~~of the specializat~~

section of specialization $t \mapsto 1$ map
image closed under $t\partial_t$

a filtration on V equiv. to a graded T-submodule V^t of $T' \otimes V$.

$$V_{\geq k} = \bigoplus_{n \geq k} V_n \Longleftrightarrow \mathrm{Im}(T \otimes V \longrightarrow T' \otimes V) = V^t$$

In this case we have

$$\begin{tikzcd} V \arrow[r, phantom, "\subset"] \arrow[rrrr, bend right, "tD"'] & T \otimes V \arrow[r, "\sim"] & V^t \arrow[r, phantom, "\subset"] & & T' \otimes V \end{tikzcd}$$

~~Other equivalent forms.~~

(j) Grading on $Q \ni 1 \in Q_0$.

$$* \quad Q \longrightarrow T' \otimes Q$$

linear map resp 1
section of $t \mapsto 1$
specialization
closed under $t\partial_t \otimes 1$

this induces

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

homom. of alg.
section of ev_1
im closed under $t\partial_t \otimes 1$.

get D ~~on~~ $RQ \longleftrightarrow t\partial_t \otimes 1$ on $T' \otimes RQ$.

get also

$$X(RQ) \longrightarrow X_{T'}(R_{T'}(T' \otimes Q)) = T' \otimes X(RQ)$$

comp. with ∂ + superstruc.
section of ev_1
im closed under $t\partial_t \otimes 1$.

can identify grading op with $L_D = L(1, D)$.

Next factor ~~[illegible]~~

$$\begin{tikzcd} Q \arrow[r, phantom, "\subset"] \arrow[rr, bend left, "t^D"] & T \otimes Q \arrow[r, "\sim"] & Q^t \arrow[r, phantom, "\subset"] & T' \otimes Q \end{tikzcd}$$

$$\begin{tikzcd} RQ \arrow[r] \arrow[d, equal] & R_T(T \otimes Q) \arrow[r, "\sim"] \arrow[d, equal] & R_T(Q^t) \arrow[r] & R_{T'}(T' \otimes Q) \arrow[d, equal] \\ RQ \arrow[r, hook] & T \otimes RQ \arrow[r, "\sim"] & RQ^t \arrow[r, phantom, "\subset"] & T' \otimes RQ \end{tikzcd}$$

Conclude $R_T(Q^t) = RQ^t$ for filt of RQ assoc. to grading.

similarly

$$\begin{tikzcd} X(RQ) \arrow[r] \arrow[d, equal] & X_T(R_T(T \otimes Q)) \arrow[r, "\sim"] & X_T(R_T(Q^t)) \arrow[r] & X_{T'}(R_{T'}(T' \otimes Q)) \arrow[d, equal] \\ X(RQ) \arrow[r, phantom, "\subset"] & T \otimes X(RQ) \arrow[u, "\wr"] \arrow[r] & X(RQ)^t \arrow[r, phantom, "\subset"] & T' \otimes X(RQ) \end{tikzcd}$$

(k) Conclude $X_T(R_T(Q^t)) \simeq X(RQ)^t$ for filtration on $X(RQ)$ assoc. to grading.

Next need $h_D = h^\phi(1, D)$

First define canonical $\phi : RQ \longrightarrow \Omega^2_\phi(RQ)$.

Relative $\phi : R_T(Q^t) \longrightarrow \Omega^2_T(R_T(Q^t))$

Everything mentioned so far is gen. nonsense

Try something elementary.

You have $Q = \oplus Q_n$ D on Q

construct D on RQ $\ni$ | derivation $D\rho = \rho D$

get grading on RQ. Then you take $L_D = L(1, D)$ on $X(RQ)$ and get grading.

Next take canon. ϕ + get $h_D = h^\phi(1, D)$

Check that $L_D = [\partial, h_D]$ $[L_D, h_D] = 0$.

All this is clear. Now you have defined everything. $Q_n, RQ_n, X(RQ)_n, D, L_D, h_D, Q_{\geq n}$, etc.

