

Contents 1984

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- January 4,5: Super-groups and super-Lie algebras.
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- January 8: Odd K -elements.
- January 9: Claims for the form $\text{tr}_E(\epsilon_E e^{D^2+[D,L]+L^2})$ where L is invertible.

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- January 10: A new proof that the Chern character cohomology class is independent of the choice of connection.
- January 11: Checking formulas of January 2.
- January 13: Some Banach algebra formulas.
- January 15, 16, 17: Odd K - classes. Notes on Clifford modules and super-vector bundles.
- January 18: Notes on a wide ranging conversation with Connes.
- January 19: The Hodge $*$ operator and Clifford multiplication. Comments on the Novikov conjecture.
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- January 25: Comment on K -theory of Clifford algebra bundles.
- January 27: Discussion of local Index formula for a family of Dirac operators with coefficients in a vector bundle.
- January 28, 29: Bott's theorem on Chern numbers. Witten's cohomology identified with localized equivariant cohomology.
- January 30: Notes on a conversation with Atiyah on Frankel's observation about a circle action on a symplectic manifold. Kirwan's work. Donaldson's proof that stable bundles have unique YM connections. Novikov Conjecture. Conversation with J. Roe.
- January 31: Notes on a conversation with Scott Petrach on Duistermatt-Heckman type proof of the Index theorem. Atiyah's summary of a talk by Berry.
- February 1: Conversation with Connes. Cyclic theory for a ring with no unit.
- February 2: Account of conversation with Katz, Gabber and Deligne.
- February 3: Conversation with Connes on $H^*(\Gamma) \rightarrow H_\lambda^*(\mathbb{C}[\Gamma])$, wave packet transform and Novikov conjecture.
- February 4: Connes method for extending cocycles from A to $\text{End}_A(E)$ where E is a projective module over A .

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- February 5: Morita invariance of cyclic cohomology.
- February 6,7: Connes construction to kill curvature.
- February 8: Connes proof of Morita invariance.
- February 9: Connes approach by tensoring an elliptic operator with a vector bundle. Relating cyclic cocycles and heat kernels.

February 10: Characterizing Dirac operators.

February 11: Constructing a cyclic cocycle belonging to a Dirac operator.

February 12: Constructing cyclic cocycles representing the Chern character of a Dirac operator. K-theory of a super-algebra.

February 17: Kasparov K -theory of graded C_k -modules.

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February 18: Check the Index theorem (using a heat kernel method) over a torus for the Dirac operator, $-i\mathcal{D} = -i\gamma^\mu D_\mu + \epsilon L$ acting on $S \otimes E$ where L is a selfadjoint operator.

February 19: Formula for the wave packet formula in Euclidean space. Bott's idea related to his work on the Lefschetz fixed point theorem.

February 20: On a rigorous proof of the classical limit formula $\text{Tr}(e^{-\beta H}) \sim \int \left(\frac{dpdq}{2\pi\hbar}\right)^n e^{-\beta H}$ where $H = \frac{p^2}{2} + V(q)$ is quantized as the operator $-h^2\nabla^2 + V(x)$. On a proof of the Index theorem using the idea that the quantum partition function has a classical limit which is an integral over the cotangent bundle.

February 21: On Connes groupoid and algebra of operators depending on \hbar .

February 22: The Index theorem for the Dirac operator interpreted as a supersymmetry operator. Partition function, Clifford algebras and Weyl algebra.

February 24: Lecture on Clifford algebras. Connes algebra of operators. Comments of Atiyah on the Index theorem.

February 25: Fundamental class in K -theory.

February 26: Attempt to derive a proof for the Index theorem for the Dirac operator using an isometric embedding.

February 27: Thom isomorphism in K -theory for complex vector bundles.

February 28: Further discussion of the Index theorem for Dirac operators and a generalisation to an assertion about maps.

1984-5

February 29: Computation of the character of the Thom class i_1 in the K -theory of a vector bundles equipped with a metric, connection and Spin^c structure.

March 1: On the Berezin determinant.

