11/15/83

<u>Getzler</u>: new proof close to Kotake

harmonic oscillator $\quad a = \frac{1}{\sqrt{2}}\left(x - \frac{d}{dx}\right) \quad [a, a^*] = 1$

$$N = a^*a = \tfrac{1}{2}(-\partial_x^2 + x^2 - 1)$$

has eigenvalues $0, 1, \ldots$

$$\chi_0 = \pi^{-1/4} e^{-x^2/2} \qquad L^2(\mathbb{R})$$

$a^{*n}\chi_0$ are n particle state

coherent states $\quad \chi_\nu = \pi^{-1/4} e^{i\nu x - \frac{x^2}{2}}$

$e^{-tN}\chi_\nu \;\overset{?}{=}$ [redacted] need $e^{i\nu(a+a^*)/\sqrt{2}}\chi_0$

$$= e^{-\nu^2/4} e^{i\nu a^*} e^{i\nu a}\chi_0 \qquad \text{Campbell Hausdorff}$$

$$= e^{-\nu^2/4} e^{i\nu a^*}\chi_0$$

$$e^{-tN}\chi_\nu = e^{-\nu^2/4} e^{-tN} e^{i\nu a^*/\sqrt{2}}\chi_0$$

$$= e^{-\nu^2/4} e^{i e^{-t}\nu a^*/\sqrt{2}}\chi_0$$

$$\therefore \boxed{e^{-tN}\chi_\nu = \chi_{e^{-t}\nu}} \qquad e^{-\nu^2(1-e^{-2t})/4}$$

Put [redacted] $k_t(x,y) =$ [redacted] $\langle x | e^{-tN} | y \rangle$

$$\int k_t(x,y)\, e^{i\nu y - y^2/2}\, dy = e^{-\nu^2(1-e^{-2t})/4}\, e^{i\nu e^{-t}x - x^2/2}$$

Calculate + get $\quad k_t(x,y) = \pi^{-1/2}(1-e^{-2t})^{-1/2} e^{-\frac{1}{2\sinh t}[\cosh t\,(x^2+y^2) - 2xy]}$

Will be identifying $\Lambda(\mathbb{R}^n) \simeq \mathrm{Cliff}(\mathbb{R}^n)$ is an $O(n)$-equiv. isom.

$$e_i \lrcorner + e_i \wedge \equiv e_i \cdot \text{ on Cliff}$$

$$\not{D}^2 = \nabla^* \nabla + \frac{R}{4}$$

$$= -\tfrac{1}{2} g^{ij} \left[\left(\partial_i + \tfrac{1}{2} A_i\right)\left(\partial_j + \tfrac{1}{2} A_j\right) - \underset{\substack{\uparrow \\ \text{doesn't appear in the} \\ \text{principal bundle}}}{\Gamma_{ij}^k \left(\partial_k + \tfrac{1}{2} A_k\right)} + R/4 \right]$$

Here A_i is in Cliff.
$A_{iab}\, e^a \cdot e^b$.

Replace $\not{D}^2$ by $A_0 = -\frac{1}{2} \sum_i \left(\partial_i + \frac{1}{4} R_{ijab}\, x^j e^a \wedge e^b \wedge \right)^2$

Replace $\lim_{t \to 0} \mathrm{Tr}|_+ - \mathrm{Tr}_- \langle 0 | e^{-t \not{D}^2} | 0 \rangle$

$$\overset{\text{to be justified later}}{=} \lim_{t \to 0} \mathrm{Tr}\left(\varepsilon \langle 0 | e^{-t A_0} | 0 \rangle\right)$$

$$= \lim_{t \to 0} \mathrm{Tr}\left(\varepsilon\, (\pi t)^{-n/2} \det\left(\frac{tR/4}{\sinh tR/4}\right)^{1/2}\right)$$

projection into Λ^n.

$$= (-2\pi i)^{n/2} \det\left(\frac{R}{\sinh R}\right)^{1/2} \Big|_{\Lambda^n \mathbb{R}^n}$$

Next replace $N = a^* a$ by $H = a^* a + \frac{1}{2}$

$$K_t(x,y) = \langle x | e^{-\frac{t}{2}\left(-\frac{d^2}{dx^2} + x^2\right)} | y \rangle$$

$$= \pi^{-1/2} (\sinh t)^{-1/2}\, e^{-\frac{1}{2\sinh t}\left[\cosh t\,(x^2+y^2) - 2xy\right]}$$

Next multi-dim'l generalization: ω_{ij} anti-symm matrix

$$\langle x | e^{+\frac{t}{2}\underbrace{(\partial_i + \omega_{ij} x_j)^2}} | y \rangle$$

$$-\frac{t}{2}\left[-\partial_i^2 + (\omega^2)_{ij} x^i x^j\right] - t\,\underline{\omega_{ij} x^i \partial_j}$$

this commutes with the rest $-\partial_i^2 + (\omega^2)_{ij} x^i x^j$

$$= e^{-t\,\omega_{ij} x^i \partial_j}\, (\pi t)^{-n/2} \det\left(\frac{t\omega}{\sinh t\omega}\right)^{1/2}$$

$$\times\, e^{-\frac{1}{2t}\frac{t\omega}{\sinh t\omega}\left[\cosh t\omega\,(x^2+y^2) - 2xy\right]}$$

Take Dirac operator for a metric, look in normal coordinates,

g_{ij} metric on R^n with normal coords at 0

Γ_{ij}^k Levi-Civita connection

$A_{i\,b}^{\,a}$ is this connection on TR^n in the synchronous frame.

$$g_{ij} = \delta_{ij} + O(|x|^2)$$

$$A_i = \tfrac{1}{2} R_{ij} x^j + O(|x|^2)$$

$R_{ij\,b}^{\;\;a}$ = Riem. curv. of $A_{i\,b}^{\,a}$