So far no alg. structure on Q. ~~We have~~ We can reformulate by introducing t.

~~How~~.

Go over this again: Take D on Q, $D1 = 0$ so can 'extend' D to a deriv. on RQ. Then $\operatorname{Ker}(D - n) = RQ_n$ is grading for RQ and $\operatorname{Ker}(L_D - n) = X(RQ)_n$ ——— $X(RQ)$.

Now can define $Q_{\geq k}$, $RQ_{\geq k}$, $X(RQ)_{\geq k}$. structure compatible with grading + filtration.

(l) Is T' [redacted] used to express this?

objects: Q, RQ, $X(RQ)$, ρ

Q_n, RQ_n, $X(RQ)_n$

$Q_{\geq k}$, $RQ_{\geq k}$, $X(RQ)_{\geq k}$

D on Q, D on RQ, L_D, ϕ, h_D

$$t^D : Q \longrightarrow T'\otimes Q \qquad Q^t \subset T'\otimes Q$$

$$t^D : RQ \longrightarrow T'\otimes RQ \qquad RQ^t \subset T'\otimes RQ$$

$$t^{L_D} : X(RQ) \longrightarrow T'\otimes X(RQ) \qquad X(RQ)^t \subset T'\otimes X(RQ)$$

more objects T, T', $R_T(Q^t)$, $X_T(R_T(Q^t))$.

So lots more objects are introduced and you get many relations

But still you have to boil it all down to a minimum amount.

Basic objects are Q, RQ, $X(RQ)$, D on $Q + RQ$, L_D, h_D

Everything else is secondary. My purpose is to do something about the filtration. The grading is only part of the story.

What do I need?

I want ~~$R_T(Q^t) \cong RQ^t$~~ to know

$Q^t \subset T'\otimes Q$ induces

~~$R_T Q^t \xrightarrow{\sim} RQ^t \subset T'\otimes RQ$~~

$$\begin{array}{ccccc} X_T(R_T Q^t) & \xrightarrow{\sim} & X(RQ)^t & \subset & T'\otimes X(RQ) \\ \cup & & \cup & & \cup \\ F^p_{I_T(Q^t)} & \xrightarrow{\sim} & (F^p X)^t & \subset & T'\otimes F^p_{IQ} \end{array}$$

(m) What else do I need?

I want to prove something about L_D, γ, h_D on $X(RQ)$ with resp. to $F^p X_{\geq k}$.

Critical points. The idea is to take $X(RQ)^t$ together with $(F^p X)^t$ and to interpret these as $X_T(R^t)$ together with $F^p_{I^t} X_T(R^t)$. Then we must take L_D, γ, h_D on $X(RQ)$, get $1 \otimes L_D, 1 \otimes \gamma, 1 \otimes h_D$ on $T' \otimes X(RQ)$, then show these restrict to, or come from a L_D, γ, h_D in the relative context.

The point maybe is ~~that~~ this

$$T \otimes Q \xrightarrow{\sim} Q^t \subset T' \otimes Q$$

$$R_T(T \otimes Q) \xrightarrow{\sim} R_T(Q^t) \longrightarrow R_{T'}(T' \otimes R)$$

Claim that canonical ϕ's exist

8/31 - 0626

So I need to check $\exists$ canonical ϕ

$$\phi : R_S A \longrightarrow \Omega^2_S(R_S A)$$

S bimod. map

$$\rho : A \longrightarrow R_S A$$

universal S-bimodule map to an S-alg respecting identities

$$\left.\begin{aligned} \rho(sa) &= s\rho a \\ \rho(as) &= \rho a s \end{aligned}\right) \quad \rho(s) = s$$

$$\omega(a_1, a_2) = \rho(a_1 a_2) - \rho a_1 \rho a_2$$

$$A/S \underset{S}{\otimes} A/S \longrightarrow R$$

S bimodule map.