March 3: Correspondances between approaches to the Index theorem in algebraic geometry and the Riemann-Roch theorem for complex manifolds.

March 4: Further discussion of the Index theorem for the Dirac operator.

March 5: Concept of a Dirac operator. Conversation with Graeme Segal.

March 6: Discussion of Riemannian geometry calculations connected with a family of Dirac operators.

March 7,8: Return to the calculation of $\text{ch}(i_1)$ where $i : M \rightarrow E$ is the zero section of a complex vector bundle.

March 8: On a formula for $\det w e^{-\bar{J}w^{-1}J}$.

March 9: Classical mechanics, quantum mechanics and classical limit formalism.

March 10: Review of the Thom class formula.

March 11: Signature operator and Gaussian-Thom form.

March 12: Discussion on the Dirac operator and $d + d^*$.

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March 13: Splitting the operator $d + d^*$ on a Kahler manifold. Notes for a lecture on the Chern character form $\text{tr}_s \left(e^{(D+L)^2} \right)$.

March 14: The Thom form on the tangent bundle.

March 15: Conversation with Atiyah on a local Index theorem for connections with non-zero torsion. Convolution algebra of differential forms.

March 16, 17: Discussion of the limiting heat kernel as an n -form on the tangent bundle.

March 20: Index theorem on a torus for the Dirac operator with coefficients in a vector bundle equipped with a connection.

March 21: Limiting heat kernels.

March 23: Review of recent work.

March 24: Classical limit for the heat kernel for the Dirac operator with values in a vector bundle with a connection.

March 25: Discussion of Friedan-Winney paper.

March 26: Fermion Lagrangian quantization.

March 27: When does a quadratic form $S(q, q')$ define a symplectic transformation $(q', p') \mapsto (q, p)$? Hyperbolic quadratic forms and orthogonal transformations.

1984-7

March 28: Path integral approach to the heat operator $e^{-\beta H}$ and the limiting heat operator.

March 29: The super-group $\mathbb{R}^{1,1}$, its convolution algebra, left and right infinitesimal translations: $\partial_\theta \mp \theta \partial_t$.

March 30: Critical points for the fermion Lagrangian.

March 31: Friedan-Winney action. Super-heat operator.

April 1: Further discussion of the limiting heat kernel.

April 2,3, 4: Motion in a uniform magnetic field.

April 6: Dirac operator, general metric, coframe w^m calculation.

1984-8

April 7: Review calculation of April 6. Getzler's filtration. Dirac operator for a constant magnetic field.

April 8: Notes on a conversation with Atiyah in Witten's office.

April 9: Problems with the idea of a classical limit for the super-heat kernel. Parallel transport using a path integral.

April 10: Graded subalgebras of $A \otimes k[h]$ where A is an algebra over k .

April 11: Further thoughts on the local Index theorem.

April 12: The Grassmannian graph construction.

April 15: An extension of the algebra of asymptotic differential algebras.

April 18: Process of going from the cyclic cohomology of $\Omega^0(M)$ to that of $\Omega^0(M, \text{End}(E))$ and restriction $\Omega^0(M) \hookrightarrow \Omega^0(M, \text{End}E)$ with applications to cyclic cohomology.

April 20: Marmonic oscillator heat kernel.

April 21: Implementation of the isomorphism taking the cyclic cohomology of $\Omega^0(M)$ to that of $\Omega^0(M, \text{End}E)$ by the cochain $\int dt \text{tr} \left(e^{D^2 + t[D, \theta] + (t^2 - t)\theta} \right)$.

April 22: Review of November and December's work on transgression. More discussion on a local Index theorem.

1984-9

April 23: Relationship between the Chern character associated with a super-connection and that coming from the Grassmannian graph construction.

April 29: Summary of recent ideas.

April 30: Constructing cocycles from odd derivations and related universal algebra.

May 2: Twisted polynomial rings and a new approach to the heat kernel of the harmonic oscillator.