(n) So suppose we have S-alg

$$\begin{tikzcd} A \arrow[d, "\rho"] & R \arrow[d] \\ R_S A \arrow[r] & R/I \end{tikzcd}$$

We need to lift

$$\begin{tikzcd} R_S(R_S A)/I_S(R_S A)^2 \arrow[d] \\ R_S A \end{tikzcd}$$

$R_S A = \Omega_S^{ev} A$ under $\circ$

$\hat{\rho} : A \longrightarrow R_S A$ S-bimodule map $1 \mapsto 1$.

S Claim: Given R an S-algebra

$\rho : A \longrightarrow R$ S-bimod resp 1.

A R Claim $\exists ! \; R_S A \xrightarrow{\rho *} R$ homom.

$\ni \rho * \hat{\rho} = \rho$

Proof:

$$\begin{tikzcd} RA \arrow[r, "\rho *"] \arrow[d] & R \\ R_S A \arrow[ur, dotted, "\exists"'] & \end{tikzcd}$$

$$\rho * (a_0 da_1 \cdots da_{2n}) = \underbrace{\rho a_0 \, \omega(a_1, a_2) \cdots \omega(a_{2n-1}, a_{2n})}$$

defined on $A \otimes_S \underbrace{(A/S) \otimes_S \cdots \otimes_S (A/S)}_{2n}$

$$\begin{tikzcd} & R_S A \oplus \Omega_S^2(R_S A) \arrow[d] \\ R_S A \arrow[ur] \arrow[r, equal] & R_S A \end{tikzcd}$$

(0) Something curious. Suppose B is an S-algebra, consider $B \oplus \Omega^2 B$ with Fedosov product. Then how do you get a quotient which is an S-algebra. Thus you want to introduce various relations. $S \subset B$ to be a subalgebra in $B \oplus \Omega^2 B$ $\iff$ $ds_1 ds_2 = 0$. You also want $B \subset B \oplus \Omega^2 B$ to be an S bimodule map. $ds\, db = db\, ds = 0$.

$\cancel{d(b_1 s) b_2}$

$\cancel{d(b_1 s) db_2}$

$$\cancel{b_1 ds\, db_2} - d(b_1 s) db_2 + db_1 d(s b_2) - \cancel{db_1 ds\, b_2} = 0$$

cocycle condition

$$s\, db_1 db_2 - d(s b_1) db_2 + \cancel{ds\, d(b_1 b_2)} - \cancel{ds\, db_1 b_2} = 0$$

OKAY this checks - ~~in other words~~

Conclude therefore that there ~~also~~ is a canonical ϕ in the case of $R_S A$ in general.

$$\begin{tikzcd}
 & R_S A \oplus \Omega^2_S (R_S A) \arrow[d] & & B \oplus \Omega^2_S B \arrow[d] \\
\cancel{R_S} A \arrow[r, "S\text{-bim}"] & R_S A \arrow[u, bend left, "S \text{ bimodule map}"] & & B \arrow[u, bend left, "S\text{-bim map}"]
\end{tikzcd}$$

(p) Outline again

X-version of Nistor construction

$X = X(RQ) \qquad F^pX = F^p_{IQ}$

~~first step~~ main steps

1. define bifiltration $F^pX_{\geq k}$ of X.

basic property: the basic heq $X \sim \Omega Q$ restricts to give heqs $F^pX_{\geq k} \sim F^p(\Omega Q_{\geq k})$

2. define L_D, γ, h_D on X and prove

$$L_D - k : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k+1}$$
$$\gamma - (-1)^k : F^pX_{\geq k} \longrightarrow F^pX_{\geq k+1}$$
$$h_D : F^pX_{\geq k} \longrightarrow F^{p-2}X_{\geq k}$$

3. filtered alg. hom.
$$Q \longrightarrow L \otimes B$$
$$Q_{\geq k} \longrightarrow J^k \otimes B$$

yields trace maps
$$l_k : X_{\geq k} \longrightarrow J^k_\# \otimes X(RB)$$

$\ni \quad l_k : F^pX_{\geq k} \longrightarrow J^k_\# \otimes F^p_{IB}$

is heq via the basic heq to the trace map
$$F^p(\Omega Q_{\geq k}) \longrightarrow J^k_\# \otimes F^p(\Omega B)$$

(g) Can you check $\gamma - (-1)^k$ maps

$F^p \Omega Q_{\geq k} \longrightarrow F^p \Omega Q_{\geq k+1}$

We know that $\gamma - (-1)^k$ maps $\Omega Q_{\geq k} \to \Omega Q_{\geq k+1}$

Check this.

$\Omega^n Q_{\geq k}$ spanned by $x_0 \, dx_1 \dots dx_n$ where $\sum \mathrm{ord}(x_i) \geq k$

modulo $\Omega^n Q_{\geq k+1}$ can assume ~~#~~ x_i homog and $\sum |x_i| = k$. Then $\gamma x_i = (-1)^{|x_i|} x_i$

and $\gamma(x_0 \, dx_1 \cdot dx_n) = (-1)^k x_0 \, dx_1 \dots dx_n$

Thus $\gamma - (-1)^k$ maps $F^p \Omega Q \cap \Omega Q_{\geq k}$ to $F^p \Omega Q \cap \Omega Q_{\geq k+1}$. This handles ~~the~~ everything in $F^p \Omega Q_{\geq k}$ except the ~~$(b \Omega^{p+1} Q)_{\geq k}$~~

$b(\Omega^{p+1} Q_{\geq k})$ part.

and the same argument applies, choosing the elements before applying b.

~~Next ingredient~~

At some point must sort out what to say about T, T' theory.

minimum to say

(n) discuss proofs.

$R^t = R_T(Q^t) = RQ^t$ $\qquad$ $I^t = I_T(Q^t) = IQ^t$

Then $X_T(R^t) = X(RQ)^t$

$F^p_{I^t} X_T(R^t) = (F^pX)^t$

D on R^t, $\overset{\text{canon}}{\phi}$ on R^t

confused about identifications + maps etc.

concentrate on minimum needed for

h_D on X^t to carry $F^pX_{\geq k}$ to $F^{p-2}X_{\geq k}$

~~to derive this~~ follows from

h_D on X^t carries $(F^pX)^t$ to $(F^{p-2}X)^t$

to derive from

h_D on $X_T(R_T(Q^t))$ carries $F^p_{I_T(Q^t)}$ to $F^{p-2}_{I_T(Q^t)}$

---

points $\left(\begin{array}{l} D \text{ on } R_T(Q^t) \quad T \text{ linear} \\ \text{canon } \phi \text{ for } — . \end{array}\right.$

give h_D on $X_T(R_T(Q^t))$

relative version of $h(F^p_I) \subset F^{p-2}_I$

say h_D carries $F^p_{I_T(Q^t)}$ into $F^{p-2}_{I_T(Q^t)}$.

—

Consider map

$T, Q^t \quad \subset \quad T', T' \otimes Q$

$R_T(Q^t) \quad \subset \quad T' \otimes RQ$

$D \quad 1 \otimes D$

$\phi \quad 1 \otimes \phi$

$h_D \quad 1 \otimes h_D$

$F^p_{I_T(Q^t)}$

(5) Again

$$\left(\begin{array}{lll} D & \text{on} & R_T(Q^t) \qquad T \text{ linear} \\ \text{canon} & \phi \text{ for} & R_T(Q^t) \end{array}\right.$$

yield h_D on $X_T(R_T(Q^t))$

relative version of $h(F_I^p) \subset F_I^{p-2}$

yields $h_D : F^p_{I_T(Q^t)} \longrightarrow F^{p-2}_{I_T(Q^t)}$

—

Consider $T, Q^t \subset T', T' \otimes Q$

$$\begin{array}{lllll}
R_T(Q^t) & \xrightarrow{\sim} & RQ^t & \subset & T' \otimes RQ \\
D & & & & 1 \otimes D \\
\phi & & & & 1 \otimes \phi \\
X_T(Q^t) & \xrightarrow{\sim} & X(RQ)^t & \subset & T' \otimes X(RQ) \\
L_D & & & & 1 \otimes L_D \\
h_D & & & & 1 \otimes h_D \\
F^p_{I_T(Q^t)} & \xrightarrow{\sim} & (F^pX)^t & \subset & T' \otimes F^p_{IQ}
\end{array}$$