May 3: Existence of the heat operator e^{tD^2} in the Weyl algebra of operators of the form $\int d\sigma f(v)e^{Dv}$.

May 4,5: Deriving the trace of e^{tD^2} using the path integral. Derivation of the Schrödinger kernel of the harmonic oscillator using the explicit metaplectic formula.

May 6: Weights in a Von Neumann algebra, KMS condition.

May 7: Computing $\text{Tr} \left(e^{-H} e^{v \cdot D} \right)$ where $H = -D_\mu^2$ and $v \cdot D = v^\mu D_\mu$ using the KMS condition.

May 8: Calculations associated with showing the existence of the heat operator.

May 9: Fundamental solution of the heat operator.

May 10: Construction of the heat operator e^{-tH} where $H = p^2 + V(q)$.

May 11: Hadamard's method of constructing the heat kernel for the Laplacian of a curved manifold.

1984-10

May 12: Volterra's method for constructing e^{-tH} . Discussion of a method of proof of the existence of the heat operator for the harmonic oscillator by deformation of the classical case where $\hbar = 0$.

May 13: The heat operator for the covariant derivation Laplacian with coefficients in a vector bundle.

May 14, 15: Working with pseudo-differential operators in \mathbb{R}^n .

May 16: Review of past few day's work. Attempt to construct e^{-tH} as a kernel on the tangent groupoid of a manifold.

May 17, 18, 19: Attempt to put together a complete proof of the existence of the heat operator depending on Planck's constant, together with an asymptotic evaluation of the trace as $\hbar \rightarrow 0$.

May 20: Review parametrix method and Fredholm theory.

May 21: Review local Index theorem for families.

May 22: Proof of Index theorem for $M = \mathbb{R}^n/\Gamma$.

May 24: On Kasparov K -theory.

1984-11

May 25: Character form and examples coming from projections. Analogue for the plane of the Hilbert operator. Fourier transform of $\frac{1}{|z|^k|z|^{2s}}$.

May 26: Operators in Kasparov's K -theory. Cocycle computation for constant coefficient operators in \mathbb{R}^n .

May 28: Discussion of the general picture for families of Dirac operators.

May 29: Laplace transform of $\text{tr}_s \left(e^{u(L^2+dL)} \right)$ where L is a family of odd-degree skew adjoint operators endomorphisms of a super-vector space. Grassmannian graph construction.

May 30, 31: Grassmannian graph approach to the local Index theorem. Representation of the k th Chern character form as $\frac{1}{2} \text{tr}_s \left(\frac{1}{\lambda-L^2} dL \right)^{2k}$. Links between the heat kernel and resolvent approaches to the Chern character.

June 2: Transgression problem from the point of view of the Grassmannian graph construction.

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June 10: Transgressing the character form $\text{ch}_k = \frac{1}{2} \frac{1}{k!} \text{tr}_s \left(\frac{\epsilon}{\epsilon^2-L^2} dL \right)^{2k}$ to the group acting on the Grassmannian graph.

June 11: Abstract discussion of character forms.

June 24: Review of the relationship between character forms defined by the super-connection formalism and that by the Grassmannian graph method, i.e., the link between the heat operator approach and the parametrix approach.

June 25: Review of Witten's approach to the determinant line bundle over a space of connections.

June 26: Baum-Connes problem: to define a good notion of equivariant cohomology.

June 28: Review of transgression situation.

June 29: Transgression calculations. More discussion of the super-connection and the Grassmannian graph formalisms.

June 30: Understanding of the relationship between the super-connection and the Grassmannian graph formalisms. Discussion of the super-character: $\text{tr}_s e^{u(L+[D,L]+D^2)}$

July 1, 2: Work on an attempt at writing paper for Gelfand on the determinant line bundle.

July 7: Description of two interesting problems.

July 9: Link between Connes S -operator and Bott periodicity. Discussion of the bilogarithm and Bott periodicity. Local Index theorem for families - odd degree case.

July 10, 11: Work on local Index theorem for Dirac operators using super-connections. The η -invariant for the Dirac operator over a circle.