Conclude that ~~$1 \otimes h_D$ [illegible]~~

$1 \otimes h_D$ on $T' \otimes X(RQ)$

~~sends~~ carries $(F^pX)^t$ into $(F^{p-2}X)^t$.

I propose to identify $R^t = R_T(Q^t)$ with $(RQ)^t$ and $I^t = I_T(Q^t)$ with $(IQ)^t$ and

$X_T(R^t)$ with $X(RQ)^t$

$F^p_{I^t}$ with $(F^pX)^t$

(t) table

$\mathbb{C}, Q$		T, Q^t		$T', T'\otimes Q$
$T\otimes Q$	$\xrightarrow{\sim}$	Q^t	$\subset$	$T'\otimes Q$
$T\otimes RQ$	$\xrightarrow{\sim}$	$R_T(Q^t) = R^t$	$\subset$	$T'\otimes RQ$
		$I_T(Q^t) = I^t$	$\subset$	$T'\otimes IQ$
$1\otimes D$		D		$1\otimes D$
$1\otimes \phi$		ϕ		$1\otimes \phi$
$t\partial_t\otimes 1 + 1\otimes D$		$t\partial_t$		$t\partial_t\otimes 1$
$T\otimes X(RQ)$	$\simeq$	$X_T(R^t)$	$\subset$	$T'\otimes X(RQ)$
$1\otimes L_D$		L_D		$1\otimes L_D$
$1\otimes h_D$		h_D		$1\otimes h_D$
$t\partial_t\otimes 1 + 1\otimes L_D$		$t\partial_t$		$t\partial_t\otimes 1$
		$F^p_{I^t} = (F^pX)^t$	$\subset$	$T'\otimes F^p_{IQ}$

Notation : $R^t = R_T(Q^t)$ $I^t = I_T(Q^t)$

better first establish canonical isomorphism

$$R_T(Q^t) \xrightarrow{\sim} (RQ)^t = \oplus\, t^k RQ_{\geq k}$$

$$I_T(Q^t) \xrightarrow{\sim} (IQ)^t = \oplus\, t^k IQ_{\geq k}$$

$$X_T(R_T(Q^t)) \simeq X(RQ)^t$$

$$F^p_{I_T(Q^t)} \xrightarrow{\sim} (F^pX)^t$$

then introduce notation R^t, I^t,

(u) bifiltration section.

recall $\overset{\text{basic ident.}}{\frown}$ $X(RQ) = \Omega Q$

$\exists \quad F^p_{IQ} = F^p \Omega Q$

and basic heq

define $\quad F^p \Omega Q_{\geq k} = F^p(\Omega Q_{\geq k})$ $\quad$ Hodge filt. of $\Omega Q_{\geq k}$

define $\quad F^p X_{\geq k} = F^p \Omega Q_{\geq k}$ $\quad$ wrt basic ident.

$F^p \Omega Q_{\geq k}$ stable under d, b, K etc. $\Longrightarrow$ $F^p X_{\geq k}$ subcx of $X(RQ)$.

$\Longrightarrow$ basic heq induces $F^p X_{\geq k} \sim F^p \Omega Q_{\geq k}$

$\mathcal{X}_{\geq k} \sim \theta(\Omega Q_{\geq k})$.

$$\gamma - (-1)^k : F^p \Omega Q_{\geq k} \longrightarrow F^p \Omega Q_{\geq k+1} \quad \forall p, k.$$

$\Longrightarrow$

$$\gamma - (-1)^k : F^p X_{\geq k} \longrightarrow F^p X_{\geq k+1}.$$

$\Longrightarrow$ $\gamma_- F^p \Omega_{\geq 2j} = \gamma_- F^p \Omega_{\geq 2j+1}$

$$\gamma_- F^p X_{\geq 2j} = \gamma_- F^p X_{\geq 2j+1}$$

$$\gamma_- \mathcal{X}_{\geq 2j} = \gamma_- \mathcal{X}_{\geq 2j+1}$$

So far only elementary considerations
T theory.

main points of T theory.

(v) find the main points of the T-theory. the reason to introduce this is to make the proofs. Present outline

define bifilt. $F^pX_{\geq k}$
state its properties:
- basic heq
- L_D, h_D behavior
- trace map

which are needed to construct the X version of Nistor bivariant Chern char.

$$Ch^{2m}(\theta,\theta') \in HC^{2m}(\mathcal{X}_A, J_{\#}^{2m+1} \otimes \mathcal{X}_B).$$

use of T theory to make the proofs conceptually clear.

T-theory Results.

First result is that when one defines $F^pX_{\geq k}$ to corres to via the basic identification, then the basic heq $X(RQ) \sim \Omega Q$ induces $F^pX_{\geq k} \sim$

proof via T-theory.

Form $Q^t = \bigoplus t^k Q_{\geq k} \subset T' \otimes Q$

subalg. Then

$$R_T(Q^t) \longrightarrow R_{T'}(T'\otimes Q) = T' \otimes RQ$$

$$X_T(R_T(Q^t)) \longrightarrow X_{T'}(R_{T'}(T'\otimes Q)) = T' \otimes X(RQ)$$

$$\Omega_T(Q^t) \longrightarrow \Omega_{T'}(T'\otimes Q) = T' \otimes \Omega Q$$

(w) Anyway what next 1428
assertions of T-theory.

First result: if we define $F^P X_{\geqslant k}$ to correspond to $F^P \Omega Q_{\geqslant k}$; then the basic heq $X(RQ) \sim \Omega Q$ induces $F^P X_{\geqslant k} \sim F^P \Omega Q_{\geqslant k}$.

How does the T-theory proof go?

Form subalgebra $Q^t \subset T' \otimes Q$
use basic identification for Q^t rel T, $T' \otimes Q$ rel T'
and naturality:

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r, hook] \arrow[d, equal, "\int(Q^t \text{ rel } T)"] & X_{T'}(R_{T'}(T' \otimes Q)) \arrow[d, equal, "\int(T' \otimes Q \text{ rel } T')"] \arrow[r, phantom, "="] & T' \otimes X(RQ) \arrow[d, equal, "1 \otimes \int(Q, Q)"] \\
\Omega_T(Q^t) \arrow[r, hook, "\text{lemma}"'] & \Omega_{T'}(T' \otimes Q) \arrow[r, phantom, "="] & T' \otimes \Omega Q
\end{tikzcd}$$

get

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r, "\sim"] \arrow[d, equal] & X^t \arrow[r, phantom, "\subset"] \arrow[d, equal] & T' \otimes X(RQ) \arrow[d, equal] \\
\Omega_T(Q^t) \arrow[r, "\sim"] & (\Omega Q)^t \arrow[r, phantom, "\subset"] & T' \otimes \Omega Q
\end{tikzcd}$$

$$\begin{tikzcd}
F^P_{I_T(Q^t)} \arrow[r, "\sim"] \arrow[d, equal] & (F^P X)^t \arrow[r, phantom, "\subset"] \arrow[d, equal] & T' \otimes F^P_{IQ} \arrow[d, "1 \otimes \int(Q, Q)"] \\
F^P \Omega_T(Q^t) \arrow[r] & (F^P \Omega)^t \arrow[r, phantom, "\subset"] & T' \otimes F^P \Omega Q
\end{tikzcd}$$