1984-13

July 12: Further discussion of the transgression process. Comment on multiplicative K -theory.

July 13: Constructing regulator maps $K_{2n-1}\mathbb{C} \rightarrow \mathbb{C}^\times$ by Karoubi's method using relative K -theory and Deligne cohomology. Discussion of parallel transport from the space of connections on the trivial bundle over S^1 to the unitary group.

July 14: Construction of left invariant character forms on the loop group.

July 15: Dirac operators and Weitzenböck formula using the principal frame bundle.

July 17, 18: Link between super-connections and the ζ -formula for ch_1 of the determinant line bundle. Odd version of Grassmannian graph.

July 17, 18: Transgression for the e_1 class. Bott periodicity from the differential point of view.

July 19: Continuous cohomology and periodicity. Dilogarithm.

July 20: Connes description of the multiplicative map on $K_1^{alg}(A)$ associated to a trace on A . Novikov conjecture and Mischenko's construction.

1984-14

July 21: On Bismut's proof of the Index theorem for the case when $M = \mathbb{R}^n/\Gamma$.

July 24: Ito's equation.

July 25: More on Bismut's paper and Ito's ideas. Review of what we know on the Index theorem and path integrals.

July 26, 27: Physicist's proof of the Index theorem using path integrals and fermion integrals. Comparison with Bismut's approach. Review of the super-group $R^{1,1}$ and Freidan-Windey. Bismut's factorization of parallel transport in $S \otimes E$.

July 28: Review construction of $\langle x_0 | e^{-hD^2} | x_0 \rangle$. Random walk on \mathbb{C}^* : $dz_t = z_t idw_t$, where w_t is Brownian motion.

July 29: Understanding the fermion integral: $\int \mathcal{D}\psi(t) e^{\int (\psi\dot{\psi} - \psi A\psi) dt}$. Idea's behind Bismut's paper. Possible approach to computing the heat kernel in a Weyl algebra.

July 30, 31: Fermion integrals and quantization. Some super-connection identities.

1984-15

August 1: Bismut's form on $\mathcal{L}M$, the loop space of M .

August 2: Interpretation of equivariant cohomology of ΩM and $\Omega BU(k)$. The $\Omega BU(1)$ case - Bismut's form is not an equivariant form but a Witten form.

August 3: Differential geometry of the normal bundle. Comment on Bismut's map

$H(\Omega^*(\Omega M)^{S^1}, d - wi_X) \rightarrow H_{DR}(M)$. Comments on 'The Index Theorem and Equivariant Cohomology of the Loop Group', by J-M Bismut.

August 4: Continuing with attempt at a local Index theorem. Review of Godbillon-Vey class and characteristic class for a codimension n foliation with trivial normal bundle.

August 5: Calculations for a 1-parameter family of connections.

August 6: $\partial_t \not{D}$ for the Dirac operator associated to a 1-parameter family of metrics.

August 7: Transverse connection on $T_{X/Y}$.

August 8: Curvature calculations.

1984-16

August 18: Berligne-Vergne proof of the Index theorem.

August 19: The Dirac operator on a Riemann surface.

August 20, 22: Bismut's Witten form on $\mathcal{L}M$.

August 23: Constructions using X/Y , and S^1 bundle over base Y .

August 24: Bismut's form is defined for an S^1 action. Analogies between cyclic theory and equivariant cohomology of the free loop space. Links between $\mathcal{L}M$ and $S^{-1}\text{Ext}_\Lambda(k^\natural, A^\natural)$.

August 29: Summary of recent ideas (generalize Bismut to Kac-Moody).

August 30: Gaussian processes.

August 31: Comment on constructing the heat operator.

September 1: Can the heat operator be constructed via $T\left(e^{\int dt\psi_t\mathcal{D}}\right) = e^{-t\mathcal{D}^2 + \theta D}$?

September 2: Applying Connes-Gezler theory.

September 5: The Clifford algebra as a deformation of the exterior algebra and an attempt at an analogous treatment of the Weyl algebra.