(x) result is

if $F^pX_{\geq k}$ is defined to corresp to $F^p\Omega Q_{\geq k}$ under basic ident $X(RQ) = \Omega Q$, then $F^pX_{\geq k}$ is a subcomplex of $X(RQ)$ and the basic heq $X(RQ) \sim \Omega Q$ induces $F^pX_{\geq k} \sim F^p\Omega_{\geq k}$

the proof is based on identifications

$$\begin{tikzcd}
F^p_{I^t} \arrow[r, phantom, "="] \arrow[d, equal] & (F^pX)^t \arrow[r, phantom, "\subset"] \arrow[d, equal] & T' \otimes F^p_{IQ} \arrow[r, phantom, "\subset"] \arrow[d, equal, "1 \otimes \mathcal{J}(Q)"] & T' \otimes X(RQ) \arrow[d, equal, "1 \otimes \mathcal{J}(Q)"] \\
F^p\Omega_T(Q^t) \arrow[r, phantom, "="] & (F^p\Omega Q)^t \arrow[r, phantom, "\subset"] & T' \otimes F^p\Omega Q \arrow[r, phantom, "\subset"] & T' \otimes \Omega Q
\end{tikzcd}$$

T' $\quad Q^t/T$, $T' \otimes Q/T'$

basic ident-

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, equal, "\mathcal{J}(Q^t/T)"] & X_{T'}(R_{T'}(T' \otimes Q)) \arrow[r, phantom, "="] \arrow[d, equal, "\mathcal{J}(T' \otimes Q/T')"] & T' \otimes X(RQ) \arrow[d, equal, "1 \otimes \mathcal{J}(Q)"] \\
\Omega_T(Q^t) \arrow[r] & \Omega_{T'}(T' \otimes Q) \arrow[r, phantom, "="] & T^* \otimes \Omega Q
\end{tikzcd}$$

ident Image of $\Omega_T(Q^t)$ is $(\Omega Q)^t$

Conclude $X_T(R_T(Q^t)) \longrightarrow X(RQ)^t \subset T' \otimes X(RQ)$.

Similarly

$$\begin{tikzcd}
F^p_{I_T(Q^t)} \arrow[r] \arrow[d, equal] & T' \otimes F^p_{IQ} \arrow[d, equal] \\
F^p\Omega_T(Q^t) \arrow[r] & T' \otimes F^p\Omega Q
\end{tikzcd}$$

(y) First have

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] & X_{T'}(R_{T'}(T'\otimes Q)) \arrow[r, phantom, "="] & T'\otimes X(RQ) \\
\cup & \cup & \cup \\
F^p_{I_T(Q^t)} \arrow[r] & F^p_{I_{T'}(T'\otimes Q)} \arrow[r, phantom, "="] & T'\otimes F^p_{IQ}
\end{tikzcd}$$

horizontal arrows induced by $Q^t \subset T'\otimes Q$

Similarly have

$$\begin{tikzcd}
\Omega_T(Q^t) \arrow[r] & \Omega_{T'}(T'\otimes Q) \arrow[r, phantom, "="] & T'\otimes \Omega Q \\
\cup & \cup & \cup \\
F^p\Omega_T(Q^t) \arrow[r] & F^p\Omega_{T'}(T'\otimes Q) \arrow[r, phantom, "="] & T'\otimes F^p\Omega Q
\end{tikzcd}$$

can you streamline this by first

$$R_T(Q^t) \longrightarrow R_{T'}(T'\otimes Q) = T'\otimes RQ$$

$$I_T(Q^t) \longrightarrow I_{T'}(T'\otimes Q) = T'\otimes IQ$$

$$\left(\begin{array}{lll}
X_T(R_T(Q^t)) \longrightarrow & X_{T'}(T'\otimes RQ) & = T'\otimes X(RQ) \\
F^p_{I_T(Q^t)} \longrightarrow & F^p_{T'\otimes IQ} & = T'\otimes F^p_{IQ}
\end{array}\right.$$

$$\left(\begin{array}{lll}
\Omega_T(Q^t) \longrightarrow & \Omega_{T'}(T'\otimes Q) & = T'\otimes \Omega Q \\
F^p\Omega_T(Q^t) \longrightarrow & F^p\Omega_{T'}(T'\otimes Q) & = T'\otimes F^p\Omega Q
\end{array}\right.$$

not clear.