September 6,7: Twisted polynomial algebra. The convolution algebra belonging to the Weyl algebra and formula for e^{-tp^2} .

September 9: Review of standard tensor calculus formulas.

September 17: Singer's suggestion that $\text{tr} T\left(e^{\int(-\dot{x}^\mu A_\mu(x) + \frac{1}{2}\psi^\mu\psi^\nu F_{\mu\nu}(x))dt}\right)$ is the trace of super-parallel transport.

September 18: Formulas in Getzler's proof of the Index theorem.

1984-17

October 15, 16: Construction of the heat operator by constructing its kernel invariantly using the filtration idea.

October 18: Formal construction of e^{-tH} at $t = 0$ using the nilpotency of $t, (t^2 = 0)$. Relations between Berline-Vergne forms and Chern forms.

October 19, 20: Attempt to understand how to construct the Thom form for a complex vector bundle E using $\mathbb{P}(E) = S/S^1$ and $\xi^n + c_1(E) + \dots + c_n(E) = 0$. Review Gaussian-Thom form coming from the super-connection formalism.

October 21: Chern classes of representations.

October 22, 23: The formula $U = \det e^{-d\bar{z}\frac{1}{\bar{w}}dz} e^{-|z|^2}$ for the Thom form. Pfaffian and related formulas.

October 26: Constructing heat operators e^{-tH} where H is a Laplacean type operator. Review of Atiyah's lecture on mixed volumes and isoperimetric inequalities.

October 27, 28: Moment map calculations. Relations between convex bodies, toral varieties and the moment map as defined by Atiyah.

October 29: Strook's remarks on Stratonovich's versus Ito's stochastic differential equations.

October 31, November 2,3: Constructing $e^{t\Delta}$ where $\Delta = \rho^{ij}\partial_i\partial_j +$ lower order terms.

November 4: The algebra of smooth kernels in the tangent groupoid.

November 5: Equivalent descriptions of a singularity.

November 6,7: Thom form and Bott's residue calculations.

November 8: Feynman's formula: $\frac{1}{ab} = \int_0^1 \frac{dt}{((1-t)a+tb)^2}$.

November 9: Singular transgression form.

November 10: Proof that $c_1(\theta(1) \otimes \pi^*E) = 0$ in $H^*(PE)$ using differential forms.

November 11: Physicist's integration process for differential functions on the loop space \mathcal{LM} .

November 12: Constructing heat operators.

November 13: Reflection positivity.

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November 14: Review of geometric approach to the heat operator.

November 15: Homogeneous distribution on \mathbb{R}^n . Kernels on the tangent groupoid.

November 17: Gaussian measure. Current operators.

November 18: Return to construction of $e^{t\mathcal{D}+\theta D}$. Bismut's separation of parallel transport in $S \otimes E$ using Brownian motion.

November 19: Summary of links between the fermion integral, Bismut's use of Brownian motion in $\text{Lie}(\text{Spin}(n))$ and Vergne's Laplacian in the group direction.

November 20: Roe's thesis.

November 22: Comments on the map $\Omega(\mathcal{L}M)^{S^1} \rightarrow \Omega(M)$.

November 26: Notes on the basic 2-form on $\mathcal{L}M$ coming from the Riemannian structure on M .

November 27-30: Integrating differential functions on $\mathcal{L}M$.

December 1: Fermion integration formulas.

December 2: On the skew form $w(f, g) = \int_0^1 fg' dt$, a generalization and the fermion integral.

December 5,6: Weiner process on the line and related problems.

December 7: Notes on Bismut's construction. Bosonic and fermionic algebras.

December 8-11: Gaussian measure.

December 12: Equivariant cohomology $H_{S^1}(\mathcal{L}BU(1))$.

December 19-26: On $\Lambda(V)$ and why $\det(w)e^{-\bar{J}\frac{1}{w}J}$ is denominator free. Pfaffian algebras and details for a paper with Matthei.

1984-19

Lecture notes on infinite determinants.

1984-20

Notes on lectures by Connes and Hurder.