So far I am discussing the idents. and I have gotten stuck a bit. Work backwards.

What we want is to conclude that

(2) $\quad X(RQ) \sim \Omega Q \quad$ induces

$$F^p X_{\geq k} \sim F^p \Omega Q_{\geq k} \qquad \forall p, k.$$

Thus we want comm. in

$$\begin{tikzcd}
(F^p X)^t \arrow[r, phantom, "\subset"] & T' \otimes X(RQ) \arrow[d, phantom, "\wr"] \\
F^p (\Omega Q)^t \arrow[r, phantom, "\subset"] & T' \otimes \Omega Q
\end{tikzcd}$$

The key is what?

~~$Q^t \subset T' \otimes Q$ induces~~

You have

$$X_T(R_T(Q^t)) \xrightarrow{\sim} X^t \subset T' \otimes X(RQ)$$

The key is that $Q^t \subset T' \otimes Q$ induces an identification

$$X_T(R_T(Q^t)) = X^t \subset T' \otimes X(RQ)$$

$$F^p_{I_T(Q^t)} = F^p X^t \subset T' \otimes F^p_{IQ}$$

$$\begin{tikzcd}
X_T(R_T(Q^t)) \arrow[r] \arrow[d, phantom, "\wr"] & T' \otimes X(RQ) \arrow[d, phantom, "\wr"] \\
\Omega_T(Q^t) \arrow[r] & T' \otimes \Omega Q
\end{tikzcd}$$

look at image of horizontal arrows get

~~[illegible]~~ $\quad X(RQ)^t \sim \Omega Q^t$

~~Next to~~

$$\begin{tikzcd}
F^p_{I_T(Q^t)} \arrow[r] \arrow[d, phantom, "\wr"] & T' \otimes X(RQ) \arrow[d, phantom, "\wr"] \\
F^p \Omega_T(Q^t) \arrow[r] & T' \otimes \Omega Q
\end{tikzcd}$$

images get $\quad F^p X^t \sim F^p \Omega Q^t$

(f) Also $I_T(Q^t) = IQ^t$

$$(D - t\partial_t)((IQ^t)^n) \subset t^{-1}(IQ^t)^{n-1}$$

$$(D - t\partial_t)(IQ^t)^{n+1} \subset \sum_0^n (I^tQ)^i (D - t\partial_t)(IQ^t)(I^tQ)^{n-i}$$

$$\subset \sum_0^n (I^tQ)^i t^{-1}RQ^t(I^tQ)^{n-1}$$

$$\subset t^{-1}(I^tQ)^{n-1}.$$

Better if $y_i \in IQ^t$, then

$$(D - t\partial_t)(y_1 \cdots y_n) = \sum_1^m y_1 \cdot y_{i-1} \underbrace{(D - t\partial_t)(y_i)}_{\in t^{-1}RQ^t} \cdots$$

$\in$

$\Delta = \frac{1}{t}(D - t\partial_t)$ OKAY for RQ^t

$$(L_D - t\partial_t)\natural(y_1 \cdots y_n\, dy_{n+1})$$

$$(\gamma - \gamma^t)(y_1 \cdots y_n)$$

$$= \sum \gamma_{y_1} \cdots \gamma_{y_{i-1}} (\gamma - \gamma^t)(y_i)\, \gamma^t_{y_{i+1}} \cdots \gamma^t_{y_n}$$

The goal is to find assertions.
& sketch the proofs
Start with
Start again. I am in the grading situation

$$Q \longrightarrow T' \otimes Q$$

$$RQ \longrightarrow R_{T'}(T' \otimes Q) = T' \otimes RQ$$

$$X(RQ) \longrightarrow X_{T'}(T' \otimes RQ) = T' \otimes X(RQ)$$